Chebyshev Approximations to the 2PN Waveform

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The 2PN waveform: Chebyshev representation

- Chronopolous & Apostolatos (*Phys. Rev. D*, **64**, 042003, 2001) have shown that 2PN signals can be detected with 1.5 PN templates without any significant loss in SNR. They find that the number of templates needed for detection is reduced by 20 – 30% w.r.t. the number of 2PN templates.
- Is the phase of the 1.5PN waveform the best approximation to the phase of the 2PN?
- The Fourier transform of the radiation from an in spiraling binary system calculated at the 2PN approximation can be written as follows (SPA)

$$\tilde{h}(f) = A f^{-7/6} \exp\{i[\lambda_0 f^{-5/3} + \lambda_1 f^{-1} + \lambda_{1.5} f^{-2/3} + \lambda_2 f^{-1/3}]\}$$

$$\Psi_{2PN}(f) = f^{-5/3}[\lambda_0 + \lambda_1 (f^{1/3})^2 + \lambda_{1.5} (f^{1/3})^3 + \lambda_2 (f^{1/3})^4]$$

- The term in square-brackets is a polynomial in the variable ($f^{1/3}$); It can be rewritten in terms of the Chebyshev polynomials in the following way

Chebyshev representation (cont.)

$$\Psi_{2PN}(f) = f^{-5/3}\,[C_0 T_0(y) + C_1 T_1(y) + C_2 T_2(y) + C_3 T_3(y) + C_4 T_4(y)]$$

$$f^{1/3} = a\,y + b\,; \quad with \quad y \in [-1,1], \;\; f \in [f_s, f_c] \Rightarrow$$

$$a = \frac{f_c^{1/3} - f_s^{1/3}}{2}, \quad b = \frac{f_c^{1/3} + f_s^{1/3}}{2}$$

$$T_0(y) = 1\,,\; T_1(y) = y\,,\; T_2(y) = 2y^2 - 1\,,\; T_3(y) = 4y^3 - 3y\,,\; T_4(y) = 8y^4 - 8y^2 + 1$$

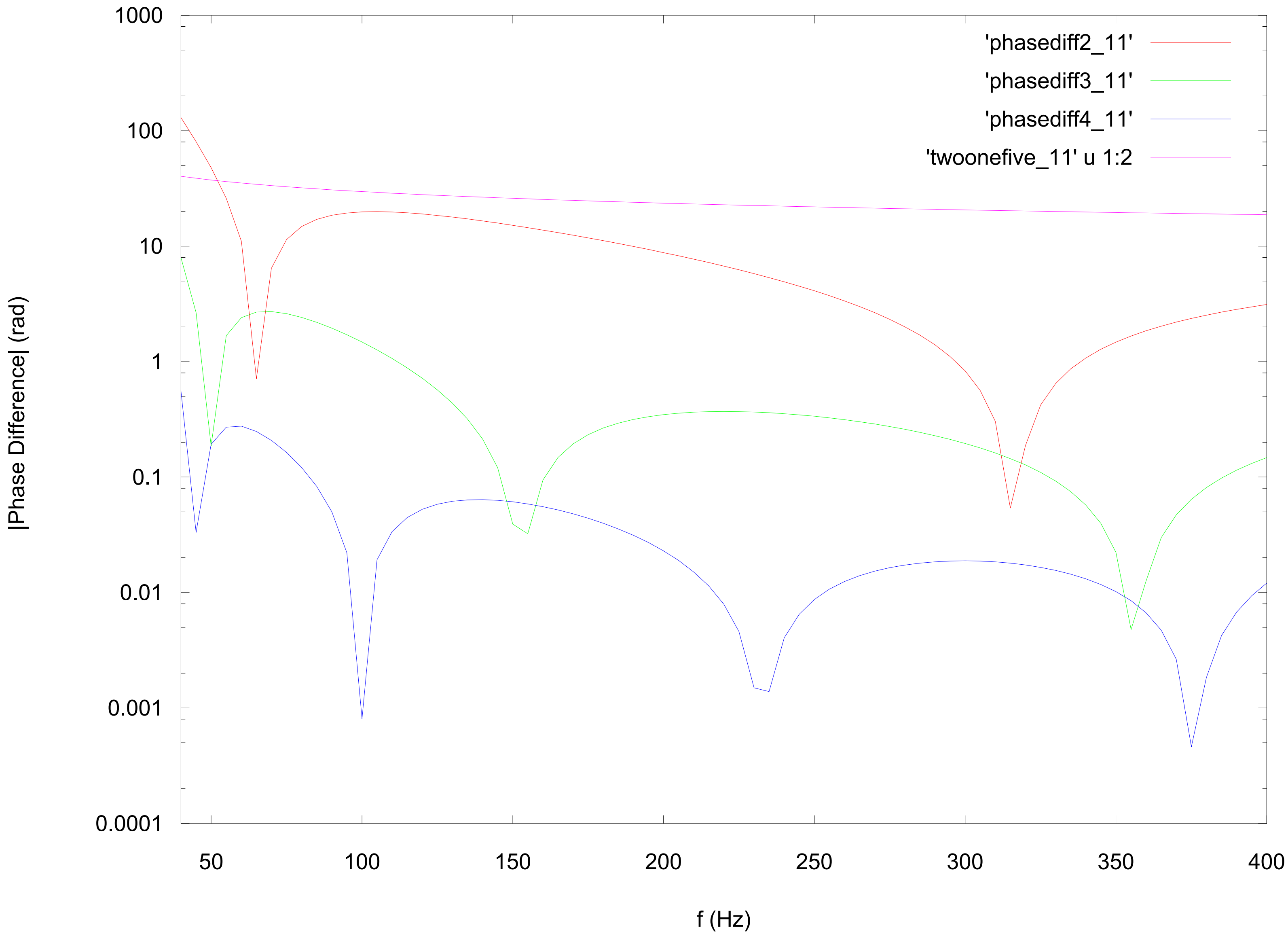
$$C_4 = \frac{1}{8}\lambda_2\, a^4 \;,\quad C_3 = \frac{1}{4}\lambda_{1.5}\, a^3 + \lambda_2\, b\, a^3 \;,\quad C_2 = \frac{1}{2}\lambda_1\, a^2 + \frac{3}{2}\lambda_{1.5}\, b\, a^2 + (3\, b^2\, a^2 + \frac{1}{2} a^4)\,\lambda_2$$

$$C_1 = 2\,\lambda_1\, b\, a + (3\, b^2\, a + \frac{3}{4} a^3)\,\lambda_{1.5} + (3\, b\, a^3 + 4\, b^3\, a)\,\lambda_2 \;,$$

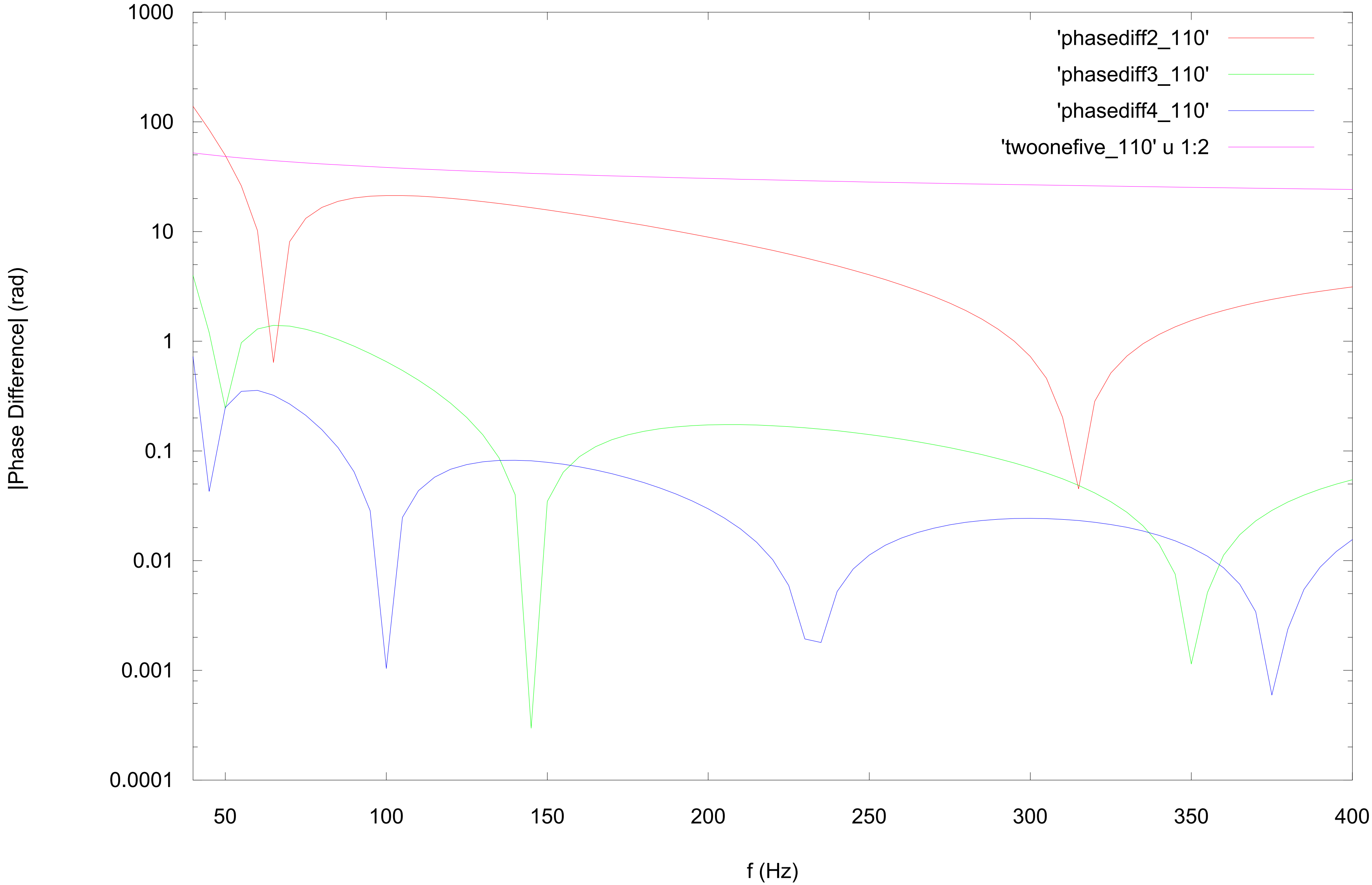
$$C_0 = \lambda_0 + (\frac{1}{2} a^2 + b^2)\,\lambda_1 + (b^3 + \frac{3}{2} b\, a^2)\,\lambda_{1.5} + (\frac{3}{8} a^4 + 3\, b^2\, a^2 + b^4)\,\lambda_2$$

- One important feature of the Chebyshev polynomials relevant to this work is that, by truncating the above polynomial to a lower degree one obtains the most accurate approximation of Ψ_{2PN} at that degree (has the smallest maximum deviation from Ψ_{2PN}).
- $\Psi_{1.5PN}$ therefore does not provide the most accurate approximation to Ψ_{2PN} !

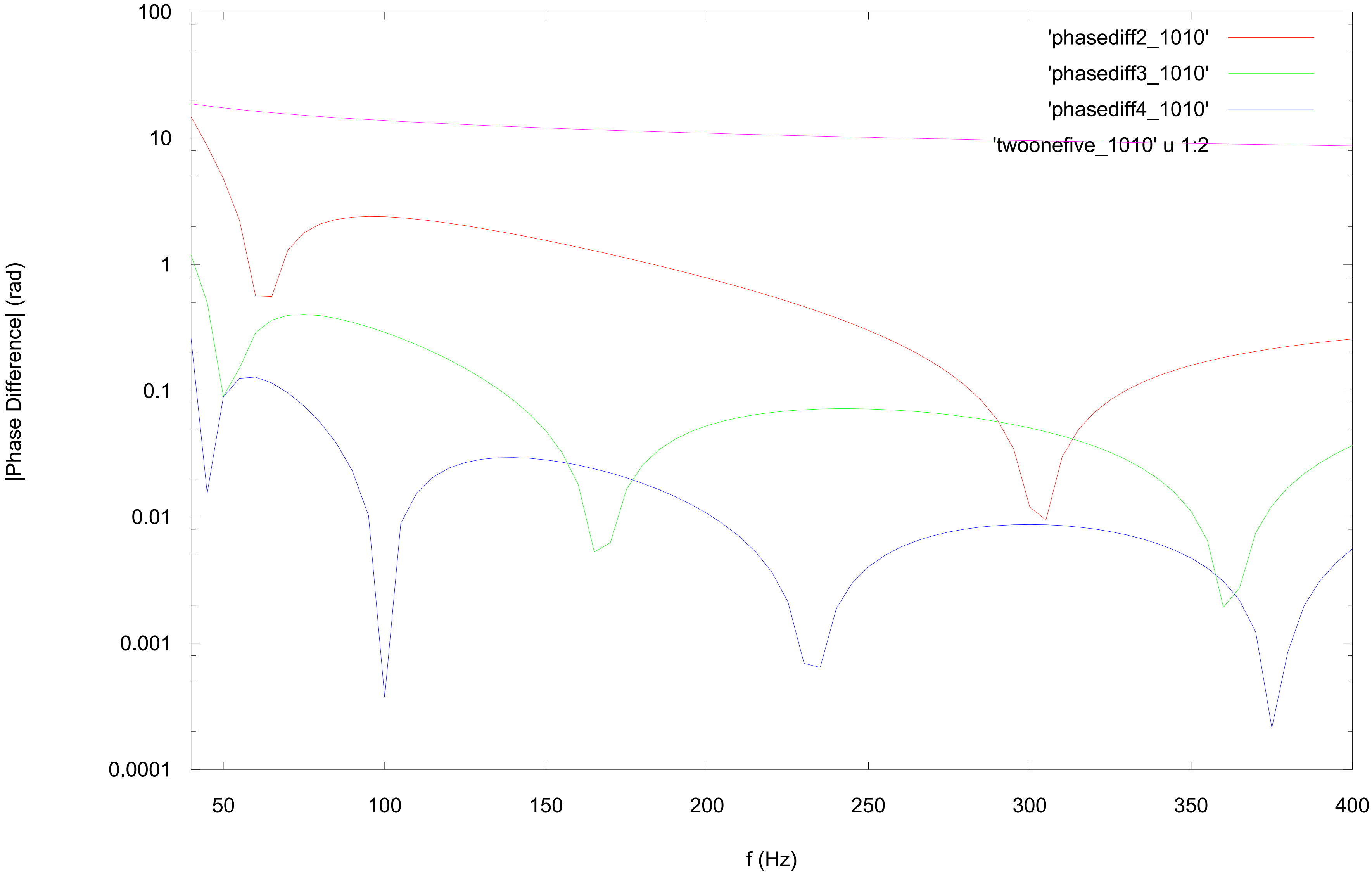
Phase Differences between the 2PN phase and its approximations – $M_1 = M_2 = 1\ M_{sun}$



Phase Differences between the 2PN phase and its approximations – $M_1 = 1\ M_{sun}$, $M_2 = 10\ M_{sun}$



Phase Differences between the 2PN phase and its approximations – $M_1 = 10\ M_{sun}$, $M_2 = 10\ M_{sun}$



Work in Progress

- What is the achievable SNR when detecting 2PN signals with templates based on the Chebyshev approximations?
- What is the number of templates needed for detecting 2PN signals with Chebyshev templates?
- What is the accuracy in the estimated masses?
- The coefficients of the Chebyshev expansion could be treated as independent from each other. One could use this representation of the phase function to search for the family of signals discussed by Buonanno et al.