MELODY/MATLAB

OBJECT-ORIENTED MODEL OF GRAVITATIONAL-WAVE INTERFEROMETERS USING MATLAB

Raymond G. Beausoleil

Stanford University/Hewlett-Packard Laboratories
13837 175th Pl. NE, Redmond, WA 98052–2180
beausol@hpl.hp.com

August 2001

LIGO-G010301-00-Z

MELODY/MATLAB OVERVIEW

- Goals and features
- Propagation model
- Object-level features
  - Interferometer configurations
  - Mirror physics: thermal loading, position, orientation
  - Four-stage resonator length pseudolocking
- Script-level features
  - Modulation schemes
  - Mirror parameters: thermal, position, orientation
  - Full interactive MATLAB functionality
- Milestones

ACKNOWLEDGMENTS

- *Stanford*
  - Eric Gustafson
  - Marty Fejer
- *MIT*
  - Ryan Lawrence
  - Peter Frischel
- *UCF*
  - Guido Mueller
- *Caltech*
  - Erika D'Ambrosio
  - Bill Kells
  - Jordan Camp
- *Glasgow*
  - Ken Strain
- *LIGO*
  - Daniel Sigg

MELODY/MATLAB GOALS

- Provide an easily usable, flexible multiplatform framework for LIGO I/II calculations and simulations
- Allow users to write scripts to drive simulations tailored to their needs (post-processing, graphics, numerical analysis)
- Easily include physical effects in mirrors: aperture diffraction, curvature mismatch, thermal lensing, thermoelastic surface deformation
- Allow translation to a lower-level language for performance
- Provide a simple interface to industry-standard software for modeling control systems (SIMULINK)

MELODY/MATLAB FEATURES

- MATLAB classes for fields, mirrors, interferometers, and detectors; driven by user-written scripts $\longrightarrow$ self-consistent solutions
- Prebuilt LIGO I/II configurations
  - Power, signal, and dual recycling
  - Arbitrary modulation schemes
  - Resonator length pseudolocking for self-contained simulations
- Mirror physics
  - Aperture diffraction
  - Mirror surface/laser wavefront curvature mismatch
  - Thermal lensing due to bulk and coating absorption (TEM_{00})
  - *Thermoelastic surface deformation (reflection, transmission)*

NEW MELODY/MATLAB FEATURES

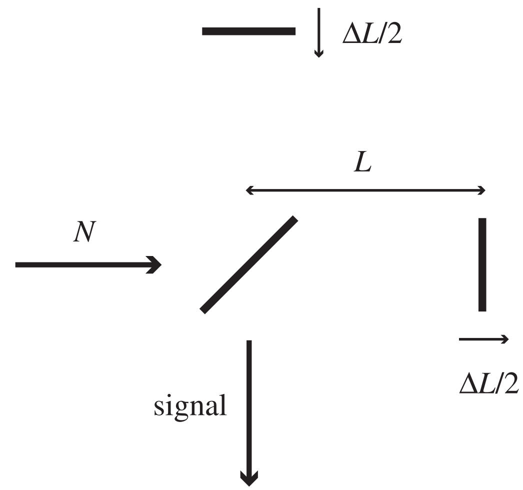
- Overall performance improvement v1.9/v1.8: 20%
- Updated model of thermoelastic surface deformation
  - Analytical calculations of thermoelastic surface deformation verified with MATLAB FEM
    - substrate absorption (symmetric)
    - HR and AR coating absorption (asymmetric)
  - Reflection *and* transmission effects included
- Write-up (manual) 90% complete

MELODY/MATLAB LIMITATIONS

- Models thermal loading due to TEM_{00} absorption only, summed over all frequency components
- Correct numerical beamsplitter treatment underway (FEMLAB)
  - Arbitrary non-normal incidence angle
  - Thermoelastic surface deformations
- Transient thermal loading not yet implemented — calculations complete, but will require nontrivial architectural changes

GRAVITATIONAL WAVE DETECTION

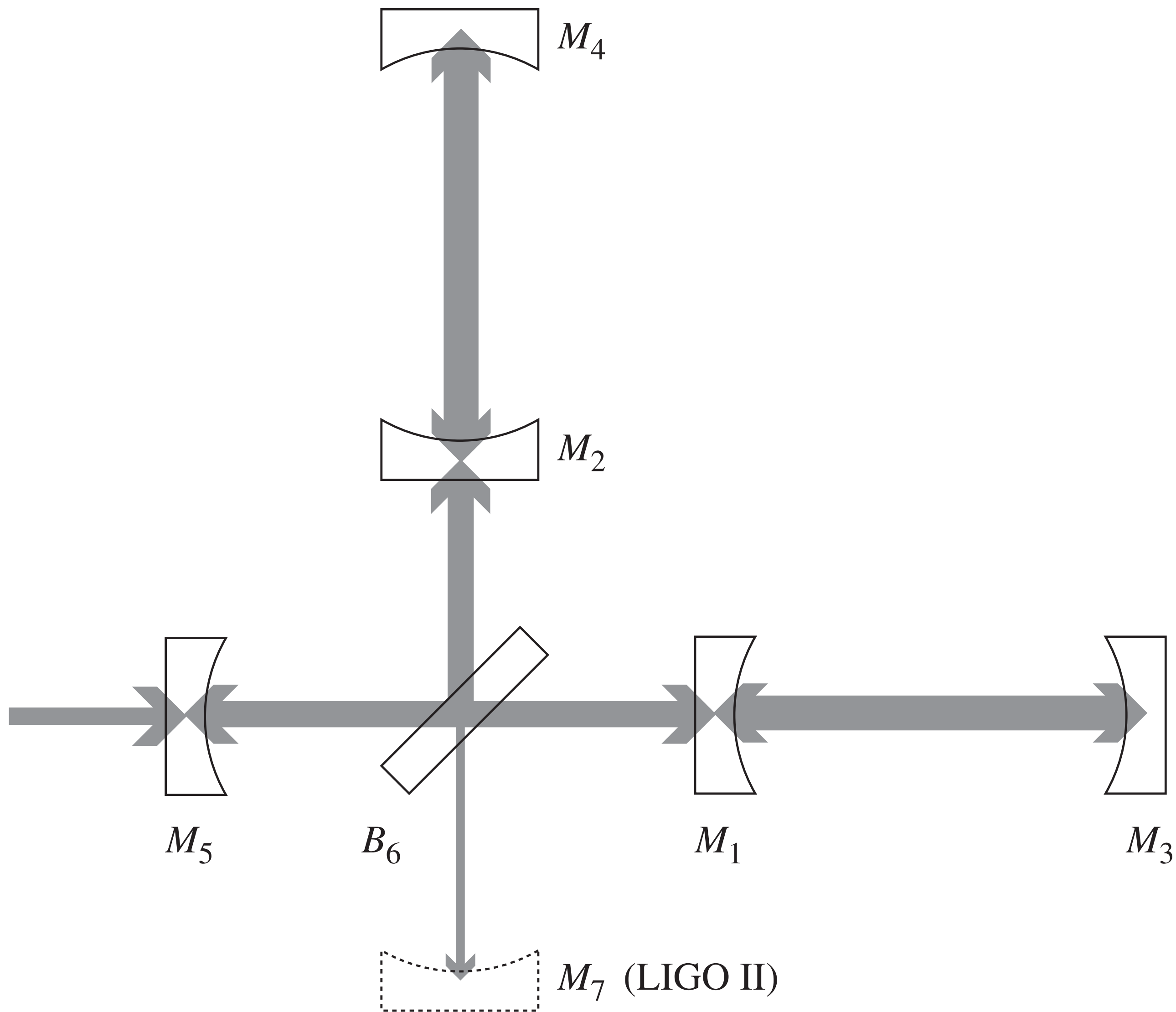
$$\Delta\Phi \approx \frac{\Delta L}{L}\frac{L}{\lambda} \equiv h\frac{L}{\lambda} > \frac{1}{\sqrt{N}}$$



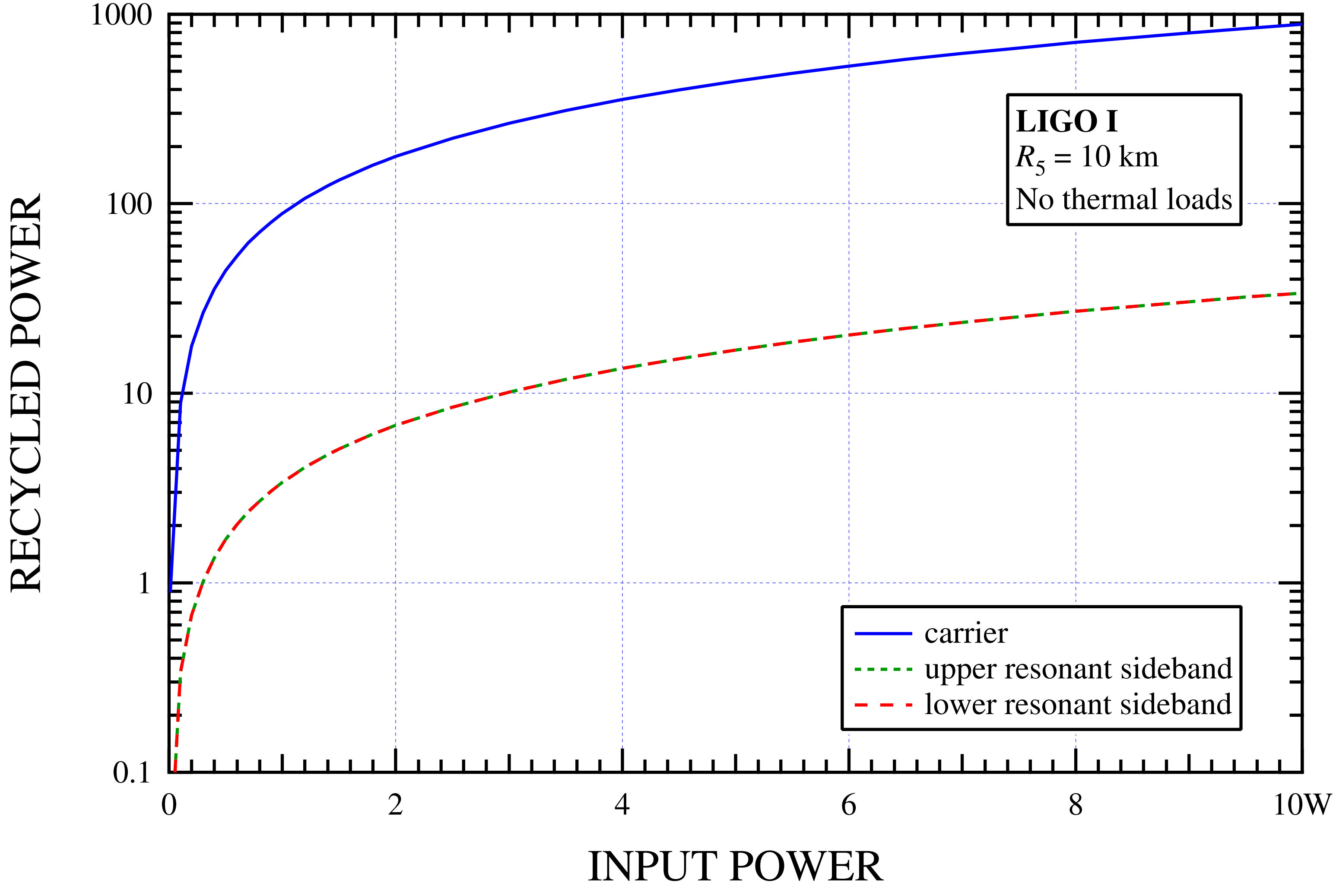
$$h_{\text{min}} \approx \frac{\lambda}{L}\frac{1}{\sqrt{N}}$$

- Use Fabry-Perot interferometers: $L \longrightarrow B\,L$
  - Improved sensitivity
  - Longer storage time $\longrightarrow$ lower signal frequencies
- Dark fringe operation $\longrightarrow$ use power recycling
  - Improved sensitivity

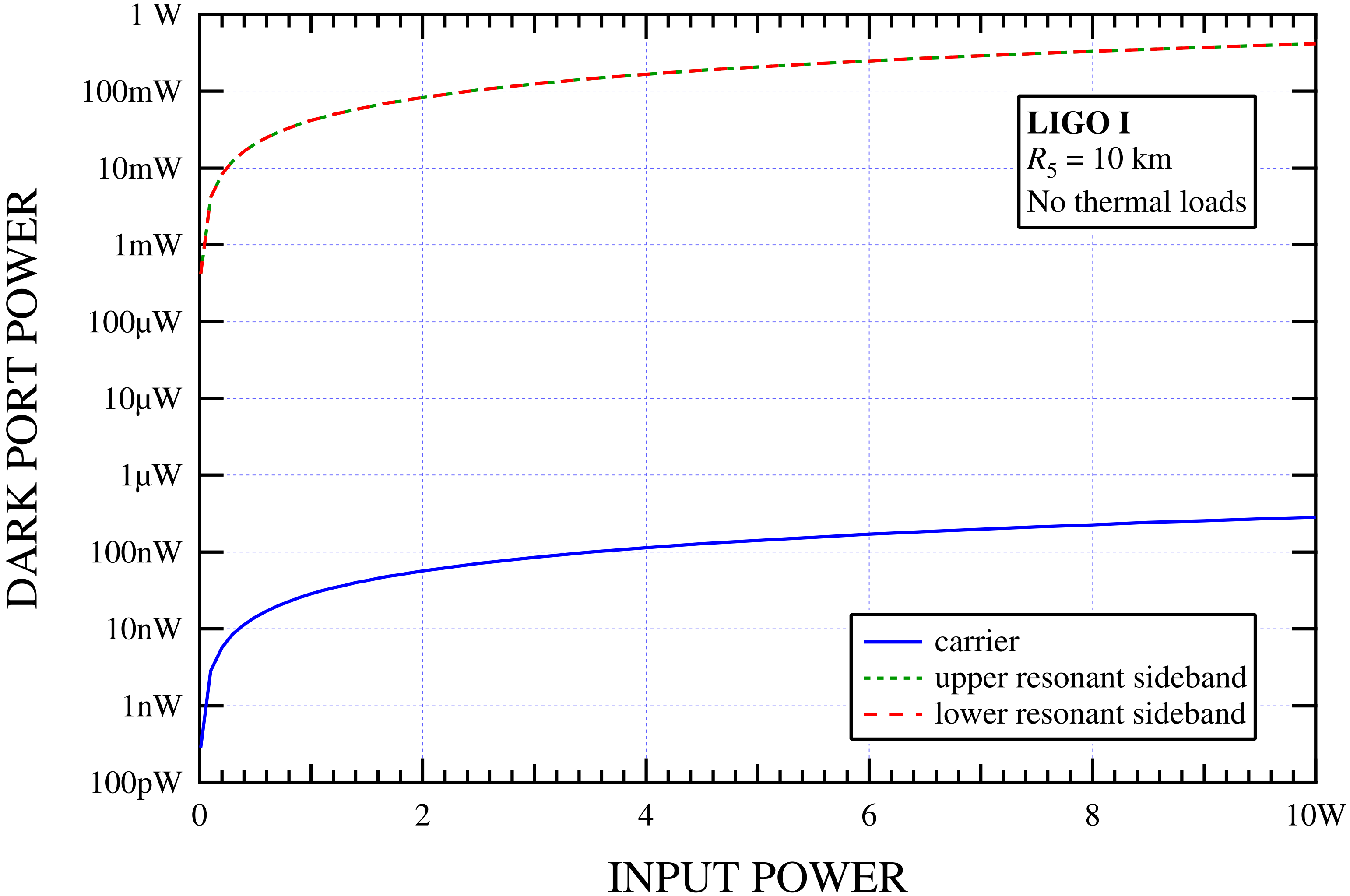
LIGO IFO CONFIGURATION



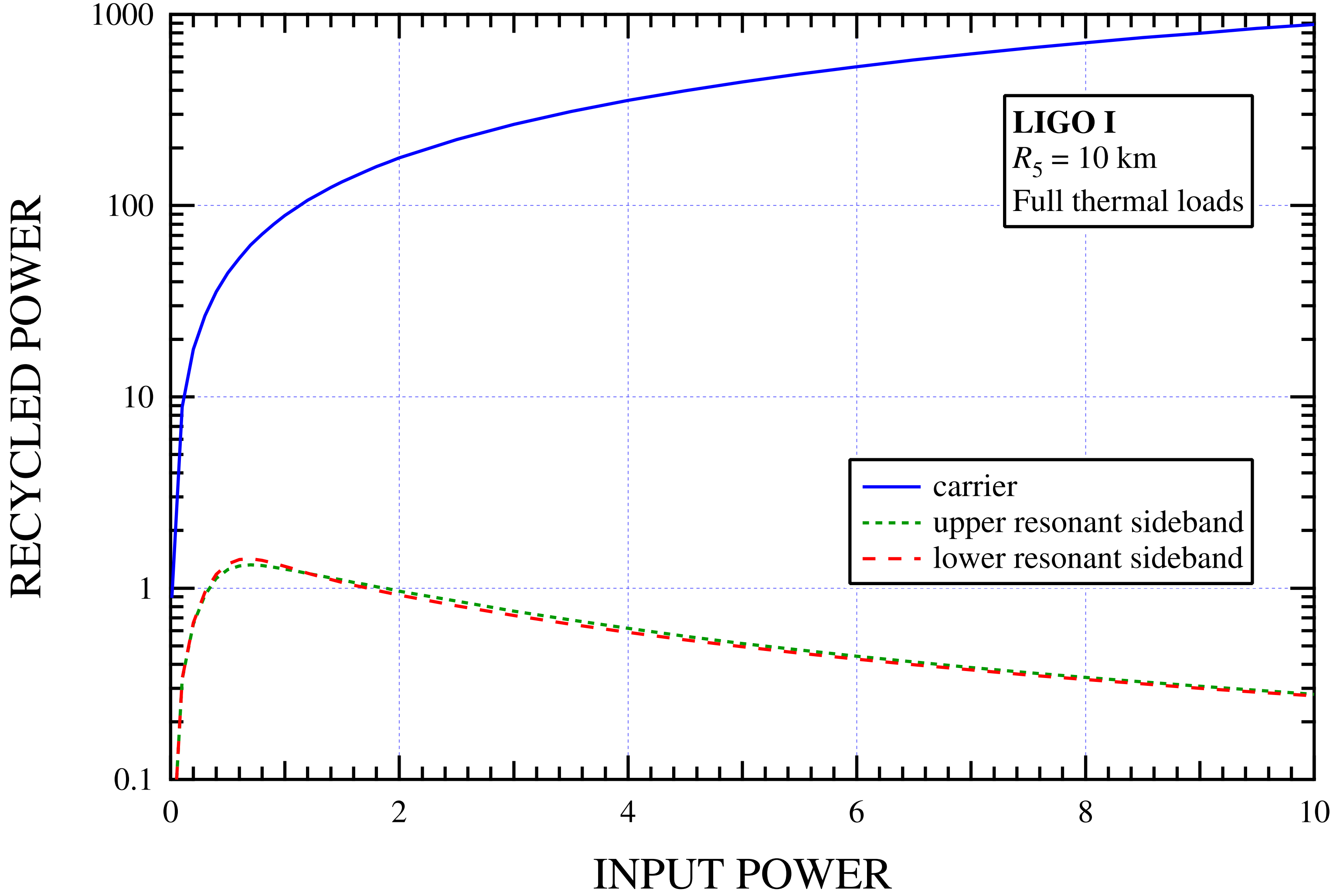
LIGO I: COLD, $R_5 = 10$ km, $E_0 \equiv$ TEM$_{00}$



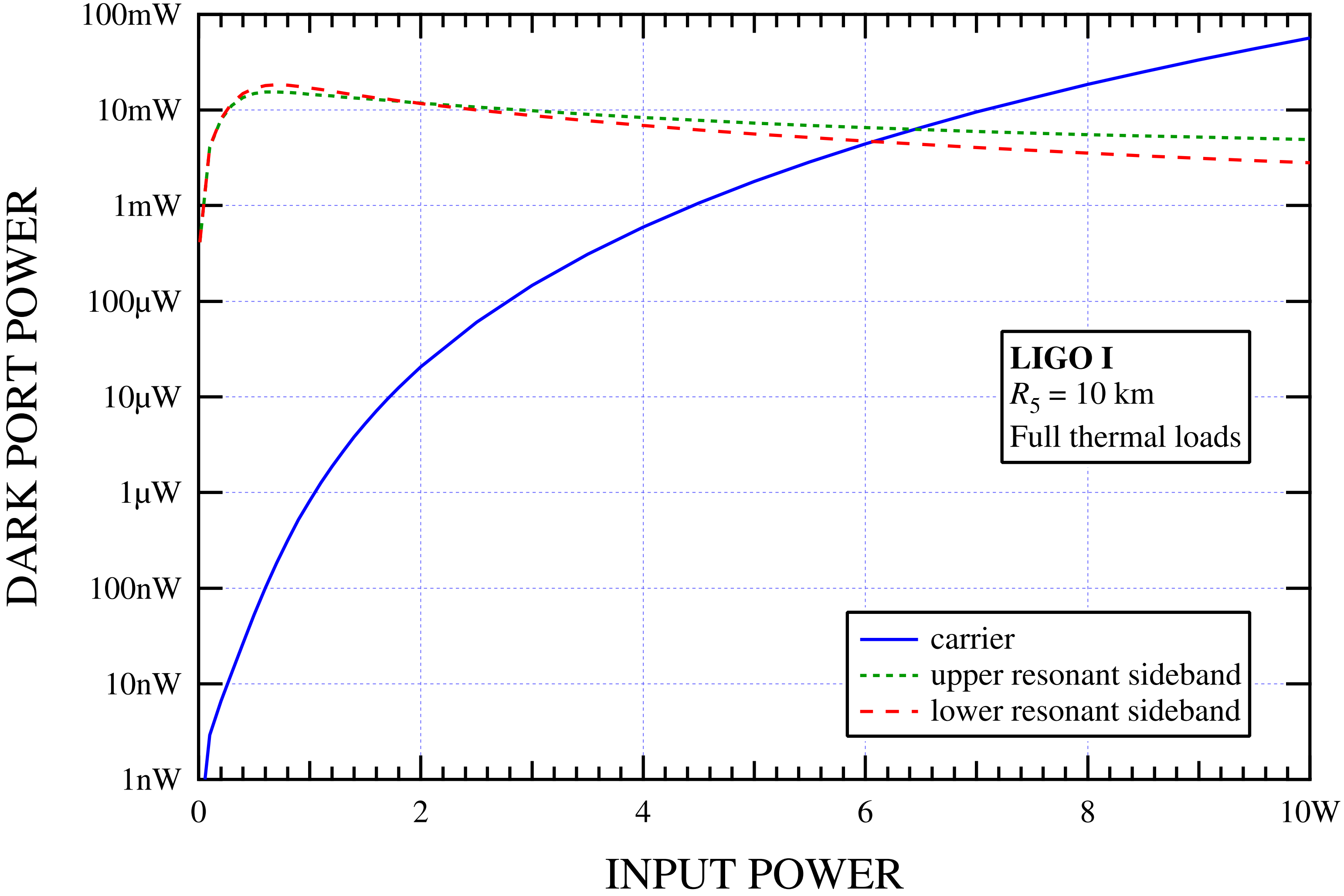
LIGO I: COLD, $R_5 = 10$ km, $E_0 \equiv$ TEM$_{00}$



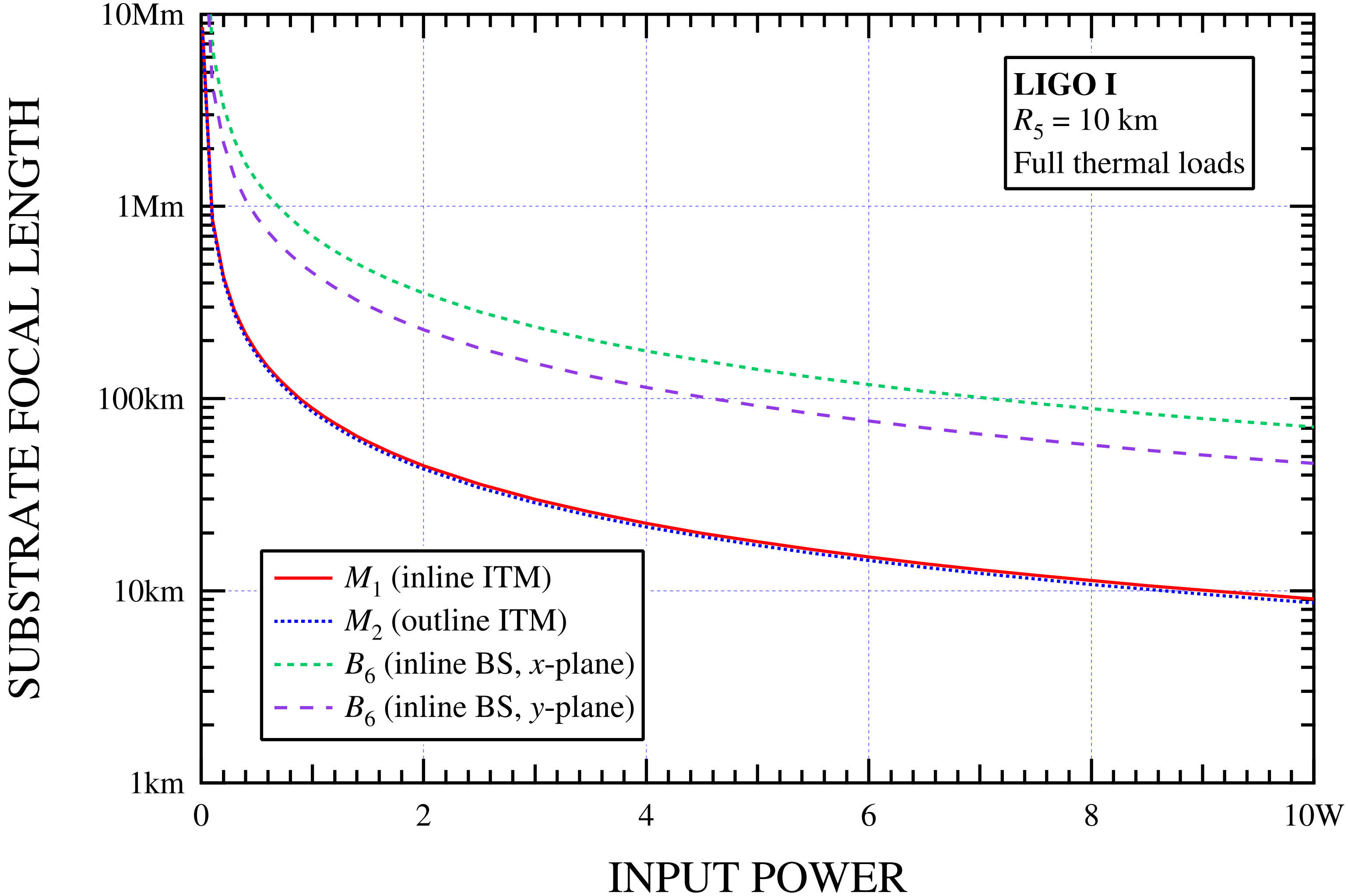
LIGO I: HOT, $R_5 = 10$ km, $E_0 \equiv$ TEM$_{00}$



LIGO I: HOT, $R_5 = 10$ km, $E_0 \equiv \mathrm{TEM}_{00}$



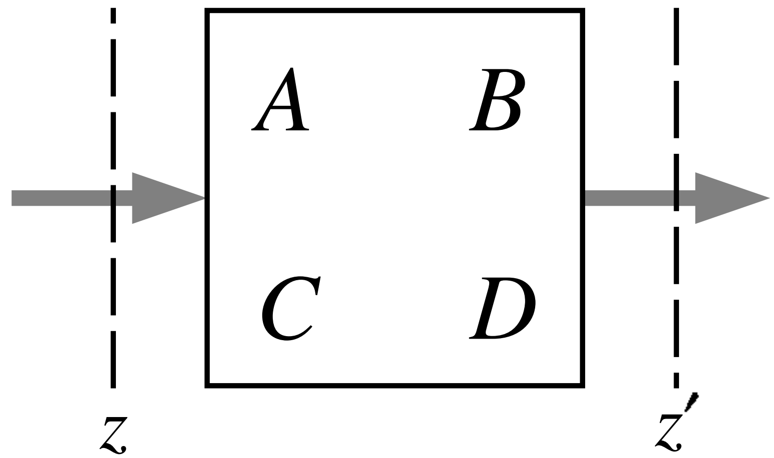
LIGO I: TFL, $R_5 = 10$ km, $E_0 \equiv$ TEM$_{00}$



FORWARD PROPAGATION: HUYGENS-FRESNEL INTEGRAL

$$\mathbf{E}(\mathbf{r},t) \equiv \mathrm{Re}\left\{\boldsymbol{\epsilon}E(\mathbf{r})e^{i(kz-\omega t)}\right\}$$

$$\nabla_\perp^2 E(\mathbf{r}) + i2k\frac{\partial}{\partial z}E(\mathbf{r}) = 0$$



Huygens Integral:

$$E(x,y,z) = \int_{\mathcal{A}_1} dx'\,dy'\,K(x,y;x',y')E(x',y',z') \equiv \hat{K}[E(x',y',z')]$$

Fresnel Approximation:

$$K(x,y;x',y') =$$

$$\frac{1}{i\lambda B}\exp\left\{i\frac{\pi}{\lambda B}\left[A(x'^2+y'^2) - 2(x'x+y'y) + D(x^2+y^2)\right]\right\}$$

UNPERTURBED BASIS FUNCTIONS

Forward and backward unperturbed basis functions:

$$\gamma_{mn} u_{mn}(x, y, 0) = \int_{\mathcal{A}_1} dx'\, dy'\, K_0(x, y; x', y') u_{mn}(x', y', 0)$$

$$\gamma^\dagger_{mn} u^\dagger_{mn}(x, y, 0) = \int_{\mathcal{A}_1} dx'\, dy'\, K^\dagger_0(x, y; x', y') u^\dagger_{mn}(x', y', 0)$$

Biorthogonality relation (Siegman), satisfied discretely:

$$\int_{\mathcal{A}_1} dx\, dy\, u^\dagger_{mn}(x, y, z) u_{m'n'}(x, y, z) = \delta_{mm'} \delta_{nn'}$$

Expand intracavity field:

$$E(x, y, z, t) = \sum_{mn} E_{mn}(t) u_{mn}(x, y, z)$$

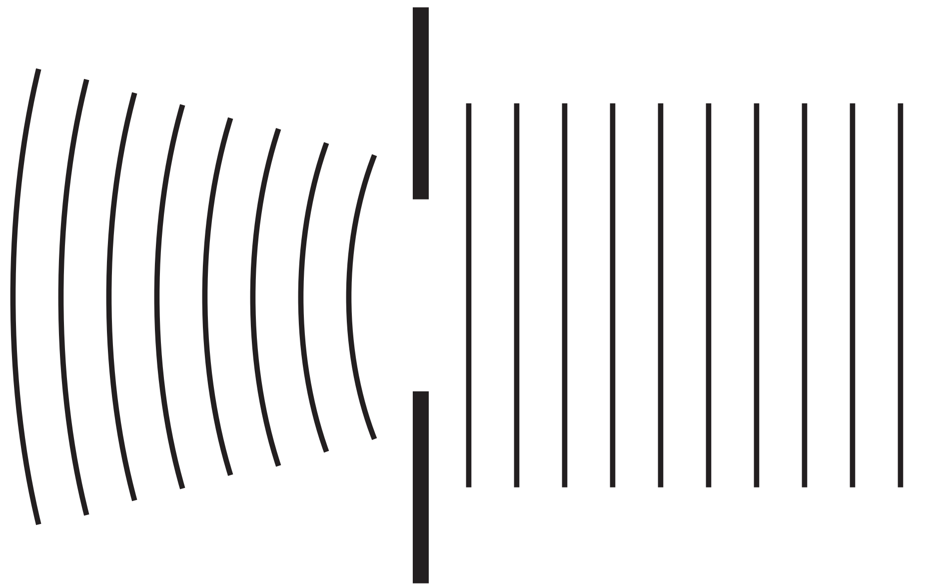
PROPAGATOR MATRIX ELEMENTS

Calculate $K_{mn;m'n'}(t)$ as the matrix element of the fully perturbed forward propagator (from reference plane $\mathcal{A}_1$ to reference plane $\mathcal{A}_2$) in the unperturbed basis:

$$K_{mn;m'n'}(t) = \int_{\mathcal{A}_2} dx\, dy \int_{\mathcal{A}_1} dx'\, dy' \times u^{\dagger}_{mn}(x,y) K(x,y;x',y';t) u_{m'n'}(x',y')$$

We compute $K_{mn;m'n'}(t)$ for each propagation region in the extended unperturbed basis of the interferometer; then construct a representation of the perturbed interferometer using matrix multiplication.

APERTURE DIFFRACTION



$$A_{mn,m'n'} = \iint_{\mathcal{A}} dx\, dy\; u^*_{mn}(x,y)\, u_{m'n'}(x,y)$$

$$\equiv \delta_{m,m'}\, \delta_{n,n'} - e^{-\alpha} I_{mn,m'n'}(\alpha),$$

where $\alpha \equiv 2(a/w)^2$, and nonzero elements of $I(\alpha)$ satisfy

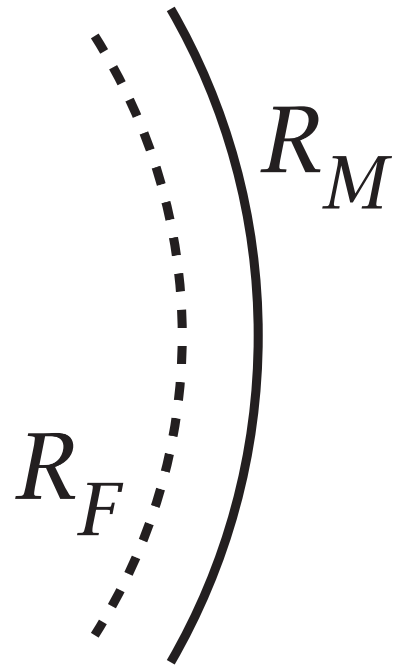
$$I_{mn,m'n'}(\alpha) \approx O\left[\alpha^{\frac{1}{2}(m+n+m'+n')}\right]$$

Unitary approximation:

$$C \cong \exp(i\,\gamma\,c),$$

where

$$\gamma \equiv \frac{\pi\,w^2}{\lambda}\left(\frac{1}{R_F} - \frac{1}{R_M}\right),$$



and, in the Hermite-Gauss basis,

$$c_{mn,m'n'} \equiv \frac{2}{w^2}\iint_{-\infty}^{\infty} dx\,dy\ \left|u_{mn}(x,y)\,u_{m'n'}(x,y)\right|\ \left(x^2+y^2\right)$$

$$= X^2_{m,m'}\,\delta_{n,n'} + \delta_{m,m'}\,X^2_{n,n'}$$

THERMAL LENSING

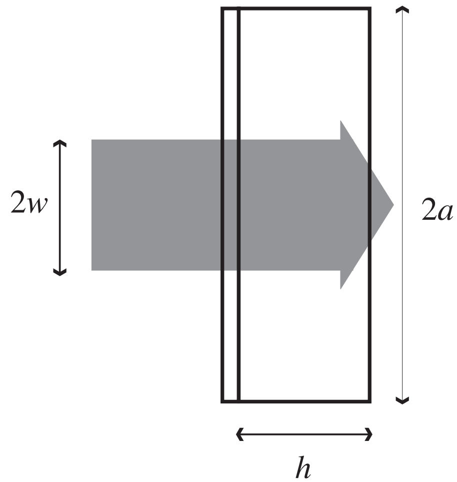
- Hello-Vinet model of substrate thermal lensing due to both substrate and coating absorption
- H-V bulk absorption result agrees with approximations:
  - Infinite half-space approximation:

$$T(r) - T(0) = -\frac{\alpha_P P}{4\pi k_T}\left[\gamma + \ln\left(\frac{2r^2}{w^2}\right) + E_1\left(\frac{2r^2}{w^2}\right)\right]$$

  - Near $r = 0$, thin lens approximation:

$$f = \frac{\pi w^2}{\alpha_P h P}\,\frac{\kappa_T}{dn/dT}$$

- Numerical implementation of astigmatic thermal loading in beam-splitter almost complete (Hermite-Gauss basis)



Reference: P. Hello and J.-Y. Vinet, J. Phys. France 51, 1267 (1990)

Coating absorption:

$$T_c(r,z) = \frac{P_c}{k_T a}\sum_{k=0}^{\infty} a^2 p_k \left[A_k \cosh\left(\zeta_k \frac{z}{a}\right) + B_k \sinh\left(\zeta_k \frac{z}{a}\right)\right] J_0\left(\zeta_k \frac{r}{a}\right)$$

Substrate absorption:

$$T_s(r,z) = \frac{P_s}{k_T h}\sum_{k=0}^{\infty} \frac{a^2 p_k}{\zeta_k^2}\left[1 - 2\tau A_k \cosh\left(\zeta_k \frac{z}{a}\right)\right] J_0\left(\zeta_k \frac{r}{a}\right)$$

ζ_k: Roots of the equation

$$\zeta\, J_1(\zeta) - \tau\, J_0(\zeta) = 0$$

Since $\tau \equiv 4\epsilon T^3 a/k_T = 0.27734$ for fused silica at room temperature,

$$\zeta_k \approxeq (k + 1/4)\,\pi, \quad k \in \{0, 1, 2, \ldots\}$$

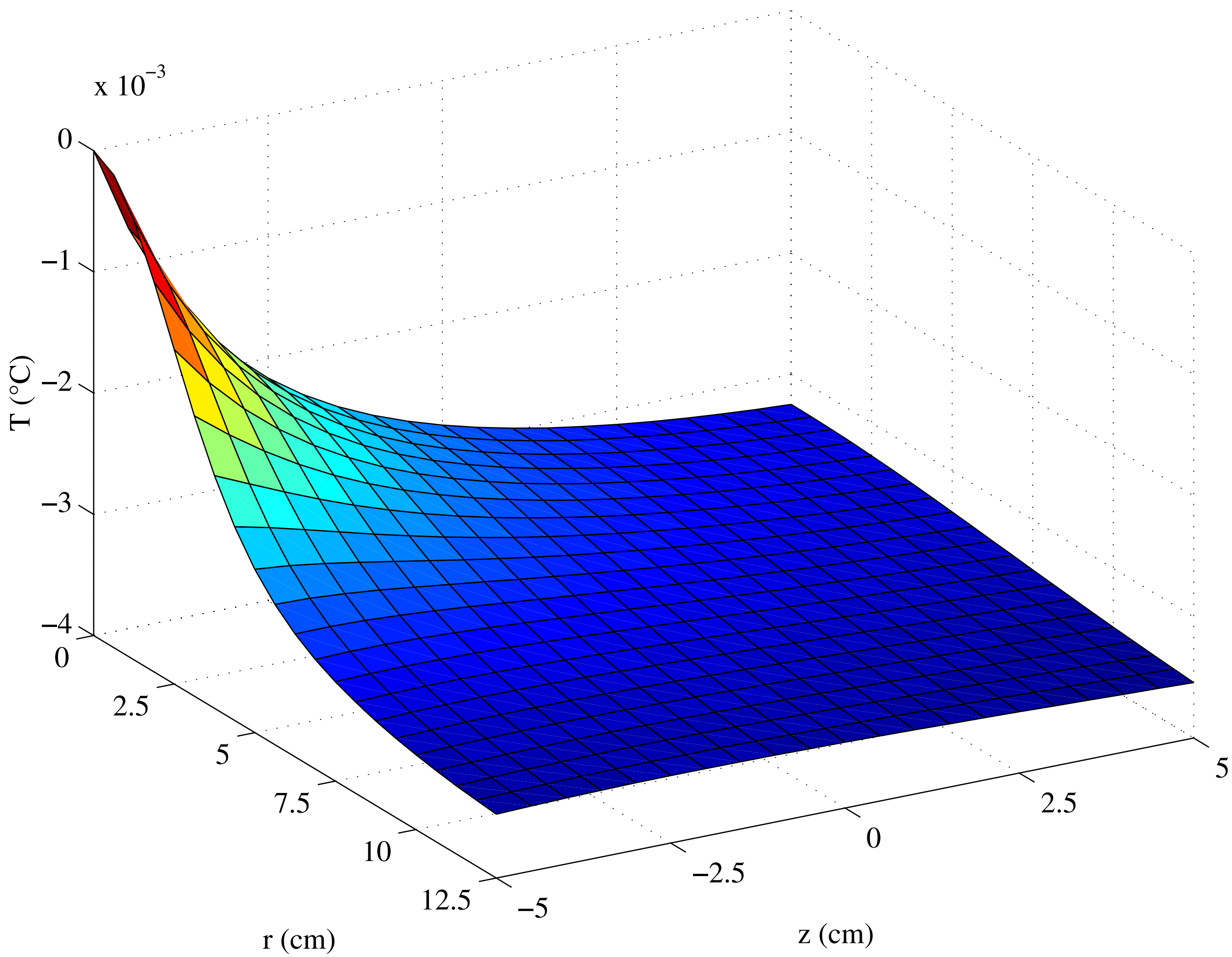
p_k: Normalized expansion coefficients

$$u_{00}^2 = \sum_{k=0}^{\infty} p_k\, J_0\left(\zeta_k \frac{r}{a}\right)$$

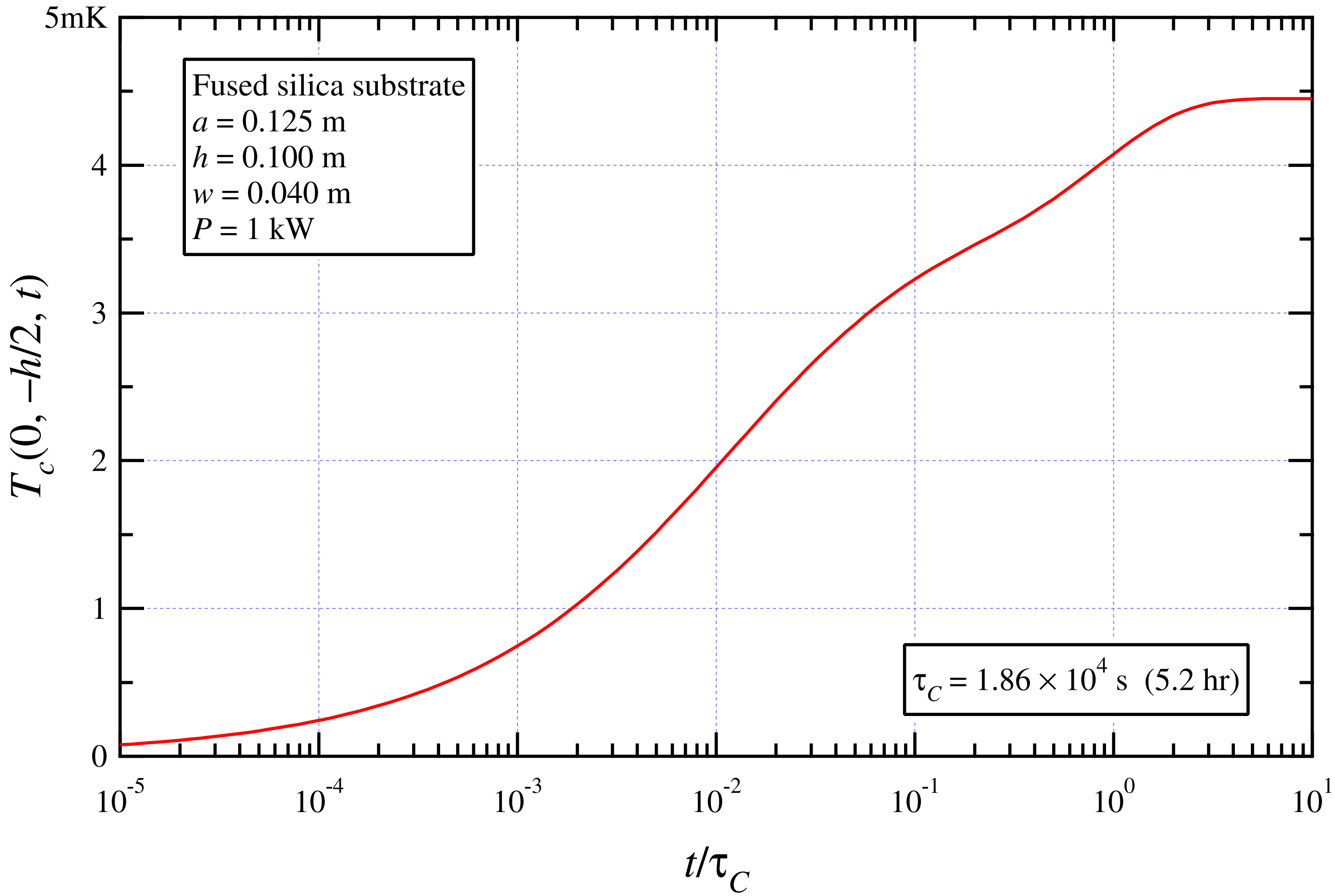
Since $(w/a)^2 \ll 1$,

$$p_k \approxeq \frac{P}{\pi a^2} \frac{\zeta_k^2}{\left(\zeta_k^2 + \tau^2\right) J_0^2(\zeta_k)} e^{-(\zeta_k w/a)^2/8}$$

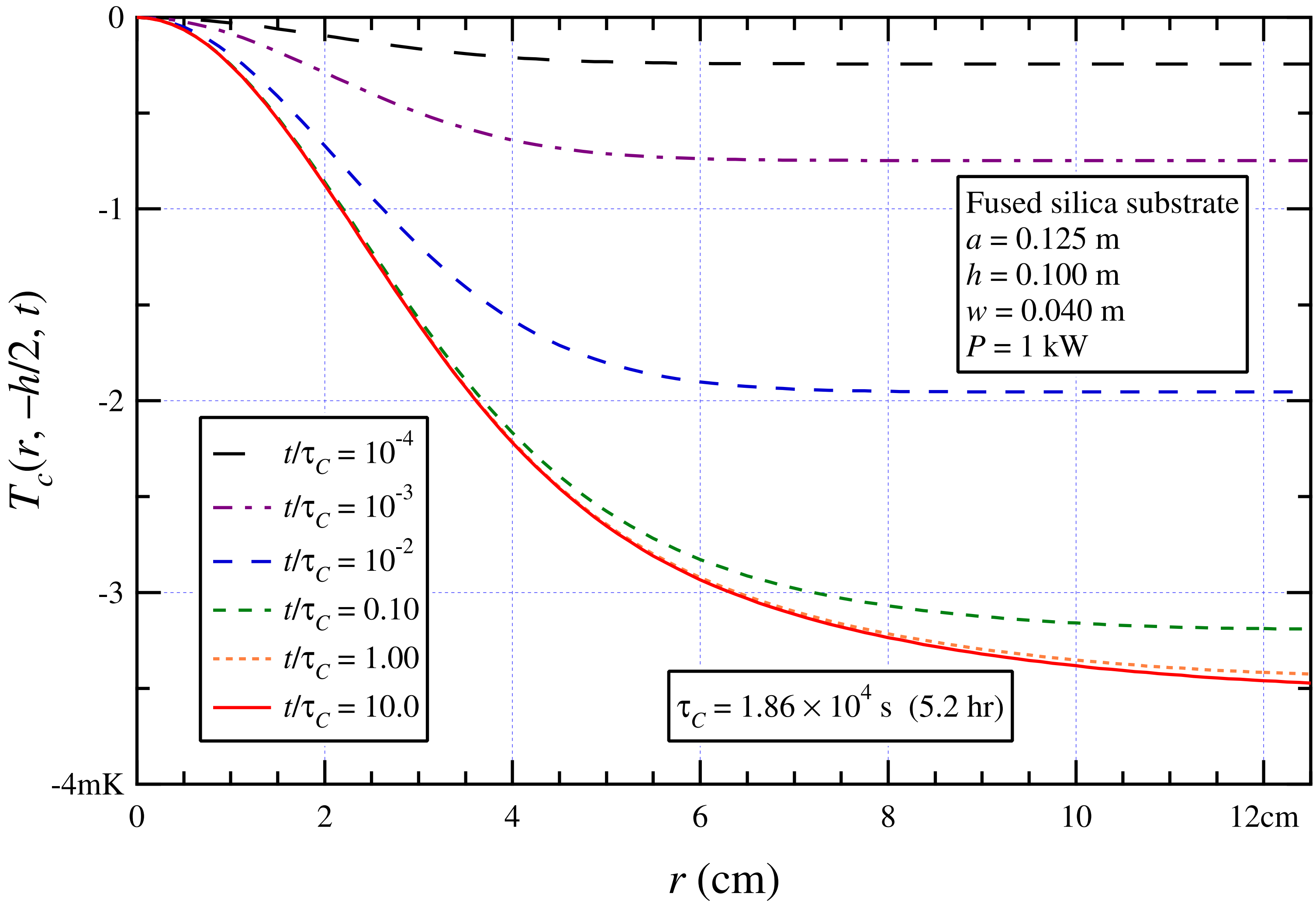
$T_C(r,z)$ FROM COATING ABSORPTION



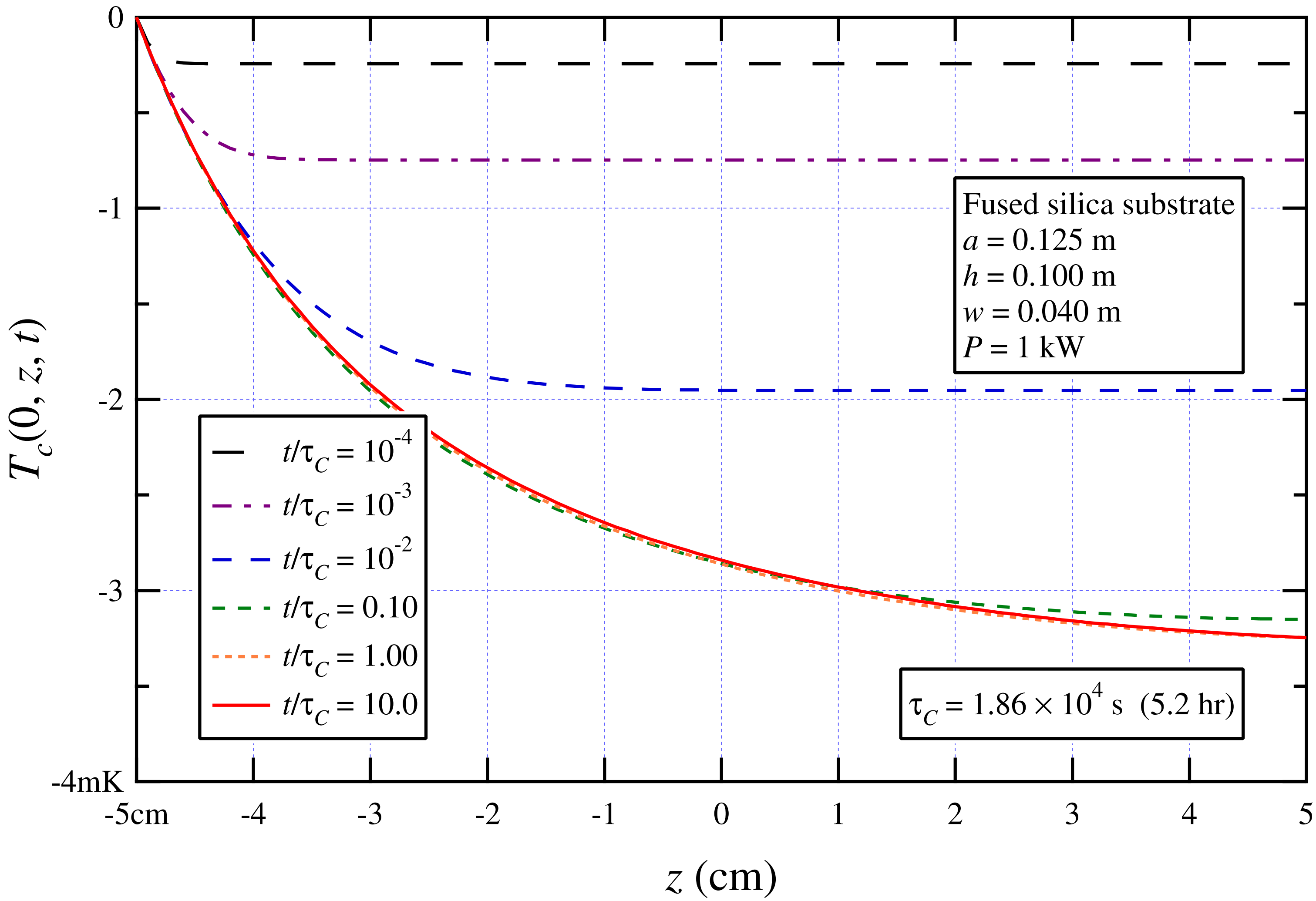
$T_C(0, -h/2, t)$ FROM COATING ABSORPTION



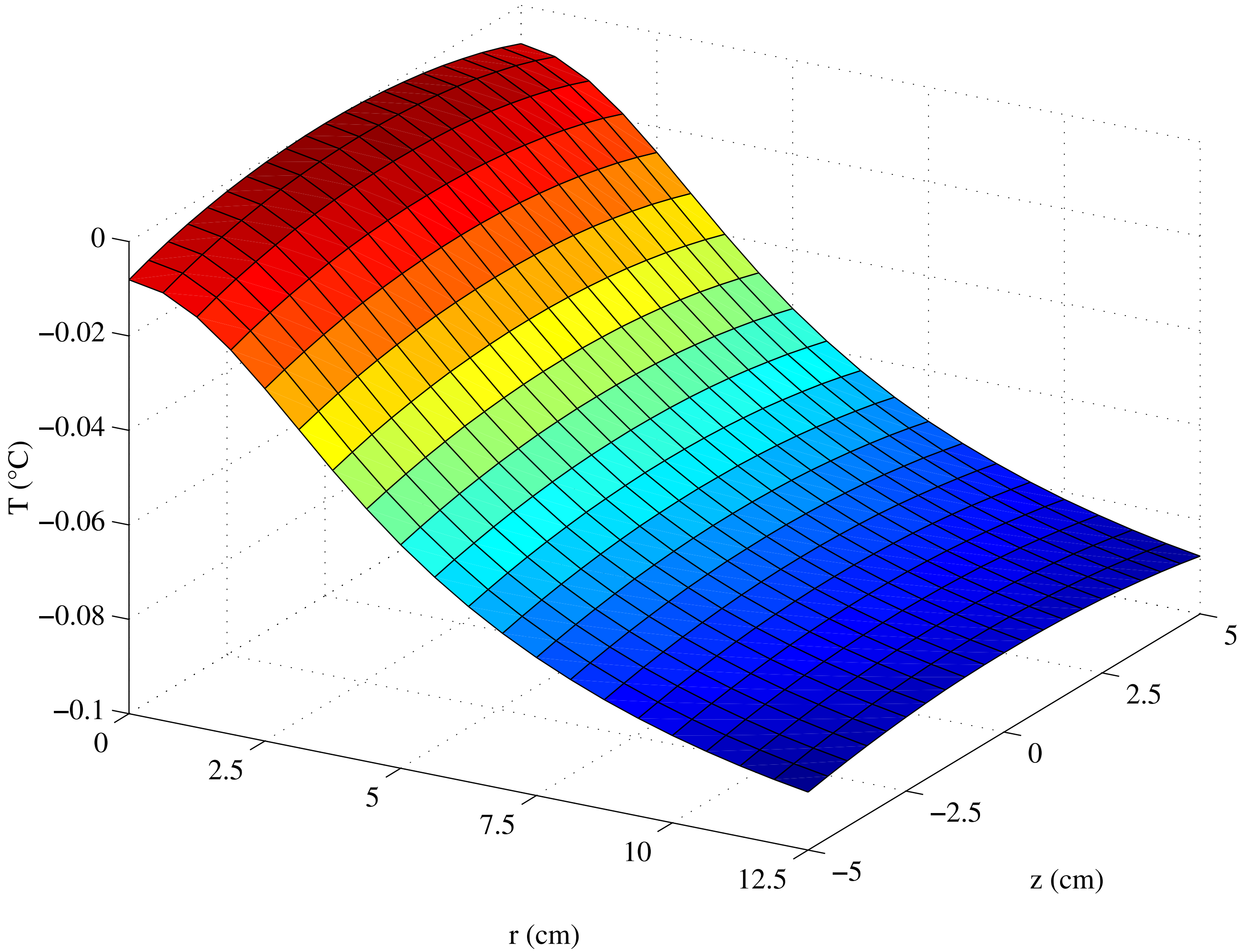
$T_C(r, -h/2, t)$ FROM COATING ABSORPTION



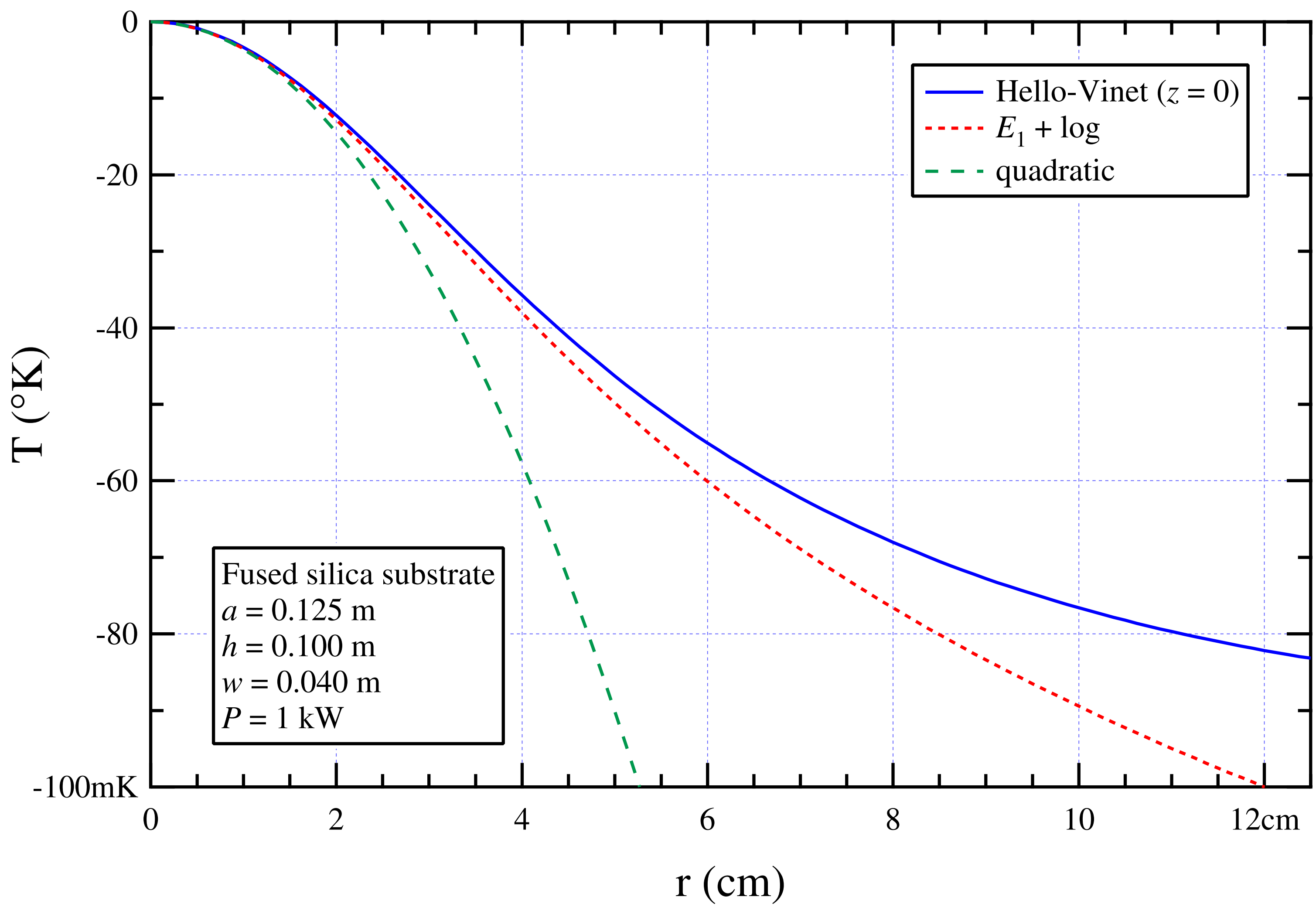
$T_C(0, z, t)$ FROM COATING ABSORPTION



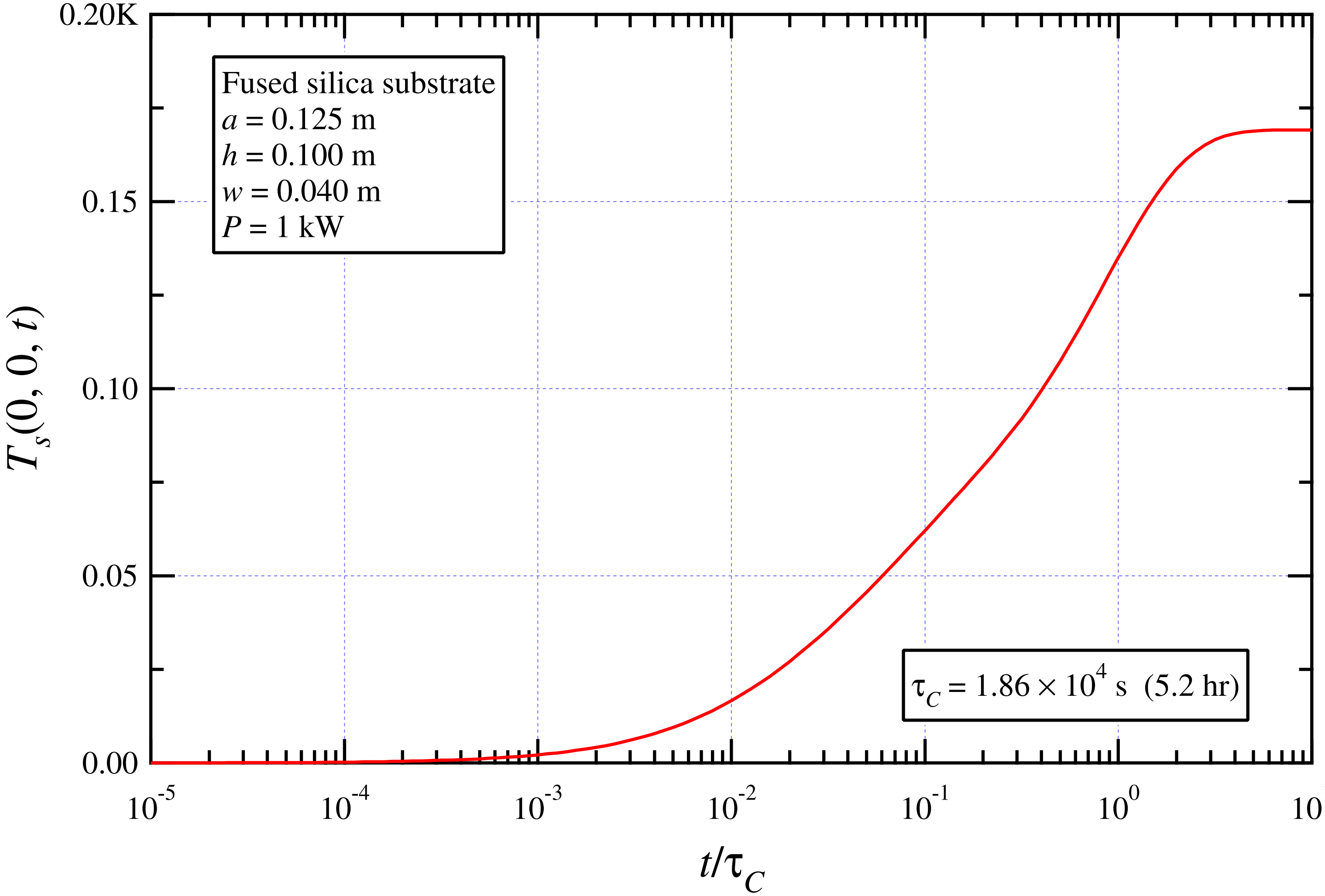
$T_S(r, z)$ FROM SUBSTRATE ABSORPTION

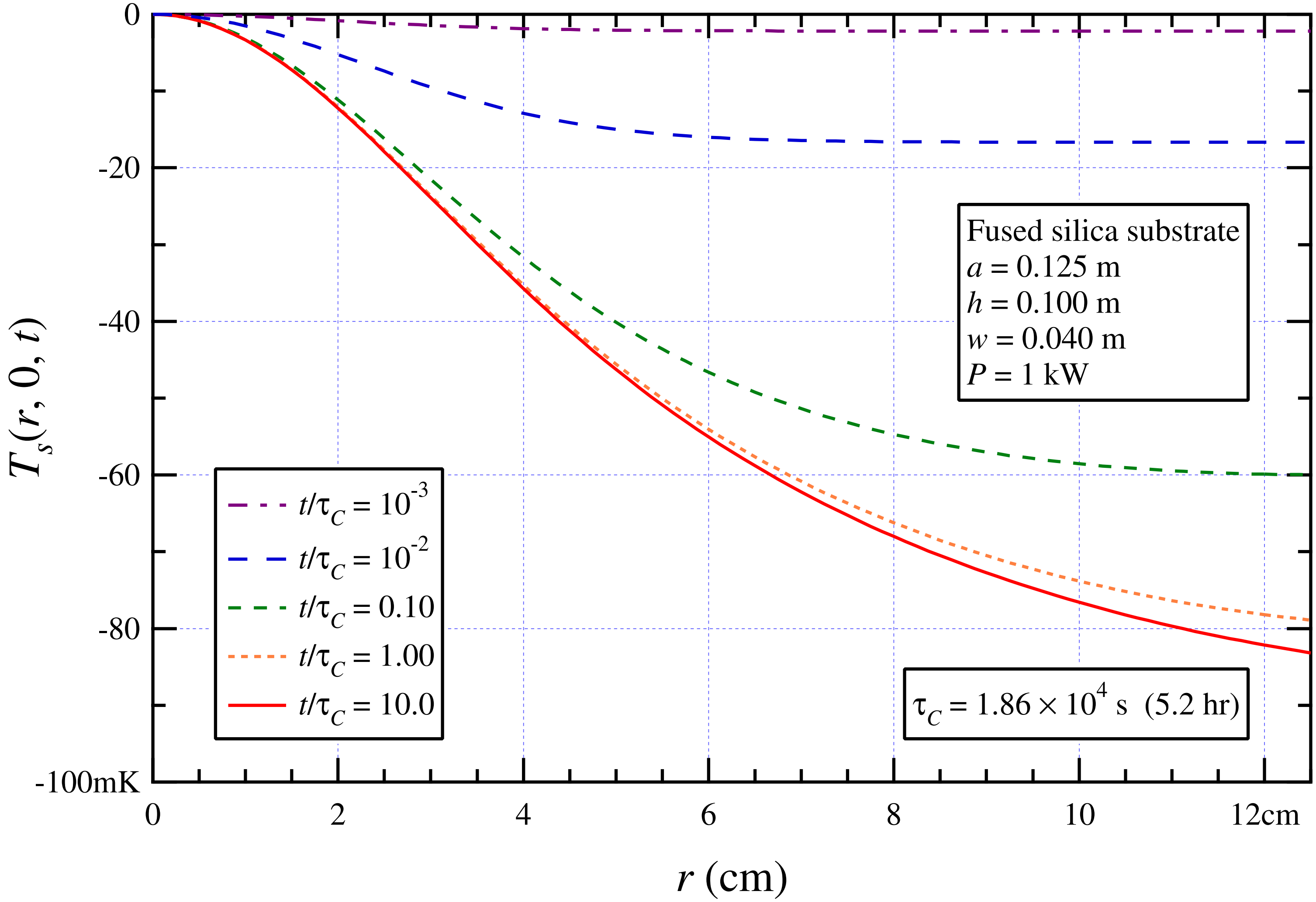


APPROXIMATE $T_S(r, 0)$ (SUBSTRATE ABSORPTION)



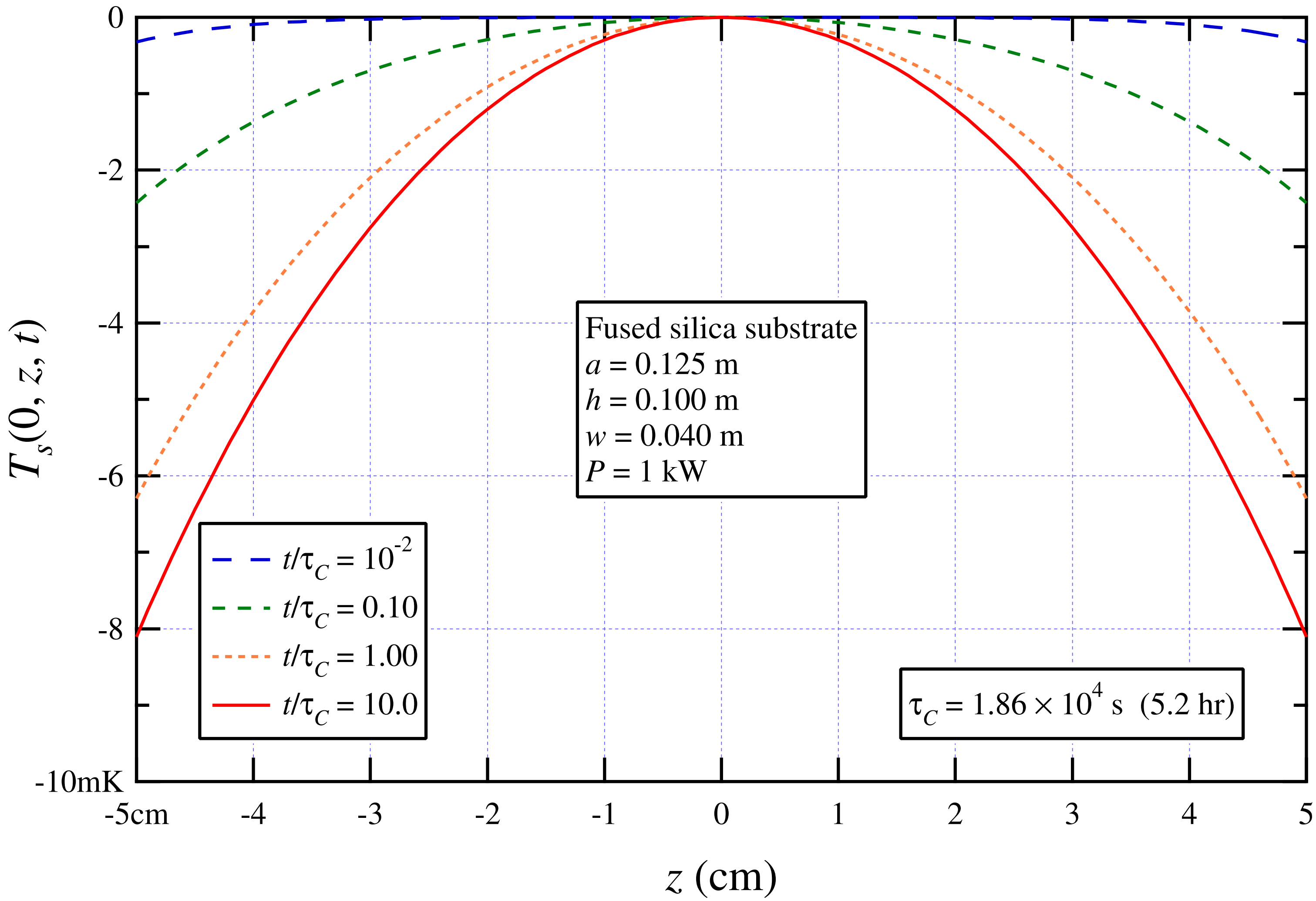
$T_S(0, 0, t)$ FROM SUBSTRATE ABSORPTION



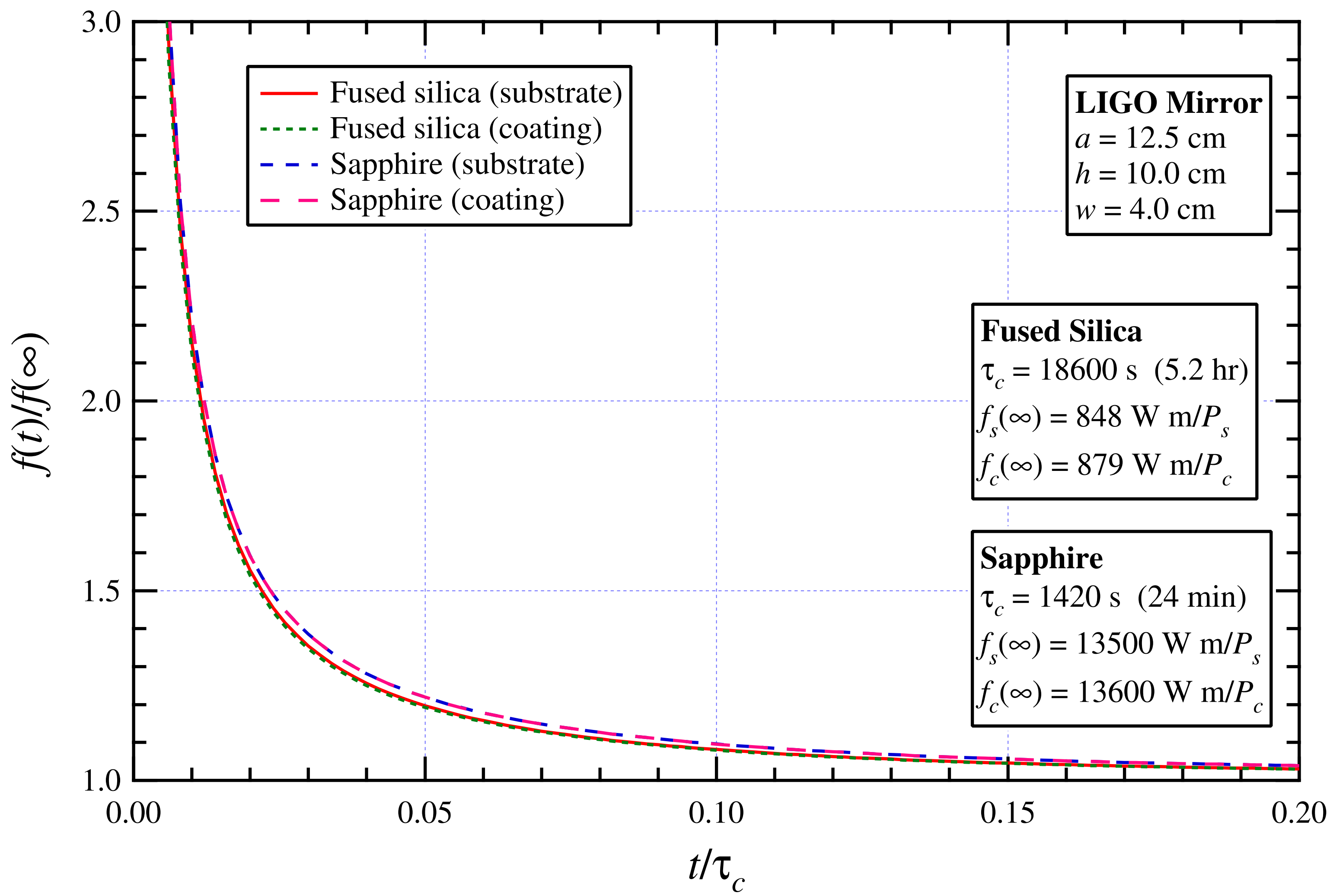


0
-20
-40
-60
-80
-100mK
$T_s(r, 0, t)$
0
2
4
6
8
10
12cm
r (cm)
Fused silica substrate
a = 0.125 m
h = 0.100 m
w = 0.040 m
P = 1 kW
$t/\tau_C = 10^{-3}$
$t/\tau_C = 10^{-2}$
$t/\tau_C = 0.10$
$t/\tau_C = 1.00$
$t/\tau_C = 10.0$
$\tau_C = 1.86 \times 10^4$ s (5.2 hr)

$T_S(0, z, t)$ FROM SUBSTRATE ABSORPTION



THERMAL FOCAL LENGTH



THERMAL LENS OPERATOR

The propagation phase perturbation due to the substrate OPD is

$$\phi(r) = \frac{2\pi}{\lambda_0} \frac{dn}{dT} \int_{-h/2}^{h/2} dz\, T(r, z)$$

where $T(r, z)$ is the *linear* sum of contributions from heating due to absorption in both coatings (HR and AR) and the substrate.

Matrix elements of the thermal lens operator (per unit power absorbed):

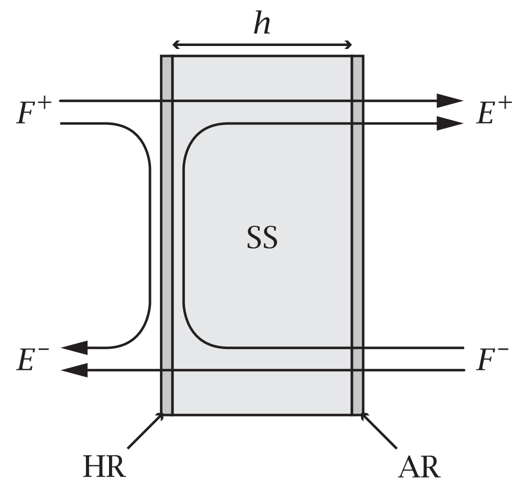
$$\Phi_{m'n';mn} = \iint_{-\infty}^{\infty} dx\,dy\, u^{\dagger}_{m'n'}(x, y)\, u_{mn}(x, y)\, \phi(r)$$

Since $\phi(r) \propto r^2$, TEM_{00} is coupled to both TEM_{20} and TEM_{02}.

FINAL THERMAL LENS MATRICES

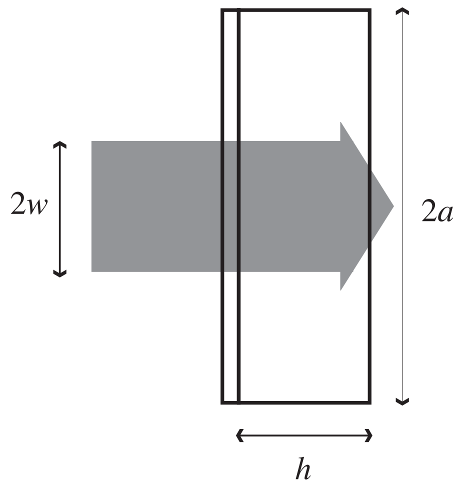
Unitary approximation:

$$S = \exp\{i\,[P_s\,\Phi_s + (P_h + P_a)\,\Phi_c]\}$$



Region	Absorbed Power
SS	$P_s = \alpha_s h\,\frac{1}{2}\sum_q \left(\lvert E^+_{00q}\rvert^2 + \lvert F^-_{00q}\rvert^2\right)$
HR	$P_h = a_{hr}\,\frac{1}{2}\sum_q \left(\lvert F^+_{00q}\rvert^2 + \lvert F^-_{00q}\rvert^2\right)$
AR	$P_a = a_{ar}\,\frac{1}{2}\sum_q \left(\lvert E^+_{00q}\rvert^2 + \lvert F^-_{00q}\rvert^2\right)$

HELLO-VINET THERMOELASTIC SURFACE DEFORMATION



Reference: P. Hello and J.-Y. Vinet,

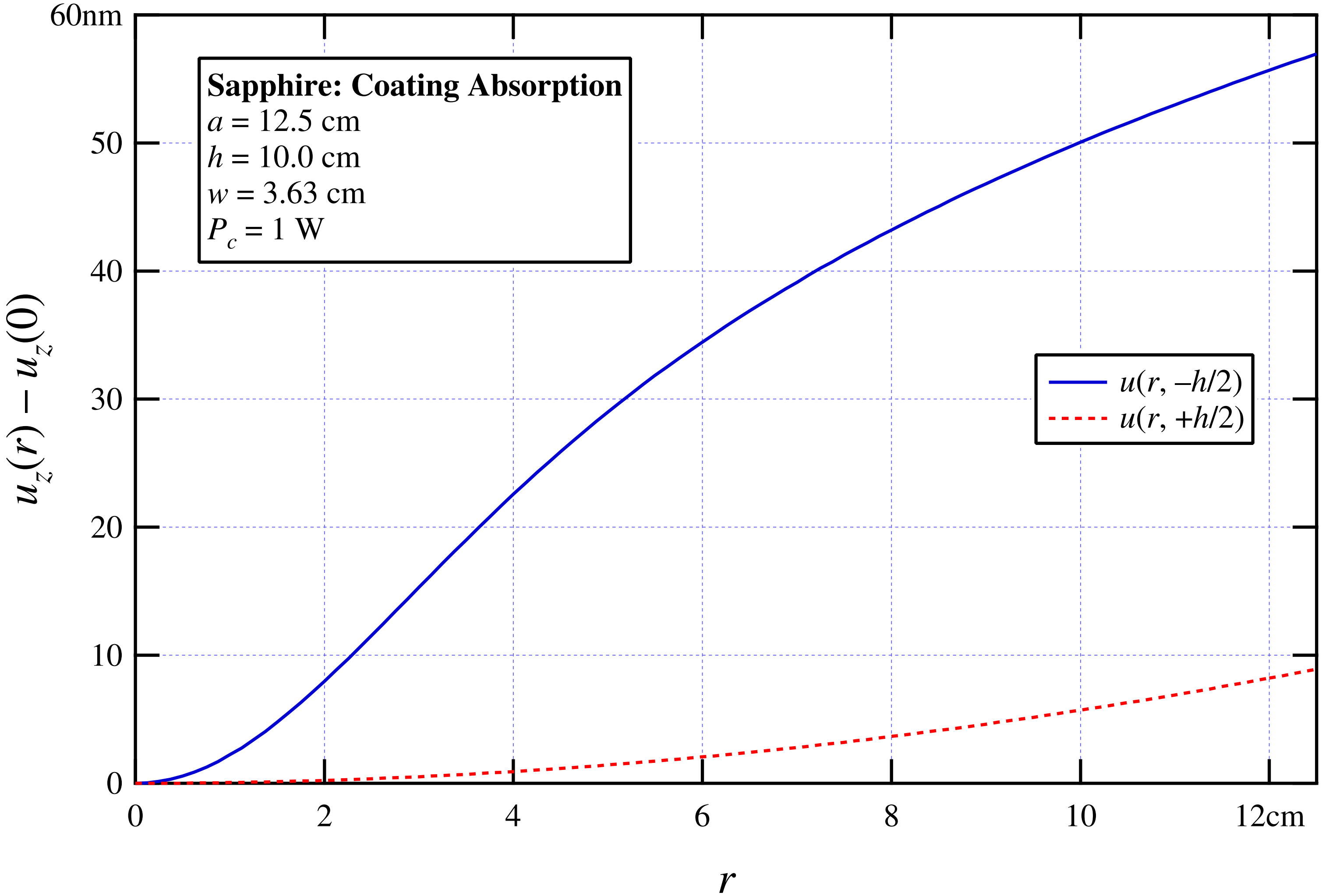
J. Phys. France 51, 2243 (1990)

$$\gamma_k \equiv \zeta_k h/2a$$

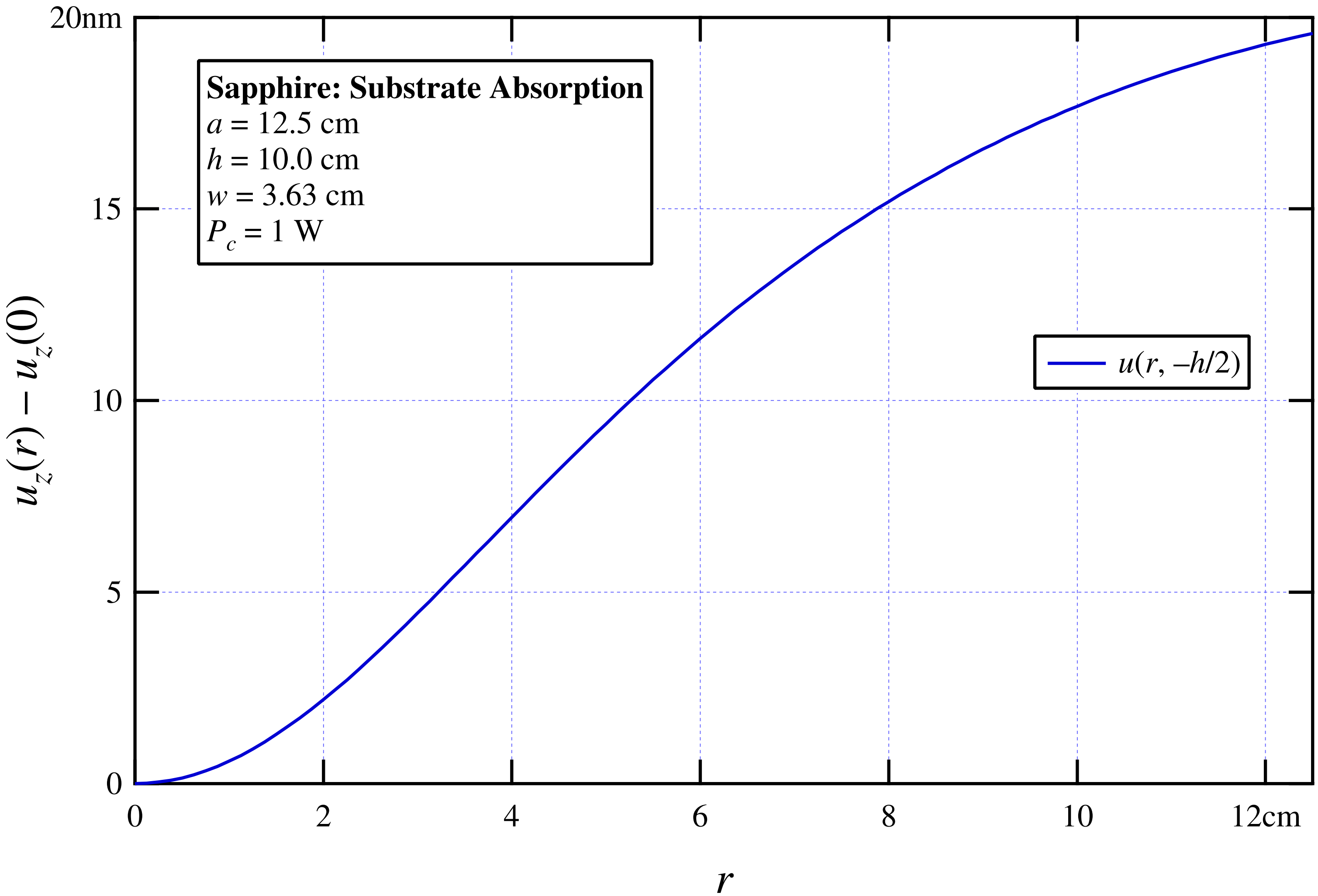
$$u_c\left(r, \pm\frac{h}{2}\right) = \mp P_c \frac{(1+\nu)\alpha_T}{k_T} \sum_{k=0}^{\infty} \frac{a^2 p_k}{\zeta_k} \left[A_k \sinh(\gamma_k) \pm B_k \cosh(\gamma_k)\right] \times J_0\left(\zeta_k \frac{r}{a}\right) - \frac{1}{2}\frac{(1-\nu)\alpha_T}{k_T} C_1 \left(\frac{r}{a}\right)^2$$

$$u_s\left(r, \pm\frac{h}{2}\right) = \mp P_s \frac{(1+\nu)\alpha_T}{k_T} \sum_{k=0}^{\infty} \frac{a^2 p_k}{\zeta_k^2} \frac{\sinh(\gamma_k)}{\gamma_k} \times \left[\tau A_k - \frac{\sinh(\gamma_k)}{\gamma_k + \cosh(\gamma_k)\sinh(\gamma_k)}\right] J_0\left(\zeta_k \frac{r}{a}\right)$$

SAPPHIRE COATING ABSORPTION



SAPPHIRE SUBSTRATE ABSORPTION



THERMAL DEFORMATION OPERATOR

The propagation phase perturbation due to the surface OPD at the HR is

$$\phi(r, \pm h/2) = \frac{2\pi}{\lambda_0} \Delta n(z)\, u(r, \pm h/2)$$

where $u(r, \pm h/2)$ is the *linear* sum of contributions from deformations due to absorption in both coatings (HR and AR) and the substrate.

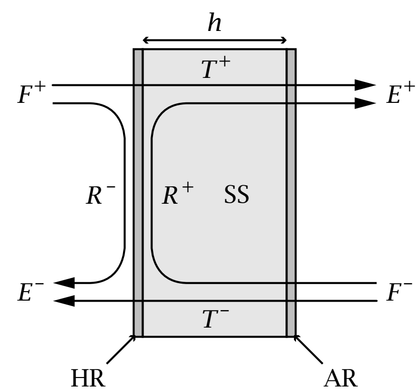
Matrix elements of the thermal deformation operator (unitary approx):

$$\begin{aligned} U_{m'n';mn} &= \iint_{-\infty}^{\infty} dx dy\, u^{\dagger}_{m'n'}(x, y)\, u_{mn}(x, y)\, e^{i\phi(r)} \\ &\equiv \exp\left[i \iint_{-\infty}^{\infty} dx dy\, u^{\dagger}_{m'n'}(x, y)\, u_{mn}(x, y)\, \phi(r) \right] \end{aligned}$$

Since $\phi(r) \propto r^2$, TEM_{00} is coupled to both TEM_{20} and TEM_{02}.

MIRROR TRANSFER MATRIX

$$\begin{bmatrix} E^- \\ E^+ \end{bmatrix} = \begin{bmatrix} T^- & R^- \\ R^+ & T^+ \end{bmatrix} \begin{bmatrix} F^- \\ F^+ \end{bmatrix}$$

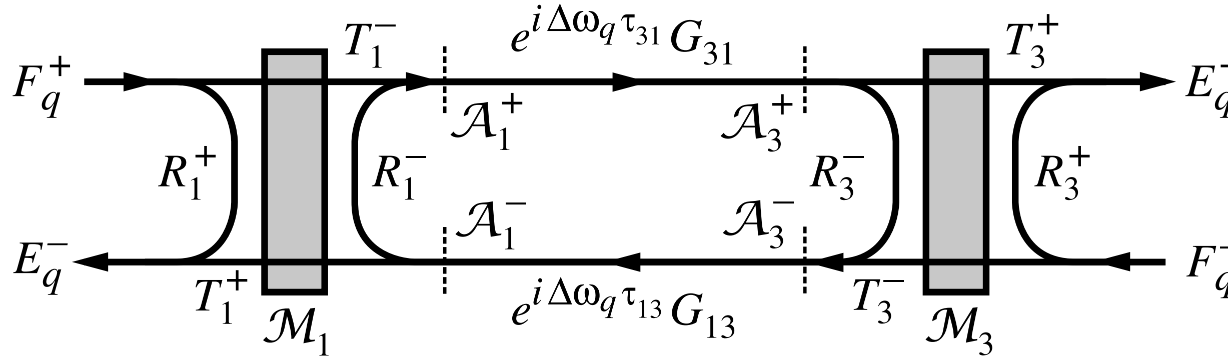


$$T^- = i\, t\, t_s\, (C^- C^+)^{1/2}\, S\, U\, A$$

$$T^+ = i\, t\, t_s\, U\, S\, (C^+ C^-)^{1/2}\, A$$

$$R^- = -r\, e^{-i\,2k\,\Delta z}\, C^-\, A, \text{ and}$$

$$R^+ = -r\, t_s^2\, e^{+i\,2k\,\Delta z}\, U\, S\, C^+\, S\, U\, A$$



Transfer matrix:

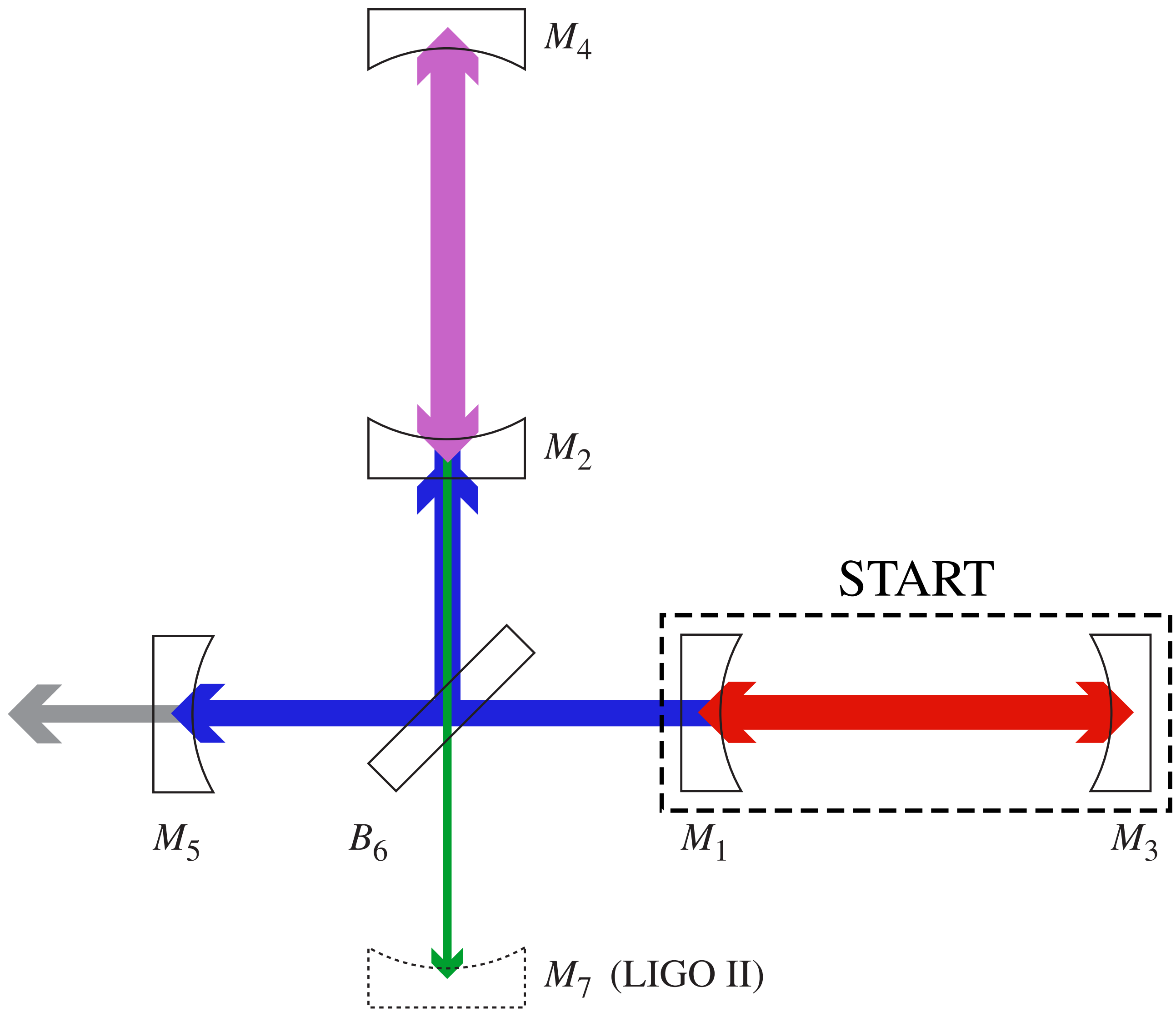
$$\begin{bmatrix} E_q^- \\ E_q^+ \end{bmatrix} = \begin{bmatrix} T_{\rm FPI}^-(\Delta\,\omega_q) & R_{\rm FPI}^-(\Delta\,\omega_q) \\ R_{\rm FPI}^+(\Delta\,\omega_q) & T_{\rm FPI}^+(\Delta\,\omega_q) \end{bmatrix} \begin{bmatrix} F_q^- \\ F_q^+ \end{bmatrix},$$

Example:

$$R_{\rm FPI}^-(\Delta\,\omega) = R_1^+ + T_1^+\,H_1^+(\Delta\,\omega)$$

$$H_1^+(\Delta\,\omega) = \left(1 - e^{i\,2\,\Delta\,\omega\,\tau_{13}} G_{13}\,R_3^-\,G_{31}\,R_1^-\right)^{-1} e^{i\,2\,\Delta\,\omega\,\tau_{13}} G_{13}\,R_3^-\,G_{31}\,T_1^-$$

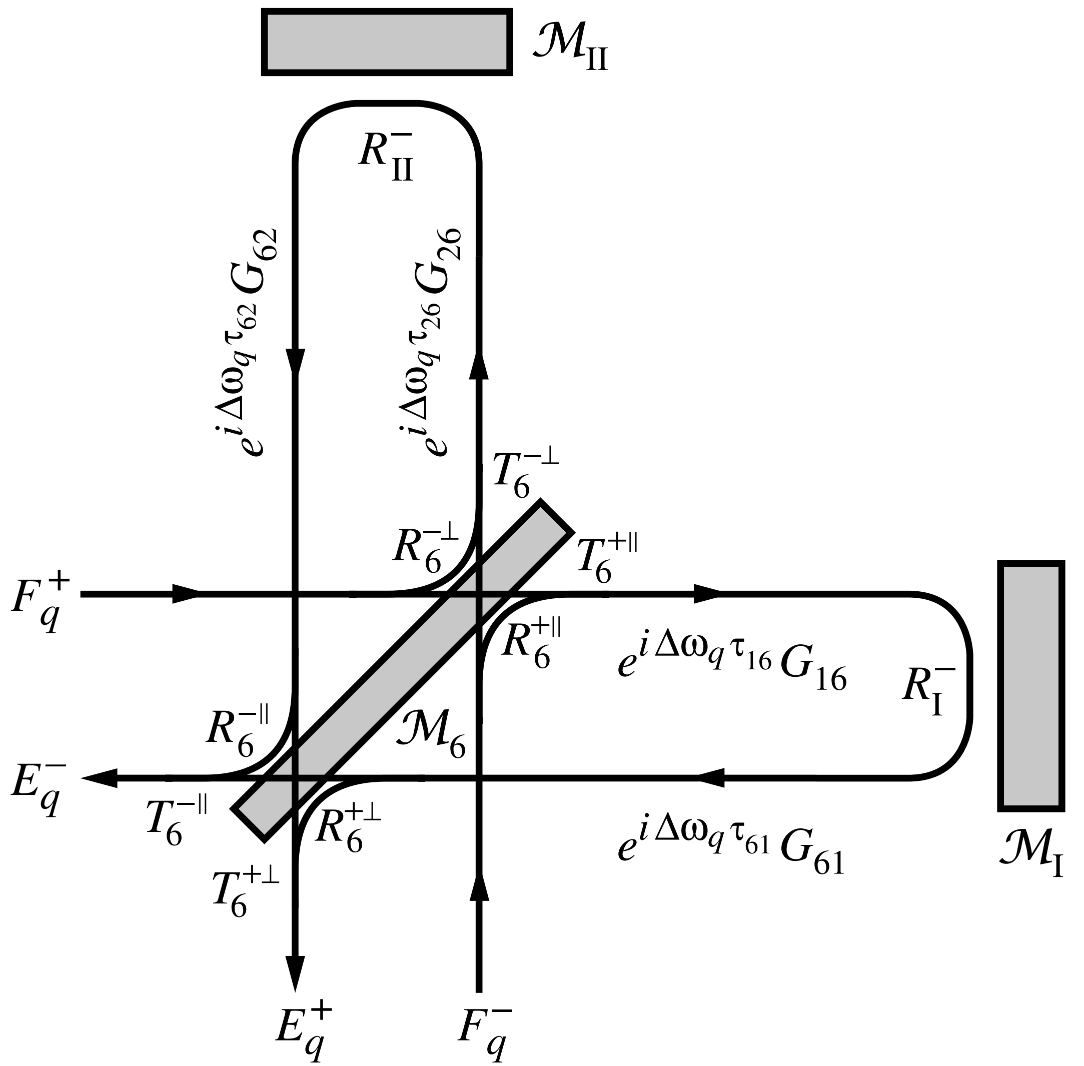
IFO COUPLING SCHEMATIC



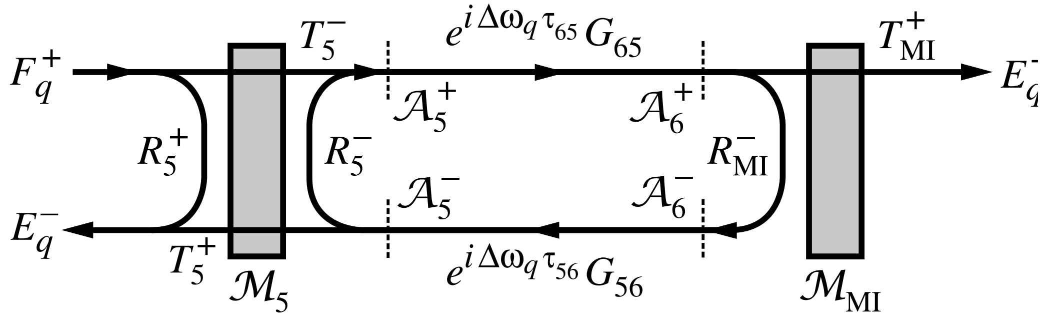
MODEL OF IFO COUPLING

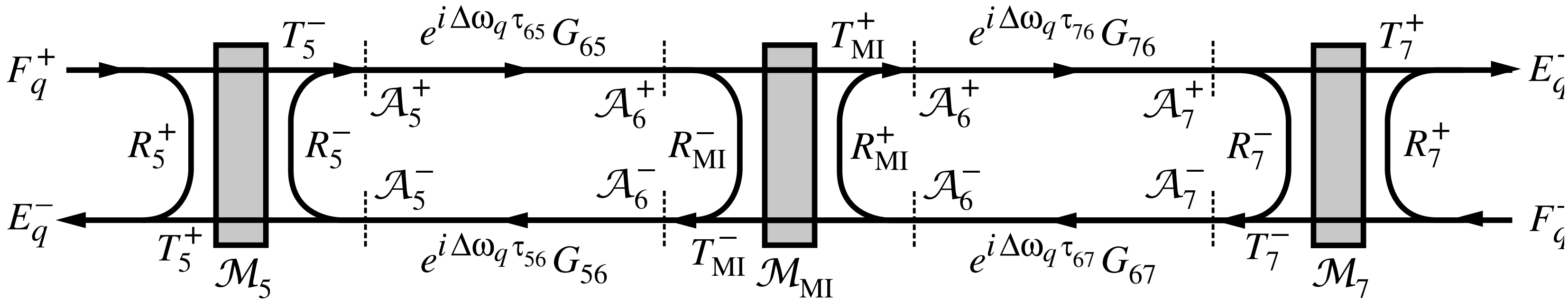
- Choose a *primary* FPI as a reference cavity; initial mirror properties define the "fundamental" unperturbed eigenmodes → basis functions
- Propagate the unperturbed basis functions from the FPI to the PRM and SRM; choose wavefront curvatures at mirror reference planes as unperturbed mirror curvatures
- Propagate basis to the secondary FPI; load FPI with fundamental basis
- Propagate out through the PRM to define a basis for the input field
- Provides a basis for the recycled fields even if recycling cavities are unstable

MICHELSON INTERFEROMETER TRANSFER MATRIX

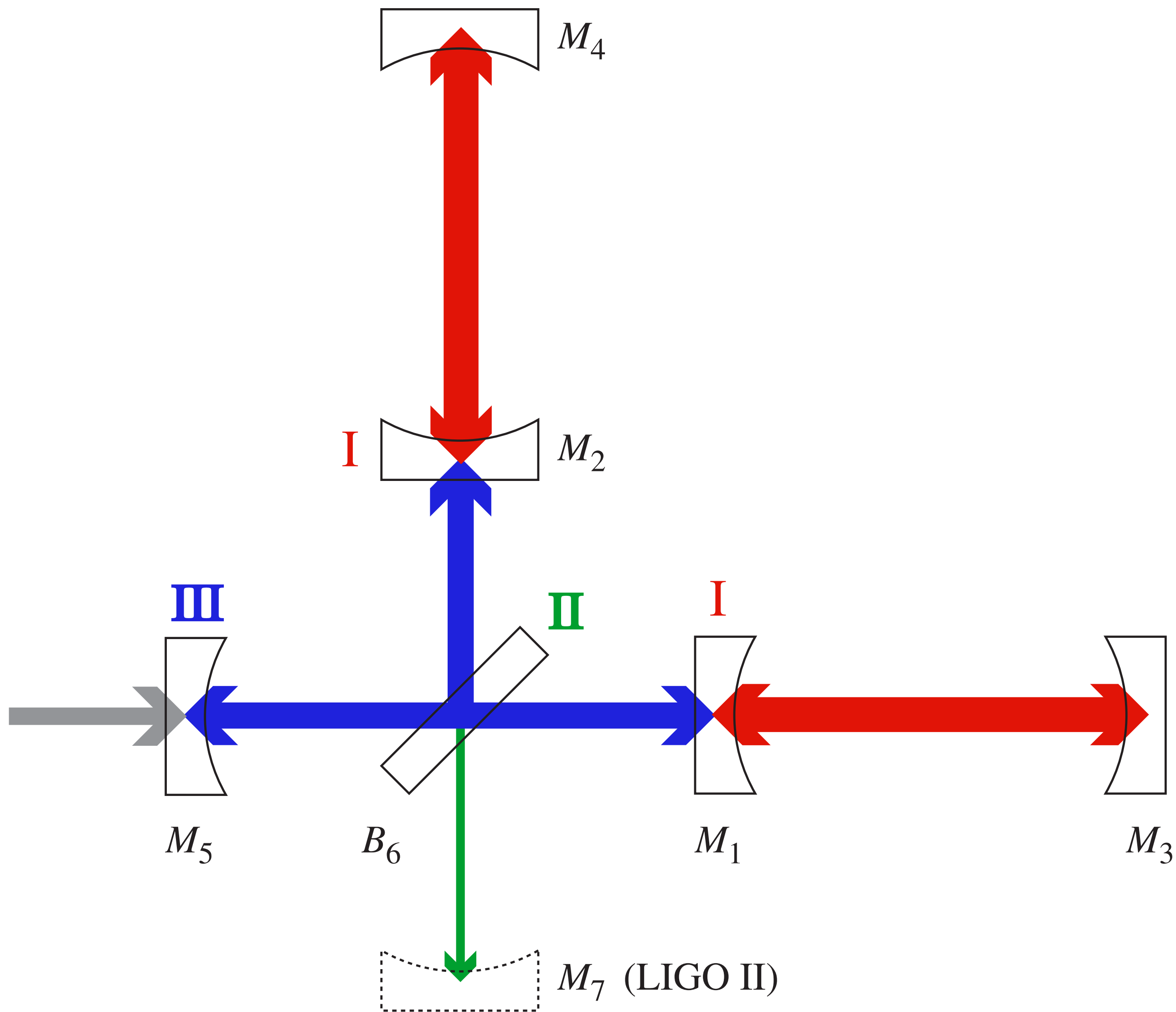


LIGO INTERFEROMETER TRANSFER MATRIX





RESONATOR LENGTH PSEUDOLOCKING

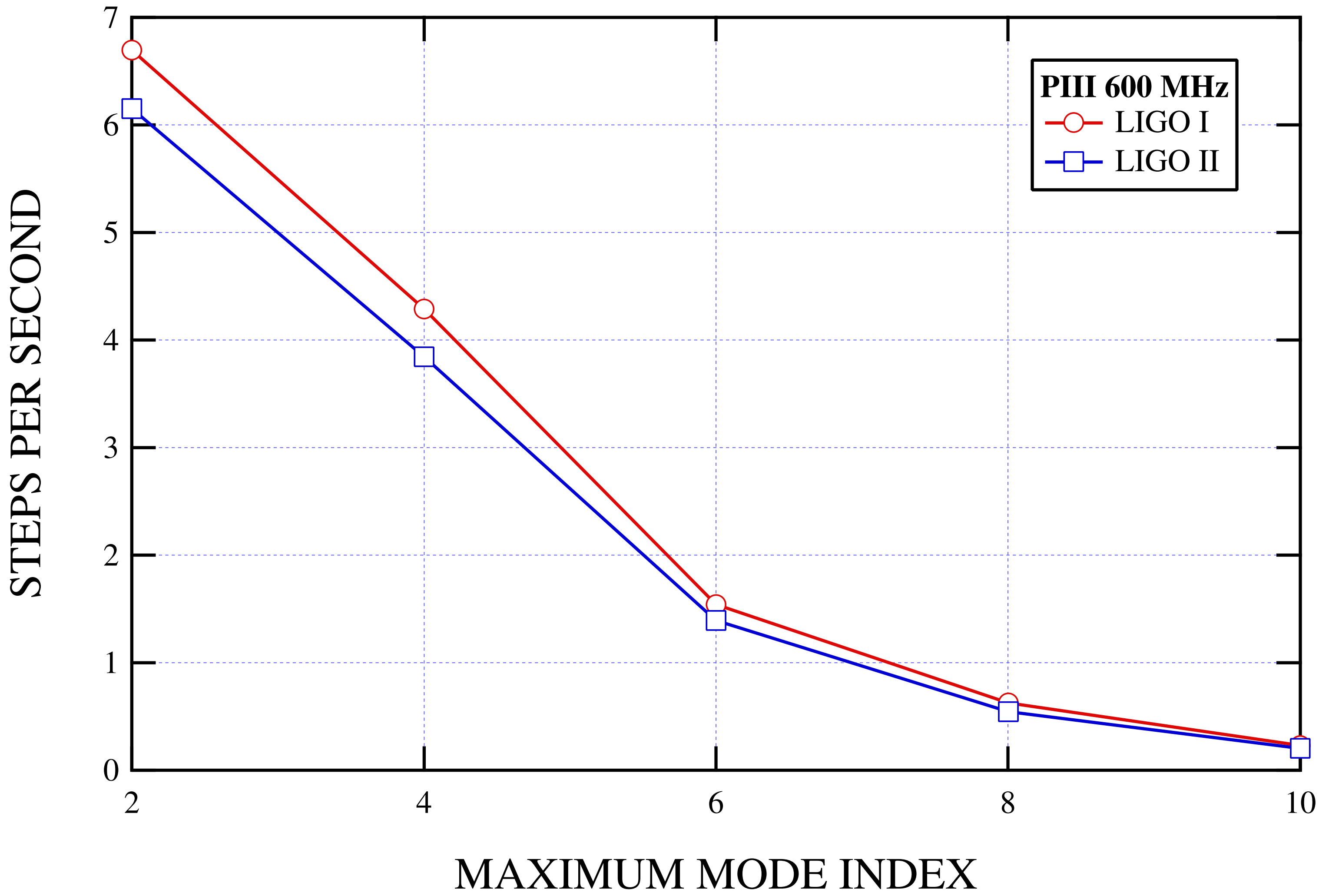


RESONATOR LENGTH PSEUDOLOCKING

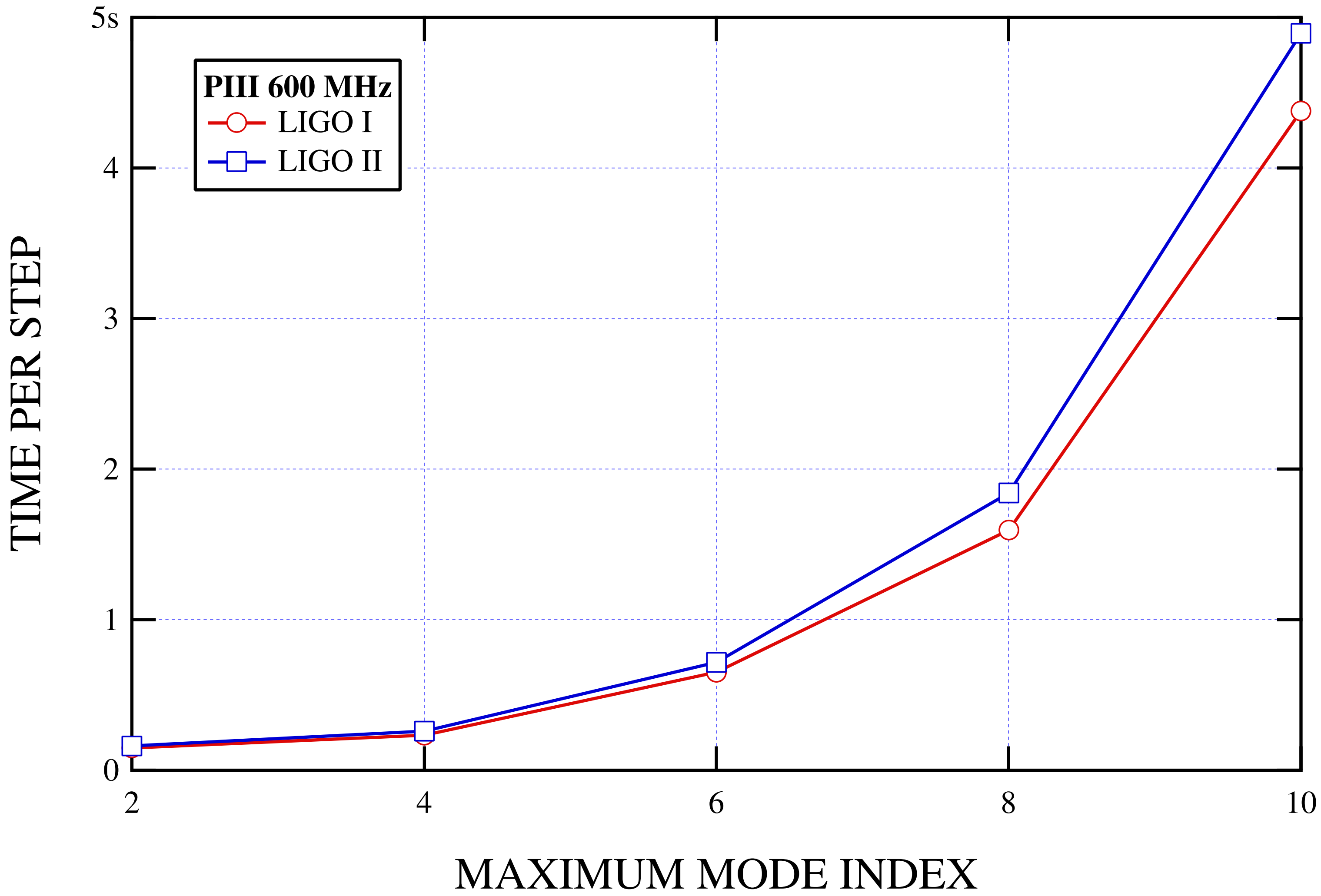
Self-contained simulations: implicit four-stage *pseudolocker*

1. *FPI* stage adjusts the positions of the FPI ITMs to maximize round-trip carrier TEM_{00} enhancement.
2. *Dark Port* stage adjusts the beamsplitter position so that the amplitude of the carrier TEM_{00} mode is minimized at the dark port.
3. *Power Recycling* stage adjusts the position of the PR mirror to maximize carrier TEM_{00} enhancement.
4. *Signal Recycling* stage adjusts the position of the SR mirror to optimize carrier TEM_{00} phase at the SR.

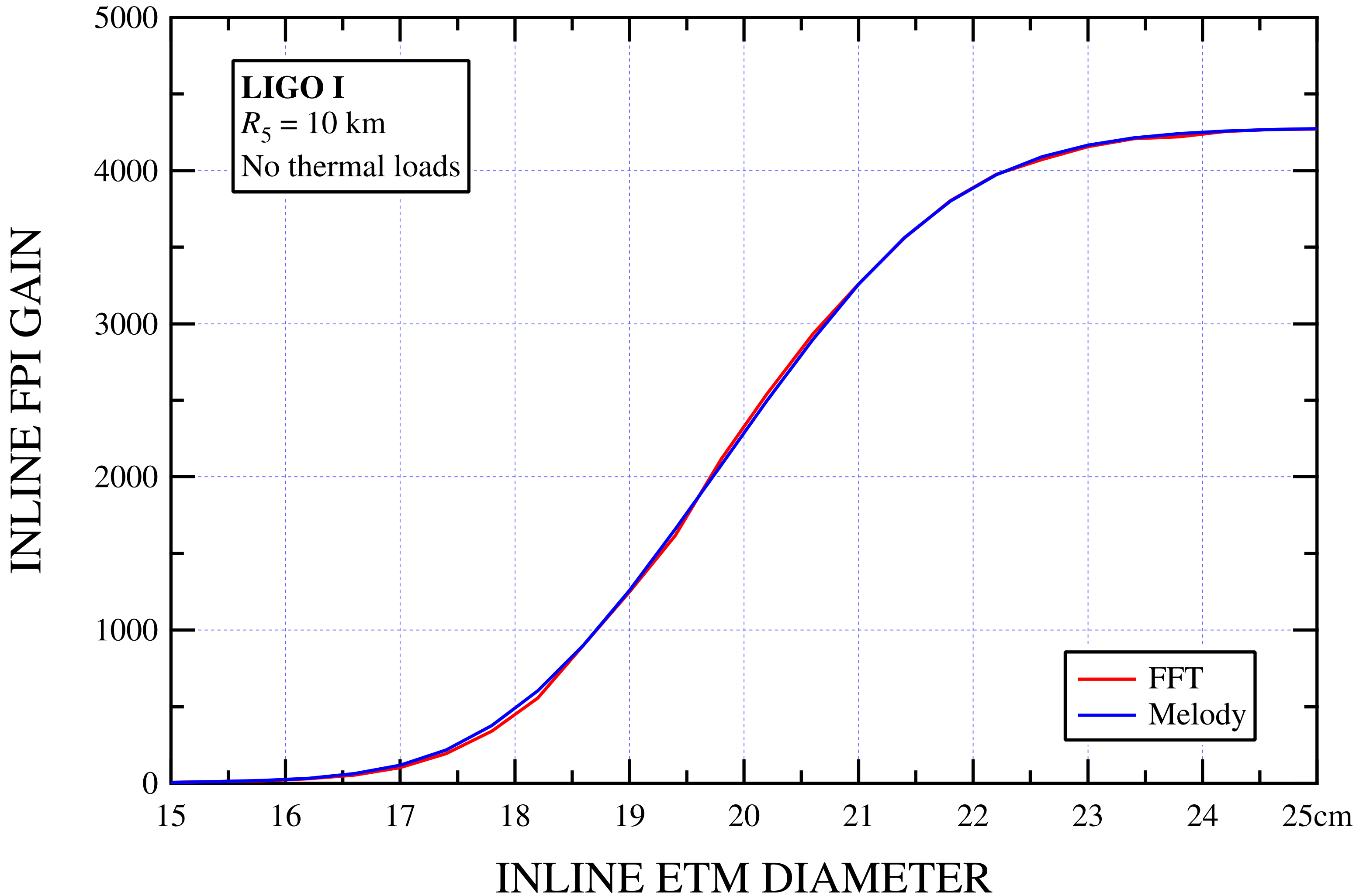
MELODY STEP RATE



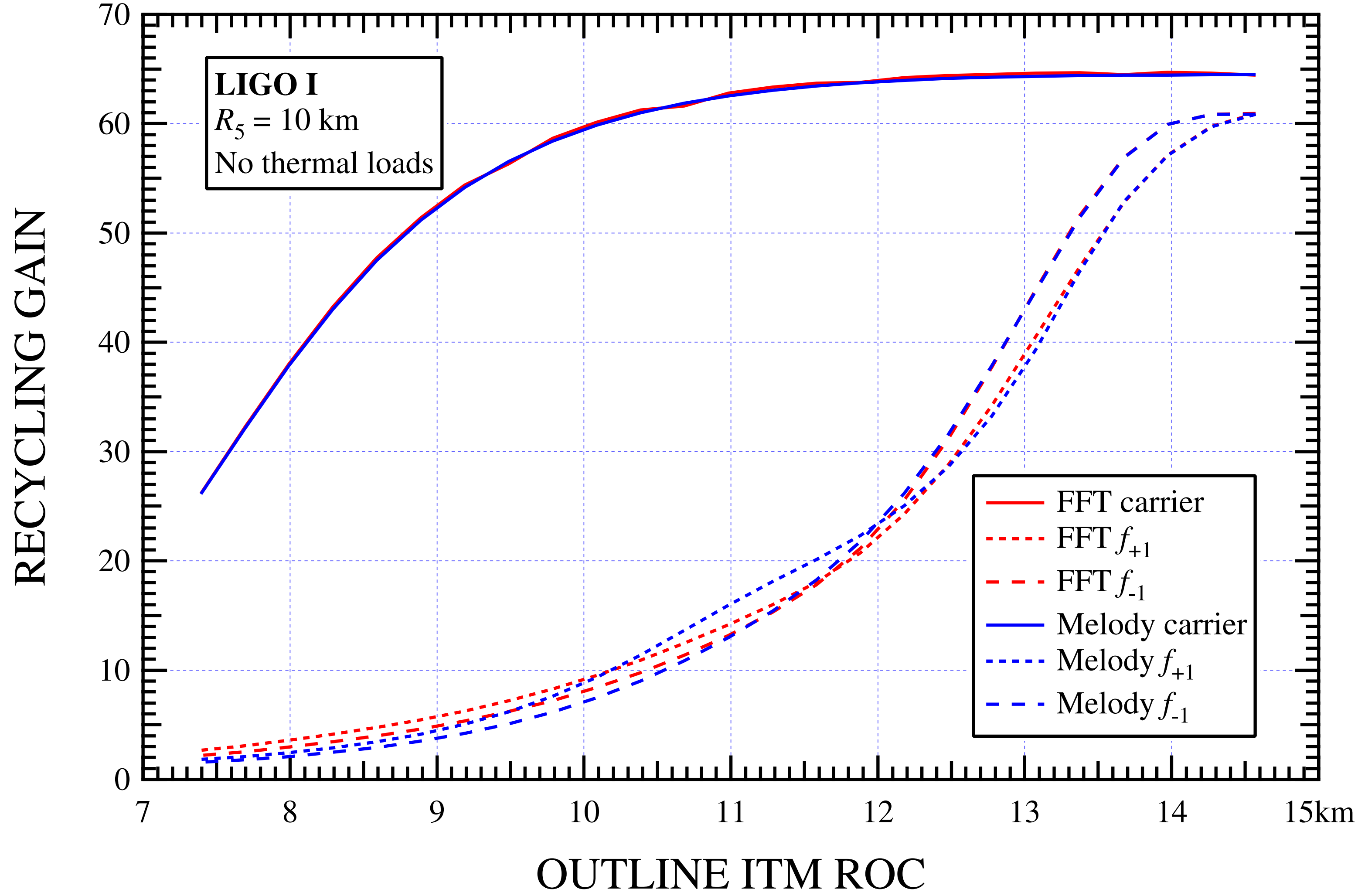
MELODY STEP TIME



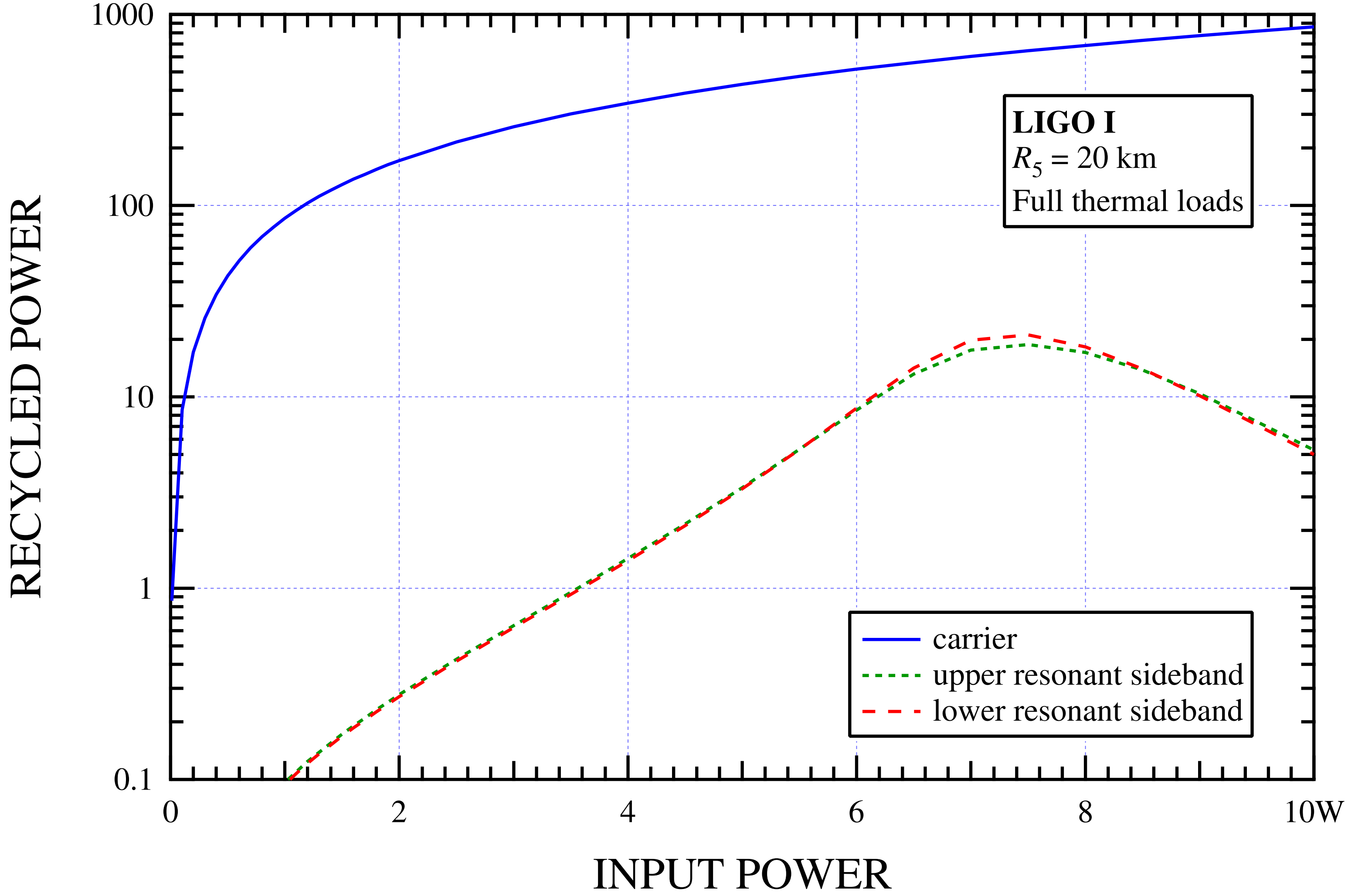
MELODY/FFT APERTURE COMPARISON



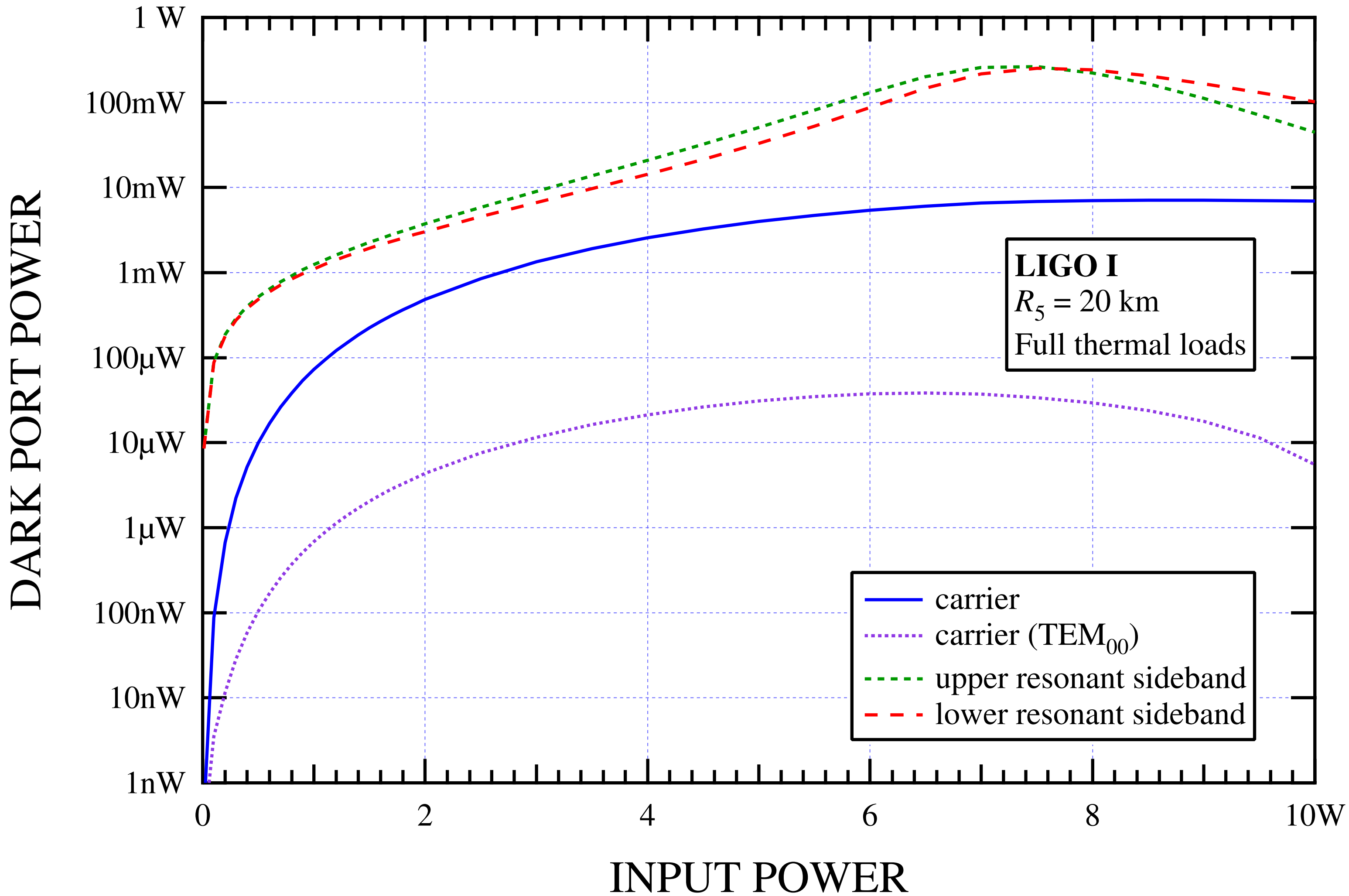
MELODY/FFT ROC COMPARISON



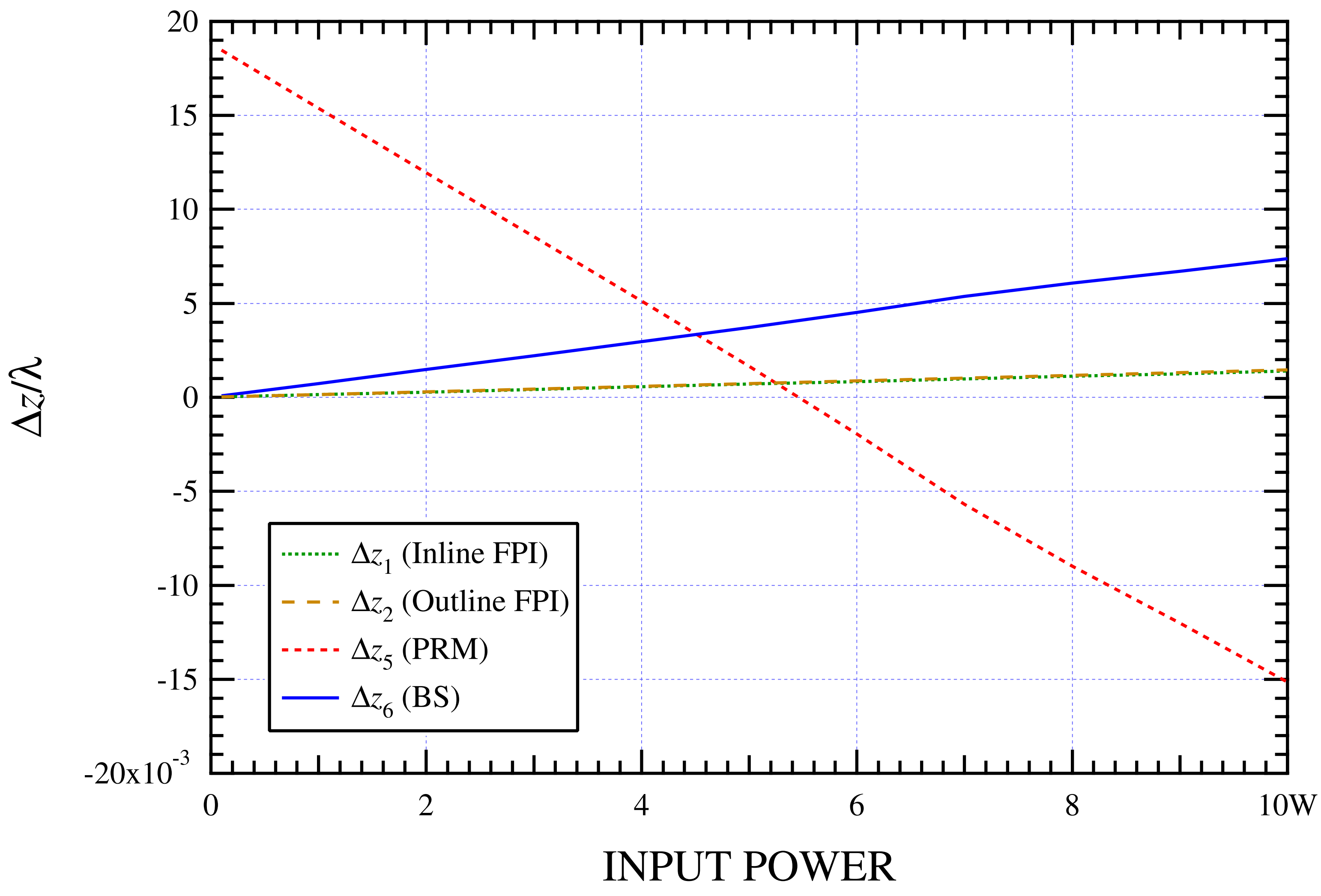
LIGO I: HOT, $R_5 = 20$ km, $E_0 \equiv \mathrm{TEM}_{00}$



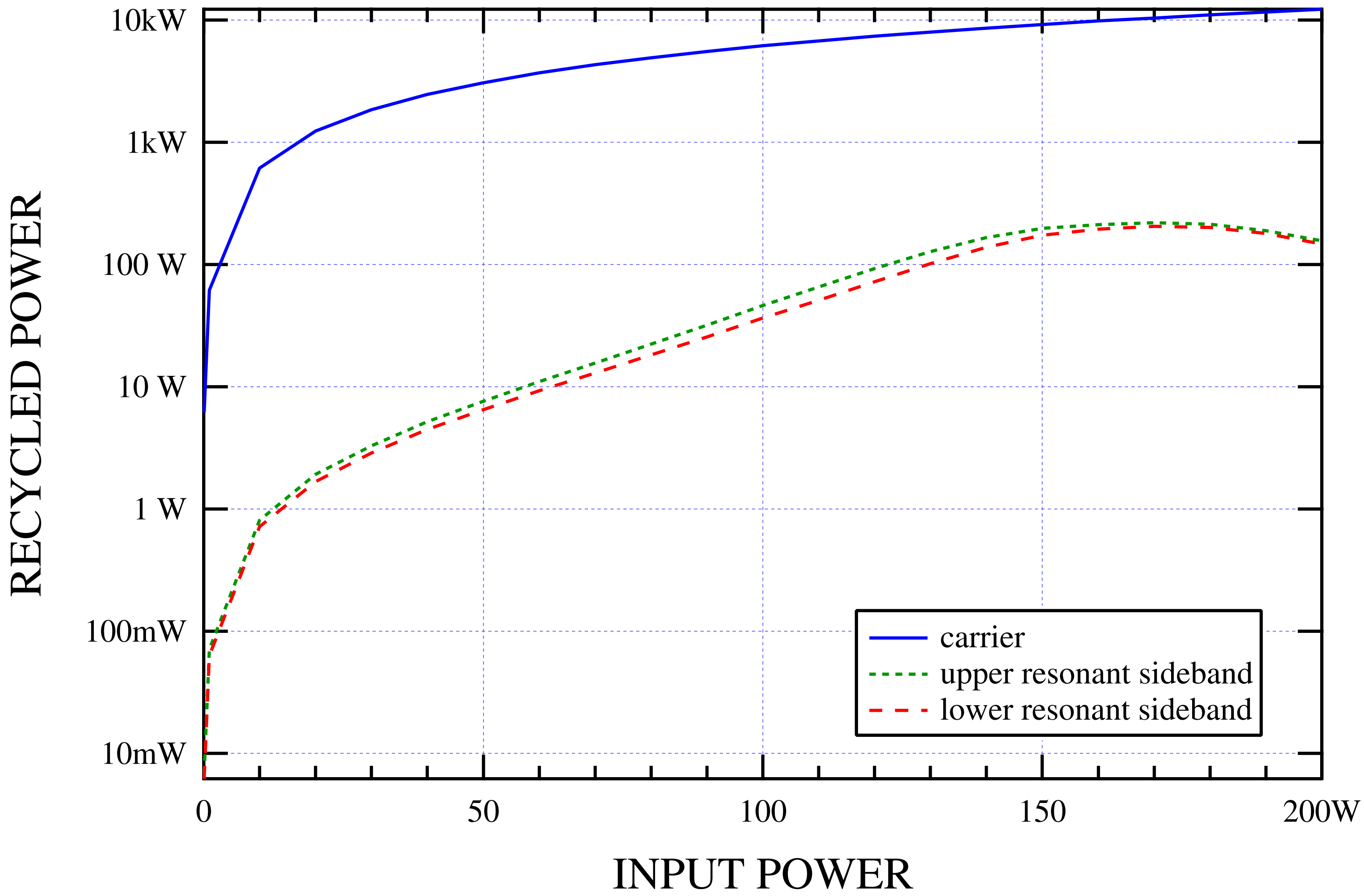
LIGO I: HOT, $R_5 = 20$ km, $E_0 \equiv$ TEM$_{00}$



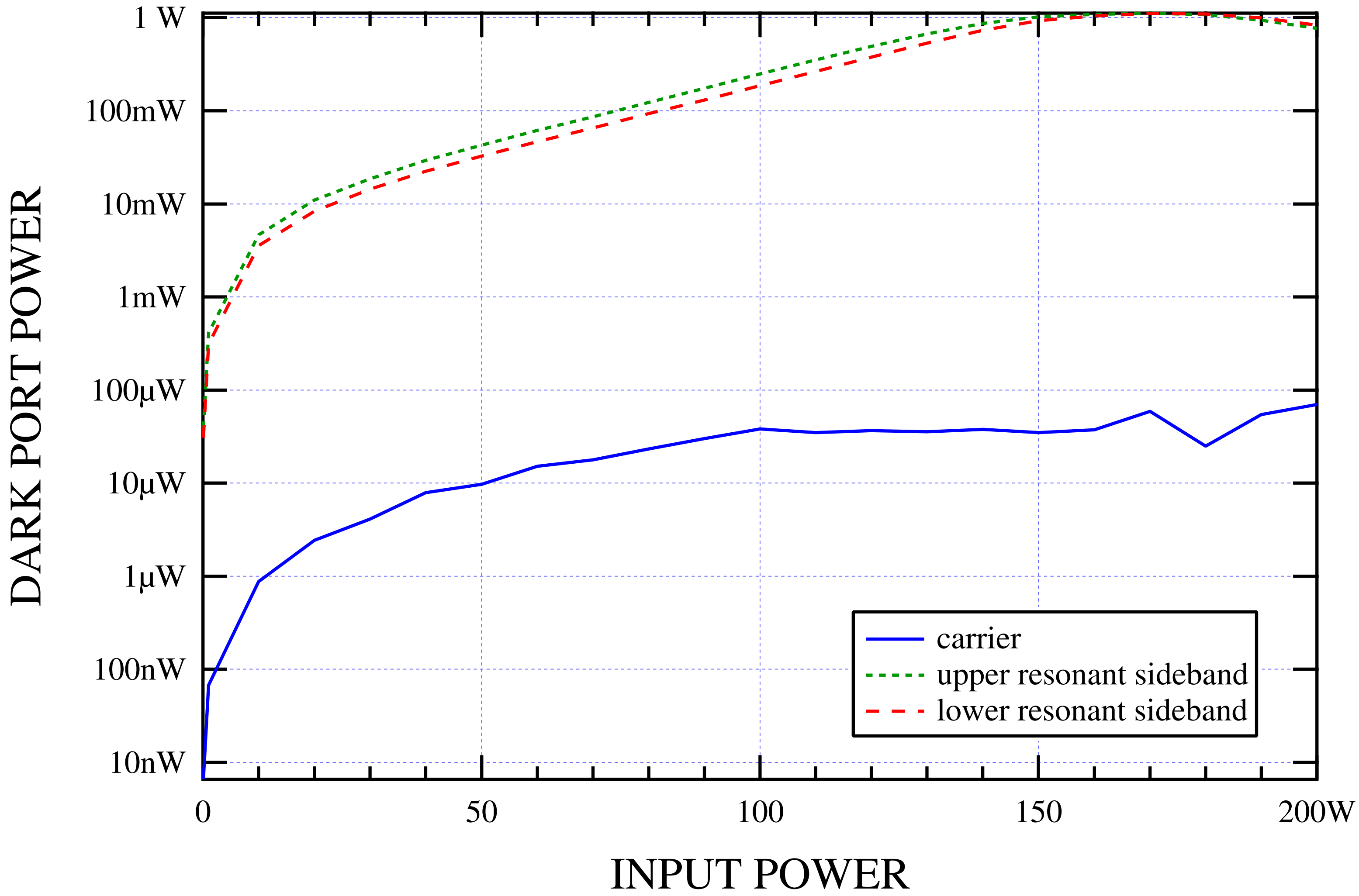
LIGO I PSEUDOLOCKER OUTPUT



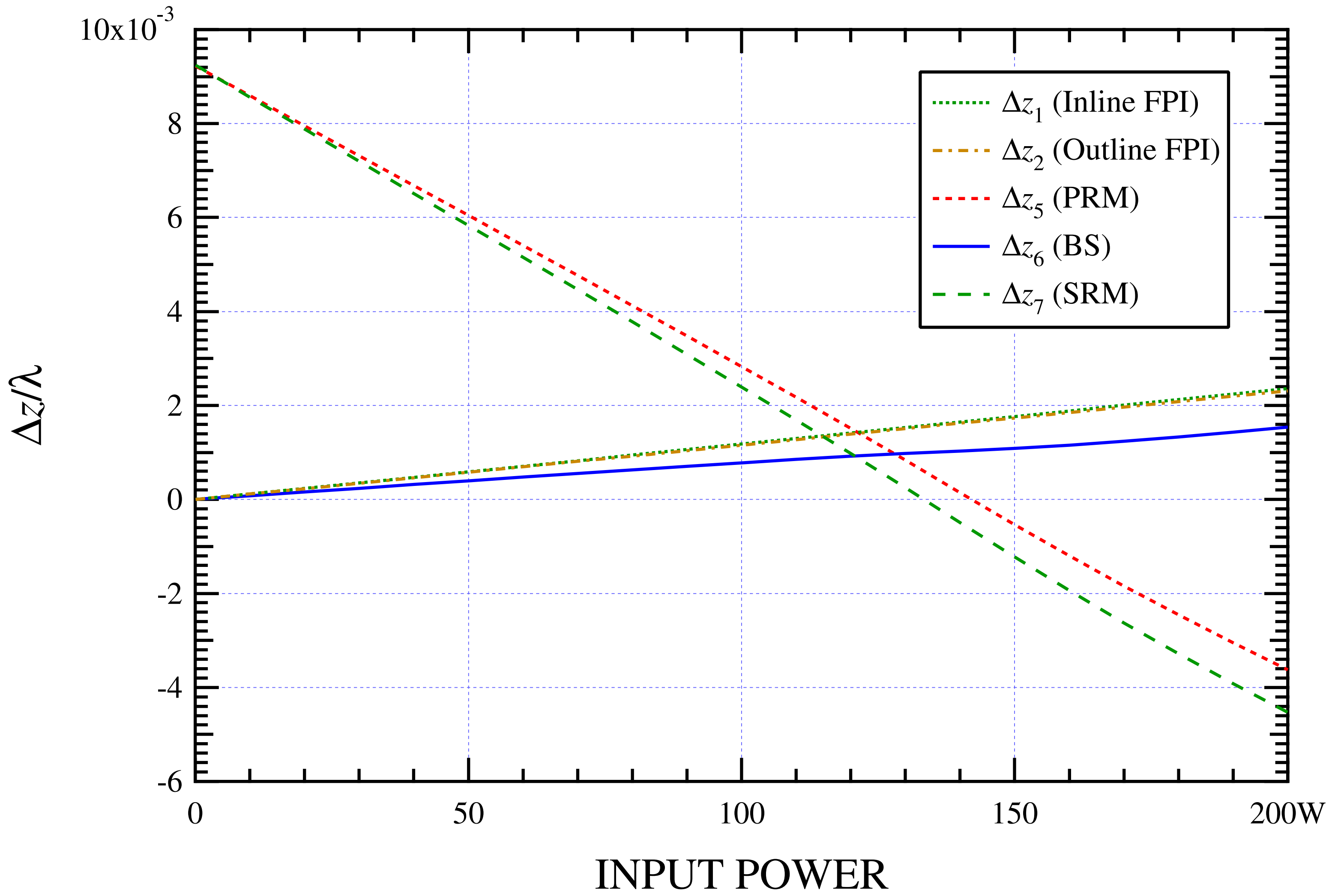
LIGO II RECYCLED POWER



LIGO II DARK PORT OUTPUT POWER



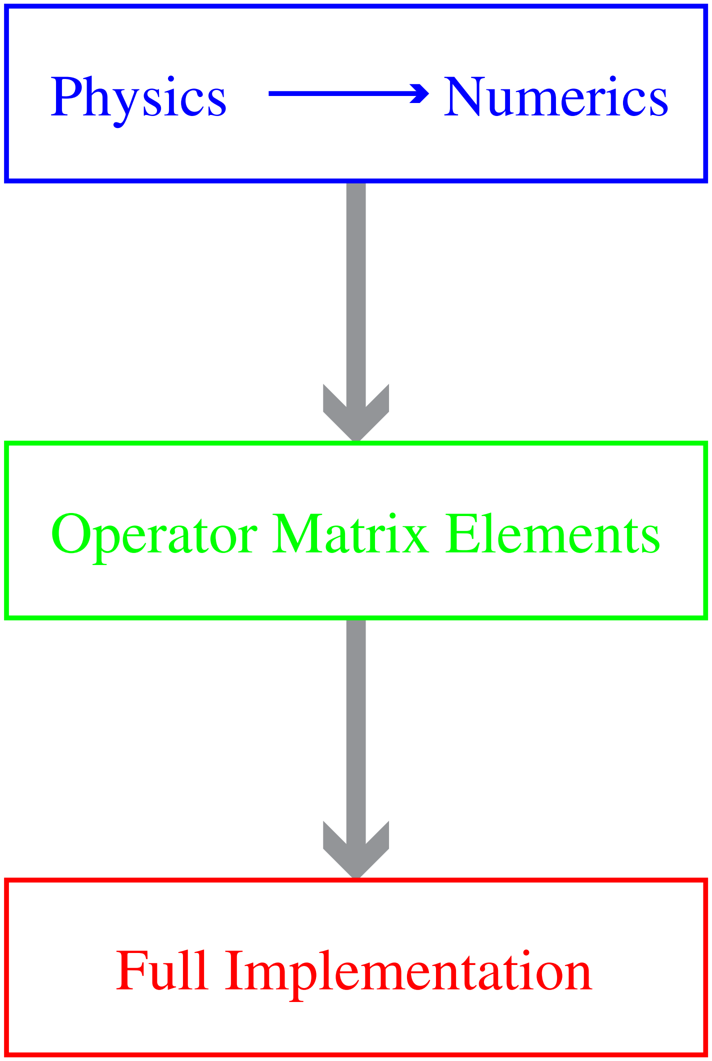
LIGO II PSEUDOLOCKER OUTPUT



MELODY/MATLAB FEATURES

- Simple object-oriented architecture in MATLAB
- Flexible modulation and resonance schemes
- Arbitrary number of spatial basis functions
- Aperture diffraction and mirror/field curvature mismatch
- Hello-Vinet mirror thermal lens and surface deformation
- Pseudolockers for LIGO I/II
- Astigmatic beamsplitter thermal lens
- Precomputation of all matrix operators available

MELODY FEATURE LIFE-CYCLE



- Transient thermal loading
- Static thermal compensation
- Mirror misalignment
- Static thermal loading
- Static thermal deformation

NEW FEATURE PRIORITIES

1. Correct model of thermal compensation (MIT)
2. IFO eigenmode extraction for mode-matching
3. Quadratic thermoelastic surface curvature reporting at runtime
4. Add artificial differential gravitational-wave signal sidebands
5. Add compensation plates to basic configuration
6. Sideband optimization: common & differential lengths, Schnupp asymmetry, modulation frequency
7. Finish numerical calculation of beamsplitter thermoelastic surface deformation $\longrightarrow$ 45° incidence

NEW FEATURE PRIORITIES

8. Demodulation routine for detector class

9. Transient thermal loading

10. Mirror misalignment operators

IMPLEMENTATION

- Use object-oriented programming in MATLAB: primitive classes, encapsulation, function/operator overloading, and inheritance
- Define classes for mirrors, Fabry-Perot and LIGO interferometers, electric fields (Hermite-Gauss, RF-modulated), and detectors
- Encapsulate classes representing simpler entities (mirrors, beamsplitters, laser fields) in classes representing interferometers
- Design simple class interfaces allowing calculations and simulations to be driven by MATLAB scripts

IMPLEMENTATION SCHEMATIC

User-defined
driver scripts

Melody architecture
and classes (@mirror, ...)

"*Qui ci sono dei mostri.*"

SCRIPT-LEVEL FEATURES

- Input powers, modulation frequencies and depths
- All mirror parameters (e.g., thermal constants, orientation and micro-position)
- All interferometer cavity lengths
- Power/signal recycling
- Iteration and solution methods
- Graphics, object storage(!), post-processing
- Full interactive MATLAB functionality

TWO-PHASE THERMAL/TEMPORAL SIMULATIONS

Characteristic time: $t_c = \rho C a^2/k_T \approx 5$ h for fused silica ($a = 0.125$ m)

THERMAL (Script-driven)

1. Run thermal relaxation code, including power-dependent optimizations (e.g., modulation depths, SRM reflectivity)
2. SAVE ligo object after stability is reached for each power level

TEMPORAL (Script-driven, SIMULINK)

1. LOAD ligo object for a specified input power
2. Perturb mirrors and simulate temporal response

SUBSET OF CLASSES

laser_field Stores all spatial components for all operating sidebands, and the frequencies of those sidebands.

mirror Maintains all perturbation matrices (e.g., thermal and angular); encapsulates mirror parameters, two laser_field objects, detectors.

beamsplitter Special case of mirror for 45° beamsplitter; uses numerical temperature distribution.

detector Demodulation detector array; almost complete.

fpi Fabry-Perot Interferometer

ligo LIGO I/II Interferometers

LASER_FIELD OBJECT DATA

```
basis: Hermite-Gauss

[f\_0, f\_1, -f\_1, f\_2, -f\_2] = [0,  2.3971e+001, -2.3971e+001, 3.5956e+001, -3.5956e+001]

TEM_00   -7.36e-01 -2.45e-02i  -7.48e-02 -5.45e-04i  -7.48e-02 -4.79e-04i  -5.35e-02 -2.04e-03i -5.35e-02 -7.46e-04i
TEM_10             0                     0                     0                     0                    0
TEM_01             0                     0                     0                     0                    0
TEM_20   -1.64e-04 +1.86e-04i   8.15e-03 -9.54e-03i   7.54e-03 -9.98e-03i  -6.69e-07 +6.32e-06i -9.64e-08 +6.47e-06i
TEM_11             0                     0                     0                     0                    0
TEM_02   -1.64e-04 +1.86e-04i   8.15e-03 -9.54e-03i   7.54e-03 -9.98e-03i  -6.69e-07 +6.32e-06i -9.64e-08 +6.47e-06i
```

- laser_field class consists of data fields (*members*) and routines which operate on those fields
- Routines fall into two broad categories:

  procedures which alter the internal state of the object but do not return results (e.g., object update procedures)

  functions which return results but do not alter the internal state of the object (e.g., overloaded arithmetic operators)

Define the propagation vector

$$k = k_0 + \Delta k_q,$$

where $\Delta k_q / k_0 \ll 1$, $\omega_0 \equiv k_0 c$, and $\Delta\omega_q \equiv \Delta k_q c$. Write the time-dependent length as

$$L(t) \equiv L_0 + \Delta L(t),$$

where $2k_0 L_0 - \varphi_{00} = 2N\pi$ and $\Delta L(t) \approx \lambda = 2\pi/k_0$. Then

$$\begin{aligned} e^{i[2kL(t)-\varphi_{00}]} &= e^{i(2k_0L_0-\varphi_{00})} e^{i[2k_0\Delta L(t)]} e^{i\left(2\Delta\omega_q L_0/c\right)} e^{i\left[2\Delta k_q \Delta L(t)\right]} \\ &= e^{i\left[2k_0\Delta L(t) + \Delta\omega_q \tau_0\right]} \end{aligned}$$

Include $\Delta L(t)$ in mirror class; implement $\Delta\omega_q \tau_0$ as a diagonal propagation matrix.

FPI OBJECT UPDATE PROCEDURE

```
% Get the total field propagating away from the
% vacuum-coating interface of m_1, and then
% propagate that field to the vacuum-coating
% interface of m_2. This is the new 'front
% field' of m_2.
e_1_r = get_field(m_1, 'front');
e_2 = fp.gouy_prop * e_1_r * fp.kz_prop;
set_field(m_2, e_2, 'front');

% Get the total field propagating away from the
% vacuum-coating interface of m_2, and then
% propagate that field to the vacuum-coating
% interface of m_1. This is the new 'front
% field' of m_1.
e_2_r = get_field(m_2, 'front');
e_1 = fp.gouy_prop * e_2_r * fp.kz_prop;
set_field(m_1, e_1, 'front');
```

LASER_FIELD MTIMES FUNCTION

```
function e_3 = mtimes(e_1, e_2)
%
...
%
if isa(e_1, 'laser_field') & ~isa(e_2, 'laser_field')
% Initialize the structure e_3 with the same basis and sidebands
% as e_1, and multiply (matrix, using *) the elements of the
% matrix e_2 by the components of e_1.
   e_3.basis = e_1.basis;
   e_3.sideband = e_1.sideband;
   e_3.component = e_1.component*e_2;
elseif ~isa(e_1, 'laser_field') & isa(e_2, 'laser_field')
% Initialize the structure e_3 with the same basis and sidebands
% as e_2, and multiply (matrix, using *) the components of
% e_2 by the elements of the matrix e_1.
   e_3.basis = e_2.basis;
   e_3.sideband = e_2.sideband;
   e_3.component = e_1*e_2.component;
else
   error('Matrix multiplication of two laser_field objects is not allowed.');
end

% Create a new laser_field object from the struct e_3.
e_3 = class(e_3, 'laser_field');
```

MATLAB/OOP REFERENCES

- Duane Hanselman and Bruce Littlefield, **Mastering MATLAB 6: A Comprehensive Tutorial and Reference** (Prentice-Hall, 2001); ISBN 0-13-019468-9
- Bertrand Meyer, **Object-Oriented Software Construction**, Second Edition (Prentice-Hall, 1997); ISBN 0-13-629155-4
- Paul F. Dubois, **Object Technology for Scientific Computing: Object-Oriented Numerical Software in Eiffel and C** (Prentice-Hall, 1997); ISBN 0-13-257808-X
- John J. Barton and Lee R. Nackman, **Scientific and Engineering C++** (Addison-Wesley, 1995); ISBN 0-201-53393-6