Fused Silica Suspension Research at Caltech, Lately

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LIGO/Caltech

(lots of help from V. Sannibale, V. Mitrofanov, J. Weel)

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LIGO

Suspensions Apparatus

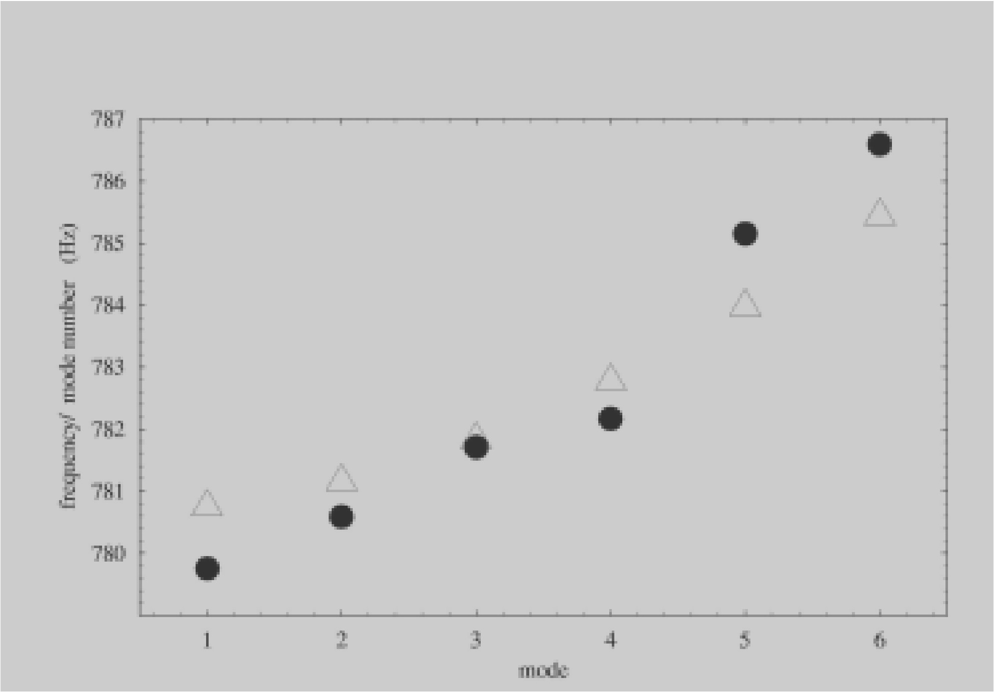
Automated fiber pulling lathe

Q-measurement rig

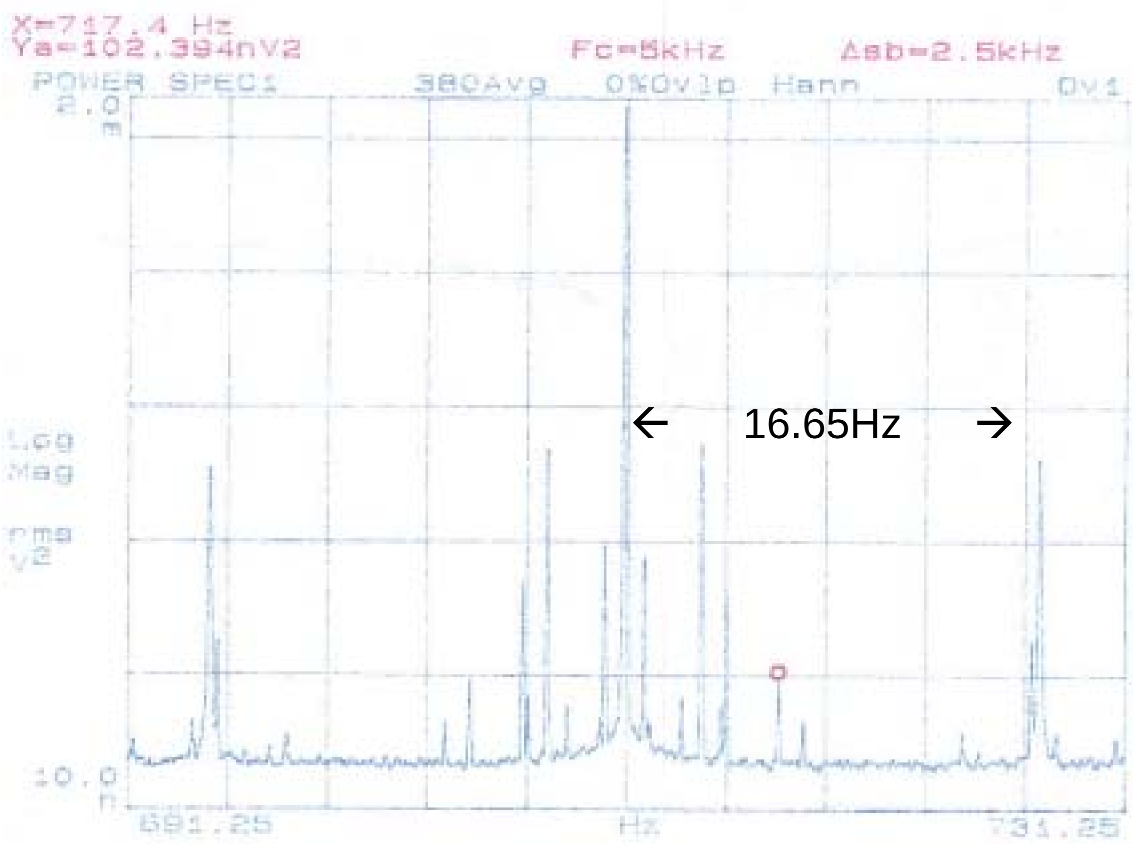
Mode frequencies enable precise determination of fiber radius

$$f_n = \frac{n}{2L}\sqrt{\frac{P}{\rho}}\left[1+\frac{2}{k_e L}+\left(4+\frac{(n\pi)^2}{2}\right)\frac{1}{(k_e L)^2}\right]$$

$$r_{fiber} = 157\mu\mathrm{m}$$



Whammy Sidebands

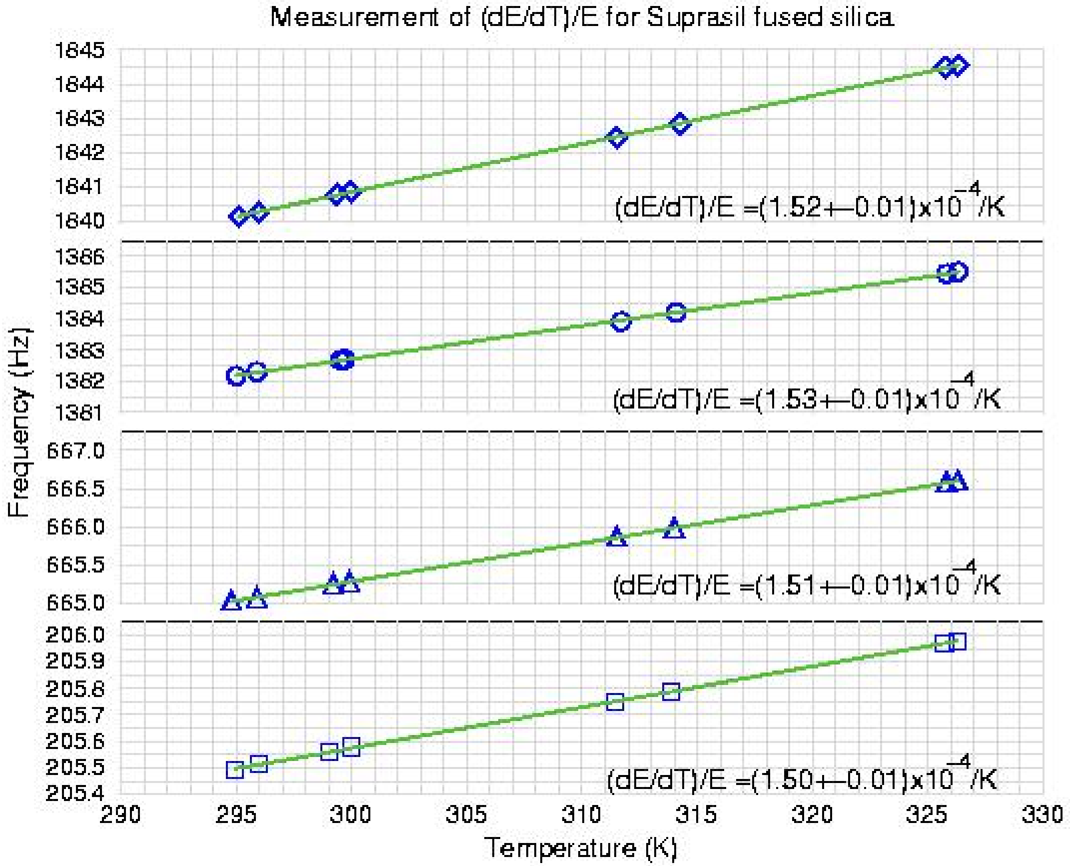


This allows measurement of Young's modulus (74.5GPa).

Temperature Shift of Unloaded Mode Frequencies Yields (dE/dT)/E

$$\frac{dE/dT}{E} = 2\frac{df/dT}{f}$$

$$= 1.52 \times 10^{-4} / \mathrm{K}$$



The Frequency Shift of the Violin Modes Yields the Dilution Factor

In general,

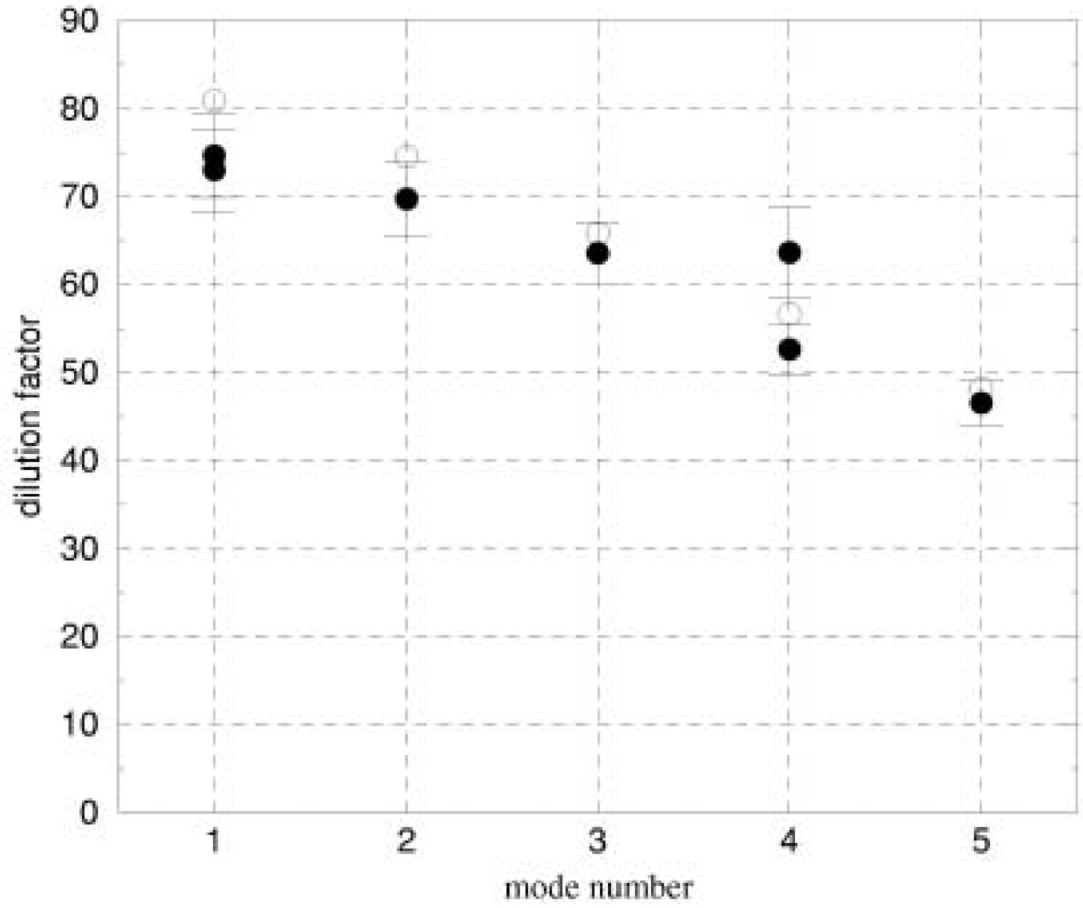
$$\frac{df_n / dT}{f_n} = -\frac{1}{2}(\alpha - u_0\beta) + \frac{1}{D_n}(\alpha + u_0\beta + \beta / 2)$$

For this suspension,

$$\frac{df_n / dT}{f_n} \approx \frac{\beta}{2D_n}$$

And the Agreement is Good

- ● data
- o theory

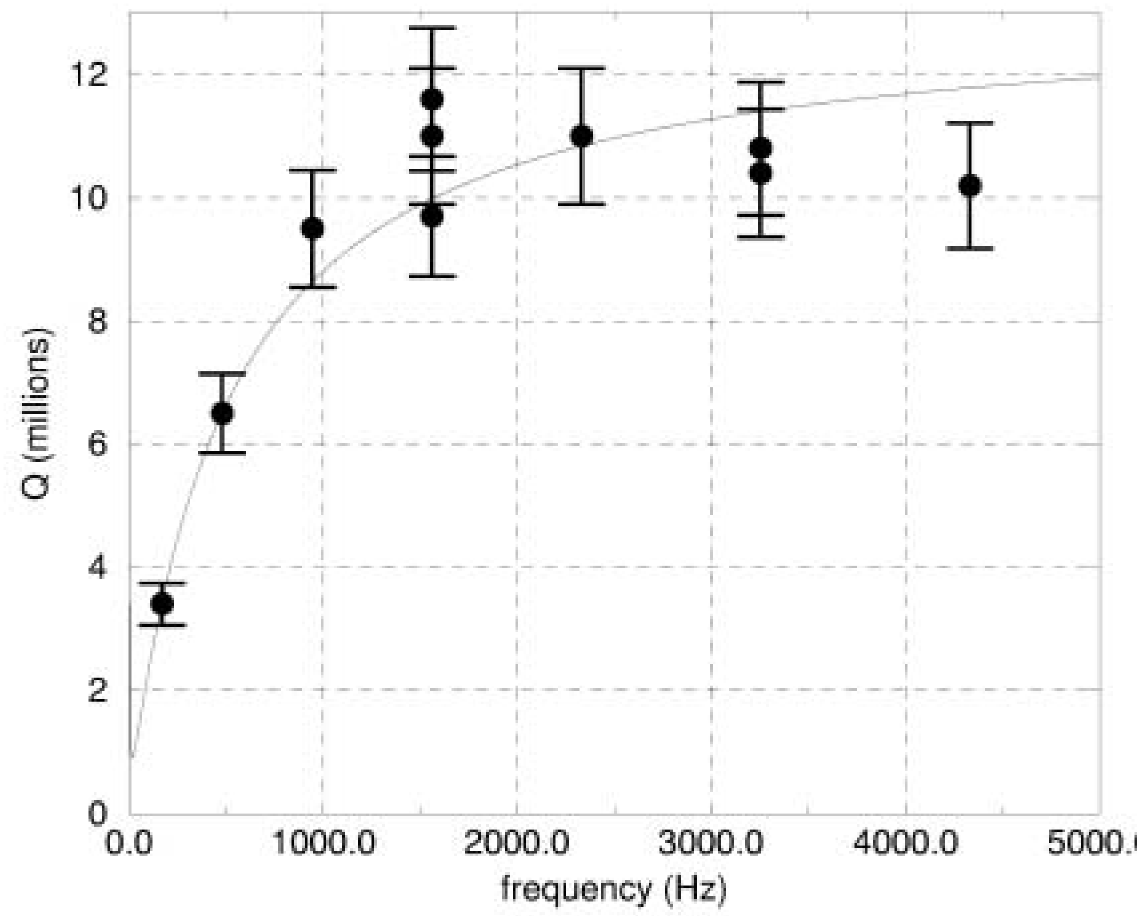


Obtain the Structural and Thermoelastic Losses from Unloaded Fiber Q's

- Best fit:

$$\alpha = 3.9 \times 10^{-7} \,/\, \mathrm{K};$$

$$\varphi = 7.6 \times 10^{-8}$$



Free fiber frequencies are very consistent with inferred diameter

$$\cos(\kappa L)\cosh(\kappa L)+1=0;$$

$$\kappa=\sqrt[4]{\omega^2\rho_L/EI}$$

mode	2	3	4	5	6	7	8
measured frequency, Hz	172.2	482.3	946.2	1561.9	2331.4	3254.2	4332.9
predicted frequency, Hz	172.1	481.9	944.4	1561.2	2332.2	3257.3	4336.7

But now E=73GPa compared to 74.5GPa for violin modes; good agreement with stress-strain law $E=E_0(1+5.75\varepsilon)$.

Put it all together to predict Q

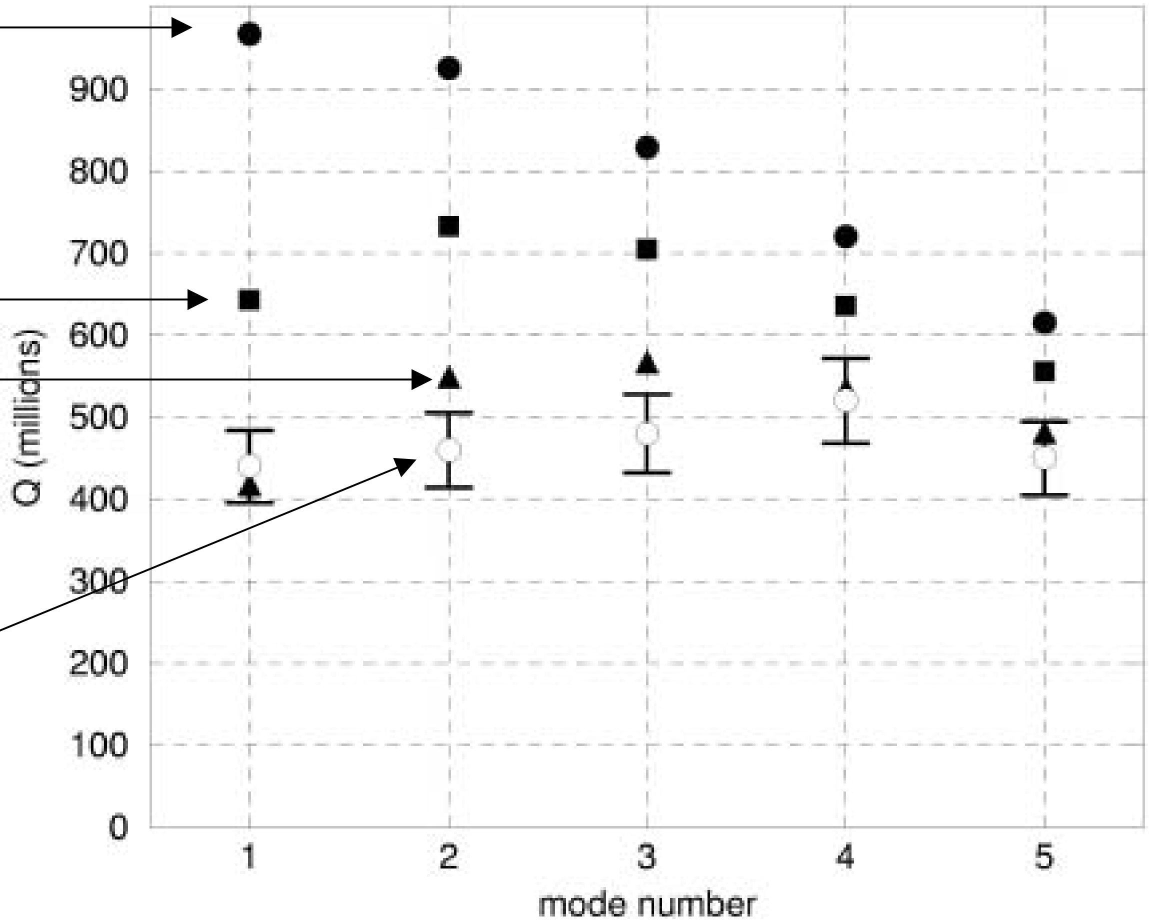
theory, with NTE

theory, w/o NTE

theory, w/o NTE

and $\alpha = 5.9$x10^{-7}/K

data



Conclusions

- Suspension fiber dynamics can be precisely quantified
- Very high Q’s possible
- Q’s not up to NTE level (and in fact more consistent with LTE theory- yikes!), but this likely due to low-frequency excess loss like recoil damping
- Work is ongoing

Dumbbell-Shaped Suspension Fibers

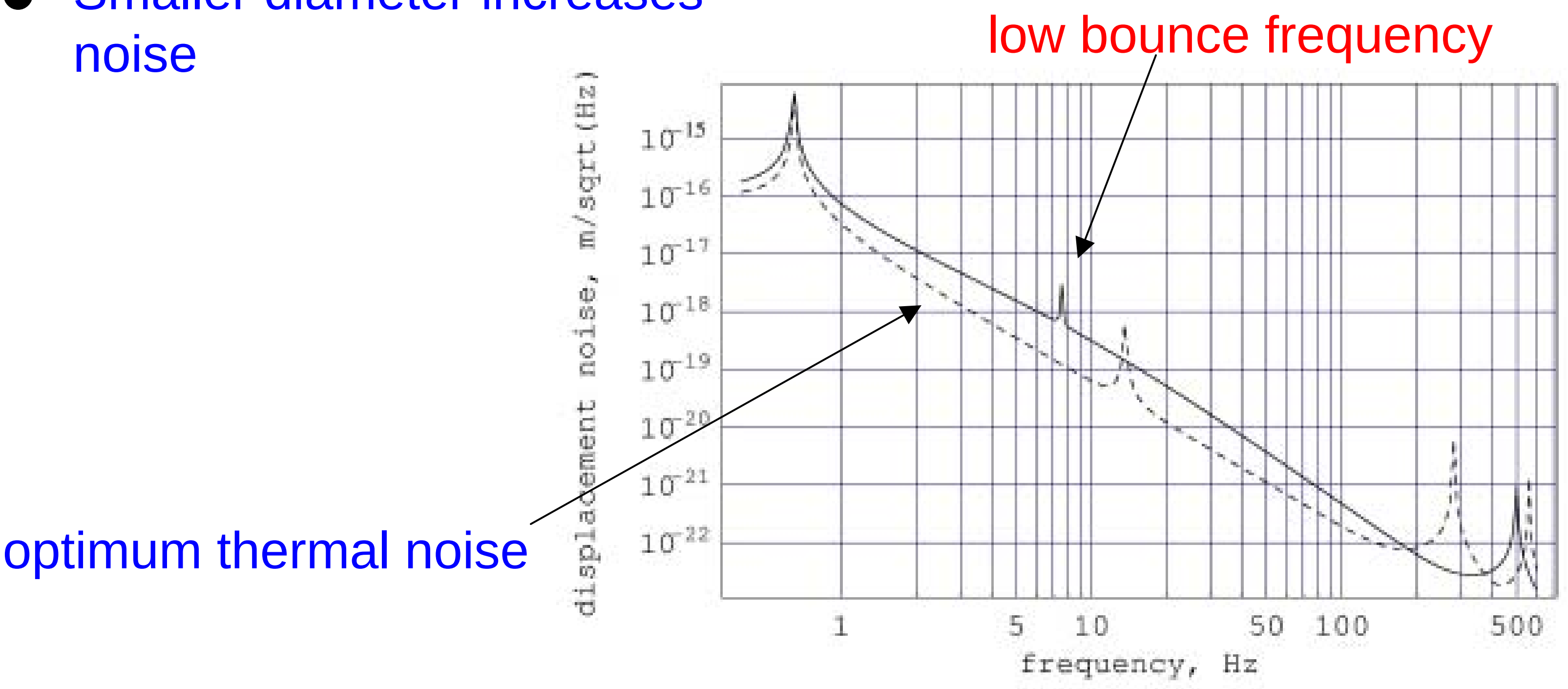
A new technique for low vertical bounce frequency and low thermal noise

LIGO-G020152-00-D

The Apparent Tradeoff

- Low thermal noise
- Optimum fiber diameter
- Smaller diameter increases noise

- Low bounce frequency
- Smaller fiber diameter better



The Ribbon Solution

- Make the cross-section as small as needed to reduce bounce frequency
- Set the ribbon thickness to increase dilution factor and push thermoelastic peak to higher frequency

But Notice the Different Dynamics of the Two Types of Motion

- Pendulum/violin motion

  Dissipative bending motion concentrated near ends of fiber

  Loss not very sensitive to middle section of fiber

- Vertical bounce motion

  Dissipative stretching motion distributed along fiber in inverse proportion to cross section

This Suggests an Obvious Solution

- Make the fiber the optimum thickness for low damping at the top and bottom, and thinner in the middle for low vertical bounce frequency-
- a DUMBBELL shape

The Equations of Motion

$$X_n(z) = A_n \cos(k_{tn} z) + B_n \sin(k_{tn} z) + C_n \cosh(k_{en} z) + D_n \sinh(k_{en} z);$$

$$k_{tn} = \sqrt{\frac{P + \sqrt{P^2 + 4EI\rho\omega^2}}{2EI}};$$

$$k_{en} = \sqrt{\frac{-P + \sqrt{P^2 + 4EI\rho\omega^2}}{2EI}};$$

where n labels the segment of the fiber.

The Boundary Conditions

$$X_1(0) = X_1'(0) = 0 \qquad \text{fiber rigidly clamped at top}$$

$$X_1(z_1) = X_2(z_1) \qquad \text{fiber smooth at boundary}$$

$$X_1'(z_1) = X_2'(z_1) \qquad \text{fiber slope smooth at boundary}$$

$$EI_1 X_1''(z_1) = EI_2 X_2''(z_1) \text{ torque continuous at boundary}$$

$$EI_1 X_1'''(z_1) - PX_1'(z_1) = EI_2 X_2'''(z_1) - PX_2'(z_1)$$

$$\text{force continuous at boundary}$$

The Boundary Conditions

$X_2(z_2) = X_3(z_2)$ fiber smooth at boundary

$X_2'(z_2) = X_3'(z_2)$ fiber slope smooth at boundary

$EI_2 X_2''(z_2) = EI_3 X_3''(z_2)$ torque continuous at boundary

$EI_2 X_2'''(z_2) - PX_2'(z_2) = EI_3 X_3'''(z_2) - PX_3'(z_2)$

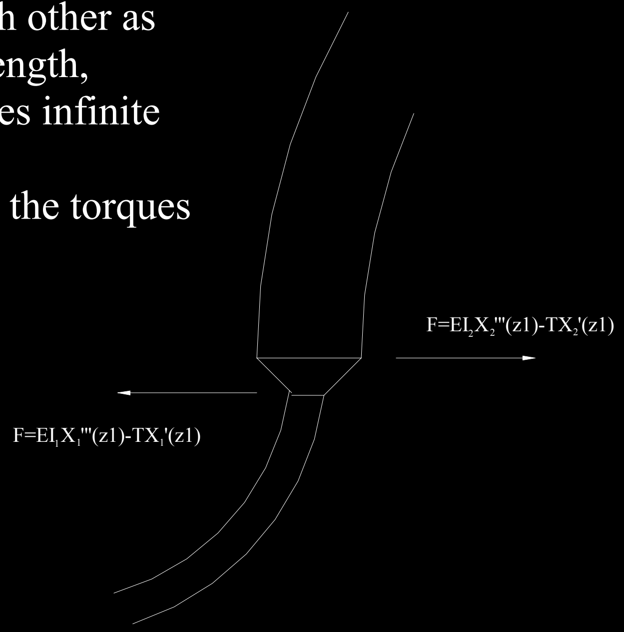
force continuous at boundary

$X_3'(z_3) = 0$ fiber slope zero at mass

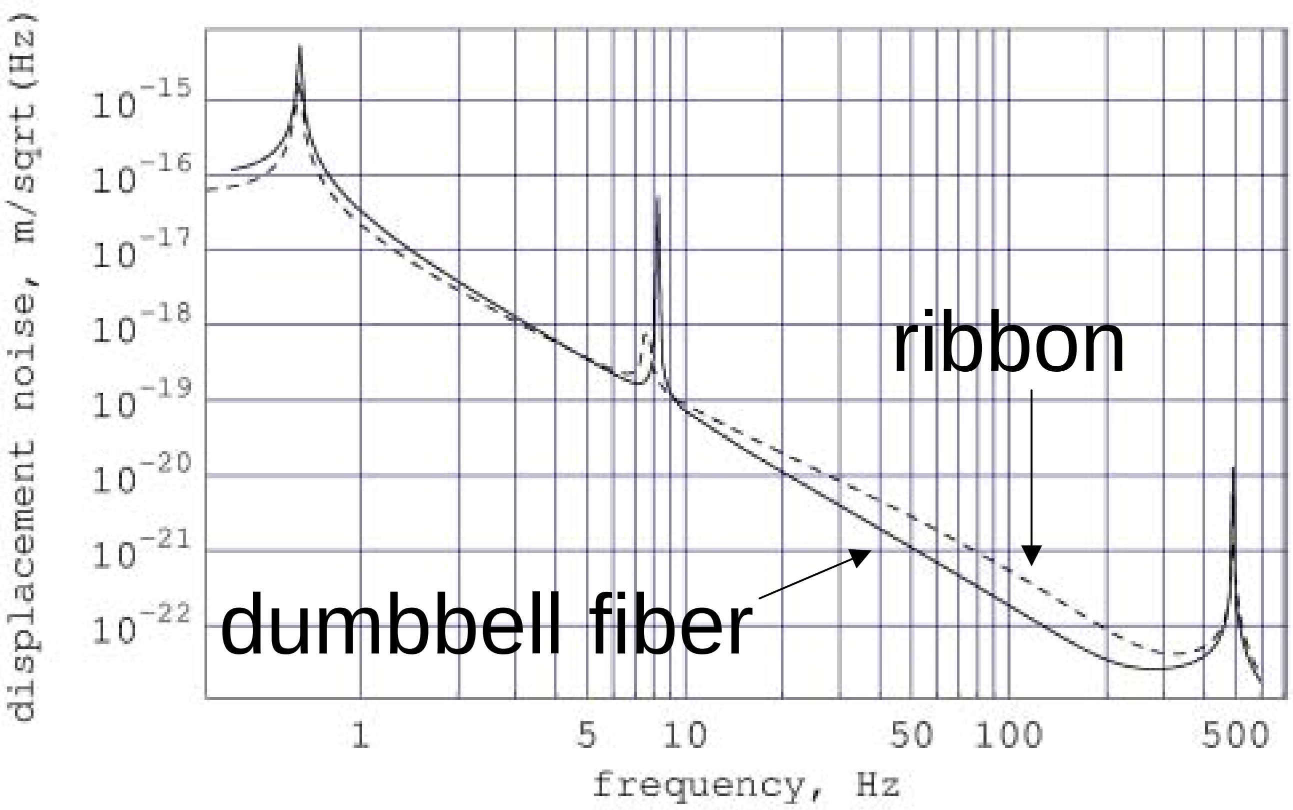
$EI_3 X_3'''(z_3) = G$ arbitrary force on fiber end

How the Boundary Conditions are Derived

Two forces must approach each other as joining segment goes to zero length, otherwise that section undergoes infinite acceleration.
A similar argument shows that the torques also become equal.



Thermal Noise Spectra: Baseline Ribbon vs. Dumbbell Fiber



Influence of Welded Pins on Suspension Q

- This program can easily model the welded pins on the ends of suspension fibers (extreme dumbbell)
- Good approximation if pins are relatively long and thin
- Used this analysis to confirm Mitrofanov and Tokmakov's estimate of Q due to lossy pins

Result of Simulation

- Mitrofanov/Tokmakov estimate:

$$Q_v^{-1} = \frac{4MgQ_p^{-1}}{Lm_p\omega_p^2}$$

  – This is based upon estimate of force exerted on pin by fiber

- Our result: this is substantially correct, although for thicker fibers the torque also plays a significant role

$$\frac{X_{pin}^{couple}}{X_{pin}^{force}} = \frac{3}{2k_e L_{pin}}$$

The Big Result

A reasonably good, reasonably short pin will not unduly influence Q or thermal noise

Fused Silica Materials Properties Database

- Many different sets of material parameters for fused silica are in use by the various LSC groups to design suspensions and predict thermal noise.
- This leads to much confusion when comparing results between groups.
- I became aware of this when analyzing violin mode data above.
- We need a common set of values shared by the community to facilitate information transfer.

Proposed Material Parameters

IN CONSTRUCTION

(I have lots of Syracuse data to digest)