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# Bicoherence and Bispectrum Monitor

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# SYNOPSIS

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- Introduction to Higher Order Statistics
  - » 1D: Correlation, Coherence, Power Spectra
  - » 2D: Bicorrelation, Bicoherence, Bispectrum
  - » 3D...
- Bispectrum diagnostic
- Gaussianity Test
- Linearity Test



# What are Higher Order Statistics?

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- 1D Statistics:

- » Correlation:  $C_{xy}(t) = \int_{-\infty}^{\infty} x(\tau) y(t + \tau) d\tau \Leftrightarrow X(f) Y^*(f) = S_{xy}(f)$

- » Power Spectral Density:  $C_{2x}(t) \Leftrightarrow X(f) X^*(f) = S_{2x}(f)$

- » Coherence:  $C_{xy}(f) = \frac{S_{xy}(f)}{\sqrt{S_{2x}(f) S_{2y}(f)}}$

- Tells us power and phase coherence at a given frequency

# Second Order Statistics

- 2D Statistics:

- » Bicumulant:

$$C_{xyz}(t, t') = \int_{-\infty}^{\infty} x(\tau) y(t + \tau) z(t' + \tau) d\tau \Leftrightarrow X(f_1) Y(f_2) Z^*(f_1 + f_2) = S_{xyz}(f_1, f_2)$$

- » Bispectral Density:

$$C_{3x}(t) \Leftrightarrow X(f_1) X(f_2) X^*(f_1 + f_2) = S_{3x}(f_1, f_2)$$

- » Bicoherence:

$$C_{xyz}(f) = \frac{S_{xyz}(f_1, f_2)}{\sqrt{S_{2x}(f_1) S_{2y}(f_2) S_{2z}(f_1, f_2)}}$$

- Tells us power and phase coherence at a coupled frequency



## Zero-lag Cumulants

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Mean

$$C_x(0)$$

Variance

$$C_{2x}(0)$$

Skewness

$$C_{3x}(0)$$

0 if Symmetric

Kurtosis

$$C_{4x}(0)$$

0 if Gaussian

Useful statistical values, but...

Skewness = 0 does not prove symmetry

Kurtosis = 0 does not prove Gaussianity

Variations in skew and kurtosis not well quantified.



# Why Higher Order Statistics?

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- For a Gaussian process:  $C_{nx}(t) = 0$ , for  $n > 2$

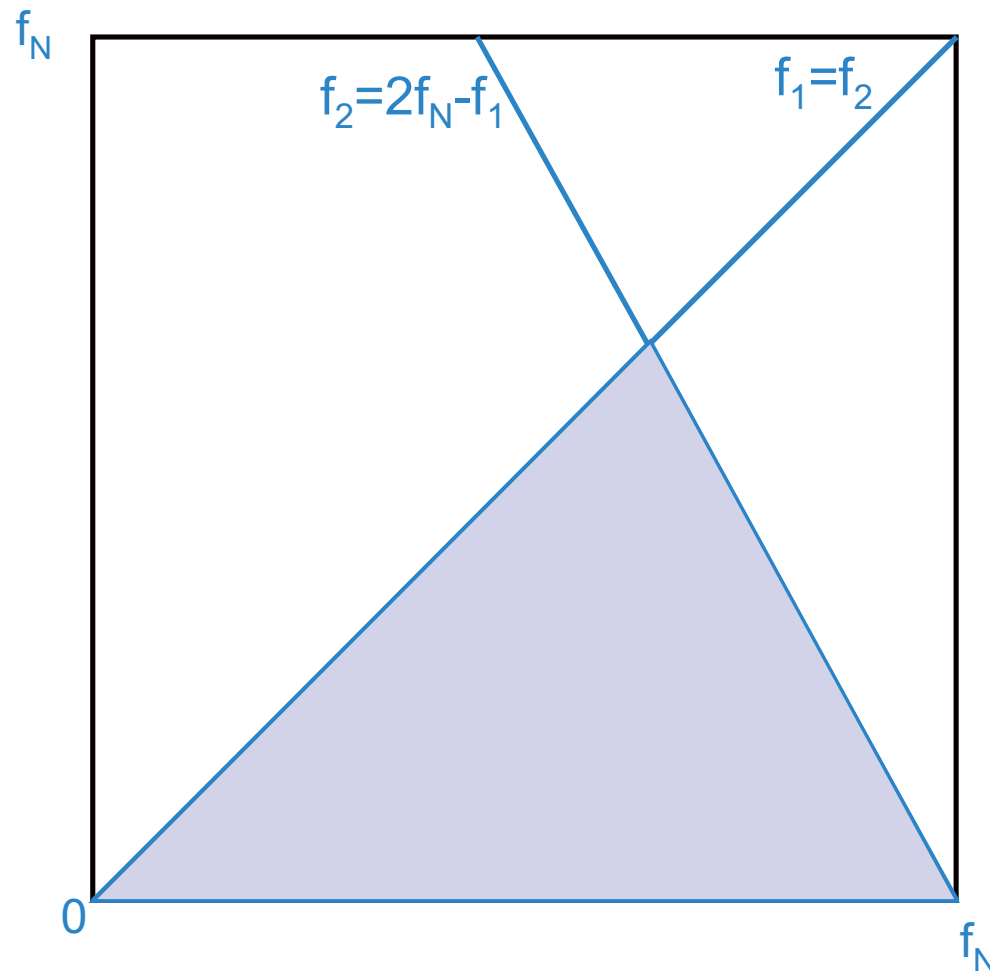
- For independent processes:

$$z(t) = x(t) + y(t), \quad C_{nz}(t) = C_{nx}(t) + C_{ny}(t) \xrightarrow{n>2} C_{ny}(t)$$

- Allows for separation of Gaussian process for  $n > 2$ 
  - » Visual check of frequency coupling and phase noise
  - » Statistical test for the probability of gaussianity and linearity
  - » Iterative process to reconstruct nongaussian signal from the higher order cumulants



# Bispectrum Unique Area





## Monitor Features

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- Performs cross-bicoherence (cross-bispectrum).  
User can enter 1–3 channels.
- Program decimates to the lowest channel rate.
- User can specify further decimation of the data.
- Choice of Bicoherence and/or bispectrum.
- Choice of direct/indirect methods for bispectrum.
- Example:

```
» XBic H0:PEM-BSC5_ACCX H0:PEM-LVEA_SEISX H2:LSC-AS_Q  
-direct -bispec -MaxFrame 900
```





## Status and Conclusions

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### Monitor Good News:

- Monitor plots Bicoherence and Bispectrum
- Real time update: increase stride (slow) or skip frames (fast)
- Handles frequencies  $< 2k$  without decimation.

### Updates Pending:

- Rao-Gabr 2-D windowing to smooth plots.
- Choice for handling rate of fast channels: reducing range or reducing resolution.
- Increasing stability.