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Modeling of Thermo-Elastic Effects in LIGO-II 16m Mode Cleaner

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Overview

purpose:

- to estimate effects of thermal deformations in optics
- to set grounds for time-domain modeling (E2E)

scope:

- temperature rise in mirror substrates
- deformation of optics due to heating
- change of radii of curvature of mirrors
- effect on modematching and stability of the cavity
- time-domain modeling

background:

- Mirror deformations and wavefront aberrations caused by c.w. high power laser beams, A.Cutolo et al., 1980
- Analytical models of thermal aberrations in massive mirrors heated by high power laser beams, P.Hello and J.-Y.Vinet, 1990
- Heating by optical absorption and the performance of interferometric gravitational-wave detectors, W.Winkler et al., 1991

Solution of Heat Equation

temperature in the mirror

$$T = T_0 + \delta T.$$

stationary heat equation (Laplace equation):

$$\nabla^2 \delta T = 0.$$

characteristic scale:

$$\chi = 4\frac{\sigma' T_0^3 a}{K}, \qquad (\chi = 0.417).$$

The roots of the characteristic equation: ζ_m,
the eigen-values: $k_m = \zeta_m / a$

The coefficients p_m, A_m and B_m:

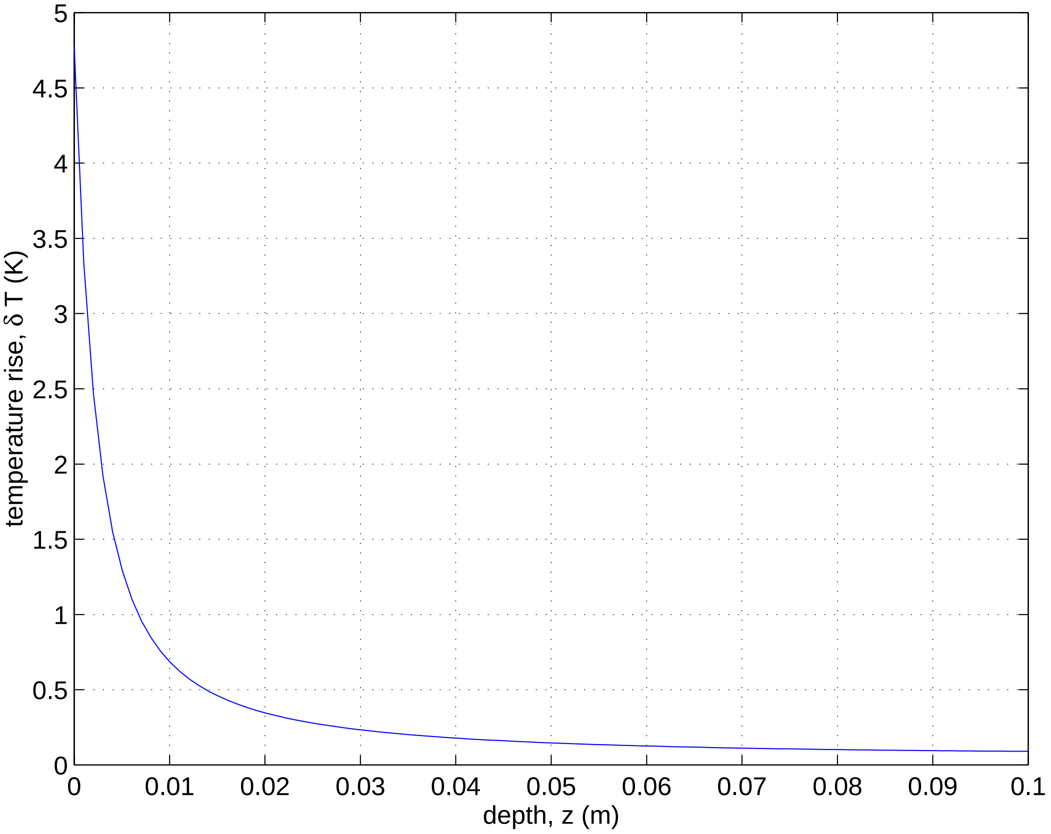
$$\begin{aligned}
p_m &= P_{\text{abs}} \frac{\zeta_m^2}{\pi a^2} \frac{e^{-\zeta_m^2 w^2/8a^2}}{(\zeta_m^2 + \chi^2) J_0(\zeta_m)^2}, \\
A_m &= \frac{p_m a}{K} \frac{(\zeta_m - \chi) e^{-3\zeta_m h/2a}}{(\zeta_m + \chi)^2 + (\zeta_m - \chi)^2 e^{-2\zeta_m h/a}}, \\
B_m &= \frac{p_m a}{K} \frac{(\zeta_m + \chi) e^{-\zeta_m h/2a}}{(\zeta_m + \chi)^2 + (\zeta_m - \chi)^2 e^{-2\zeta_m h/a}}.
\end{aligned}$$

The temperature rise is given by

$$\delta T(r,z) = \sum_m \left(A_m e^{k_m z} + B_m e^{-k_m z} \right) J_0(k_m r).$$

Temperature Rise in the Substrate

Temperature rise along the axis of symmetry (z-axis).



Deformation of Cylindrical Mirror

Heating of the MC mirrors leads to change of their radii of curvature.

input laser power P_{in}.

power absorpted in the mirror: $P_a = \mathcal{L} G P_{in}$,
where $G \approx 650$ is the cavity gain and $\mathcal{L} = 10^{-6}$ is the absorption losses in the coating.

The sagita of the mirror surface (over spot size w):

$$s_0 = R - \sqrt{R^2 - w^2},$$

where R is the radius of curvature.

result of Winkler et al:

$$\delta s = -\frac{\alpha P_a}{4\pi K}.$$

the sagita of the deformed mirror is

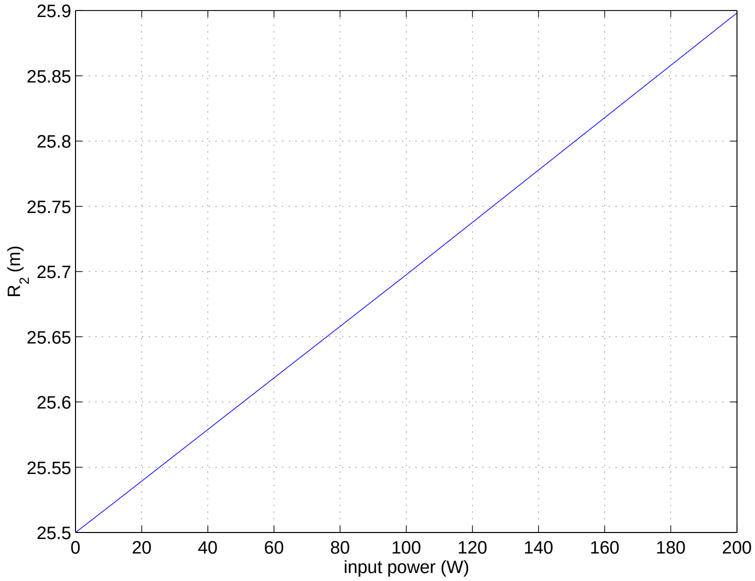
$$s = s_0 + \delta s.$$

the effective raius of curvature:

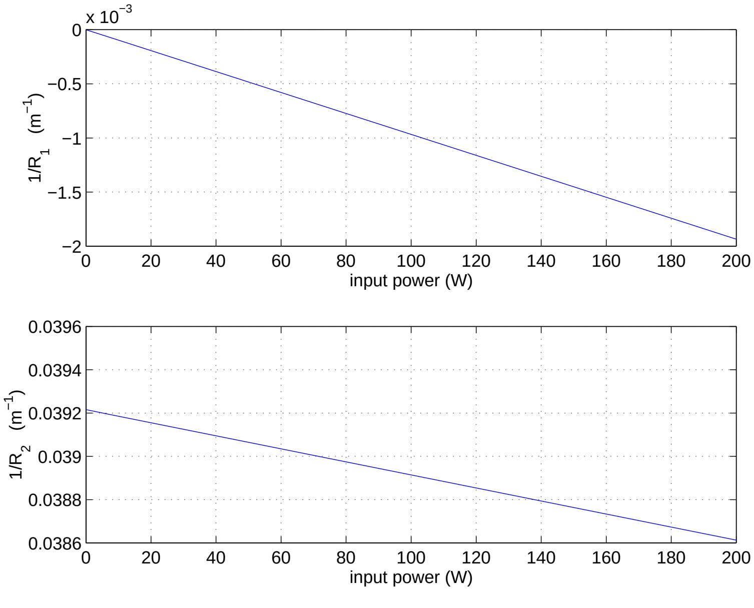
$$R = \frac{s^2 + w^2}{2s}.$$

Change of Radii of Curvature

Radius of curvature of MC3 as a function of laser power.



Curvature of MC mirrors as a function of laser power.



Stability of Fabry-Perot Cavity

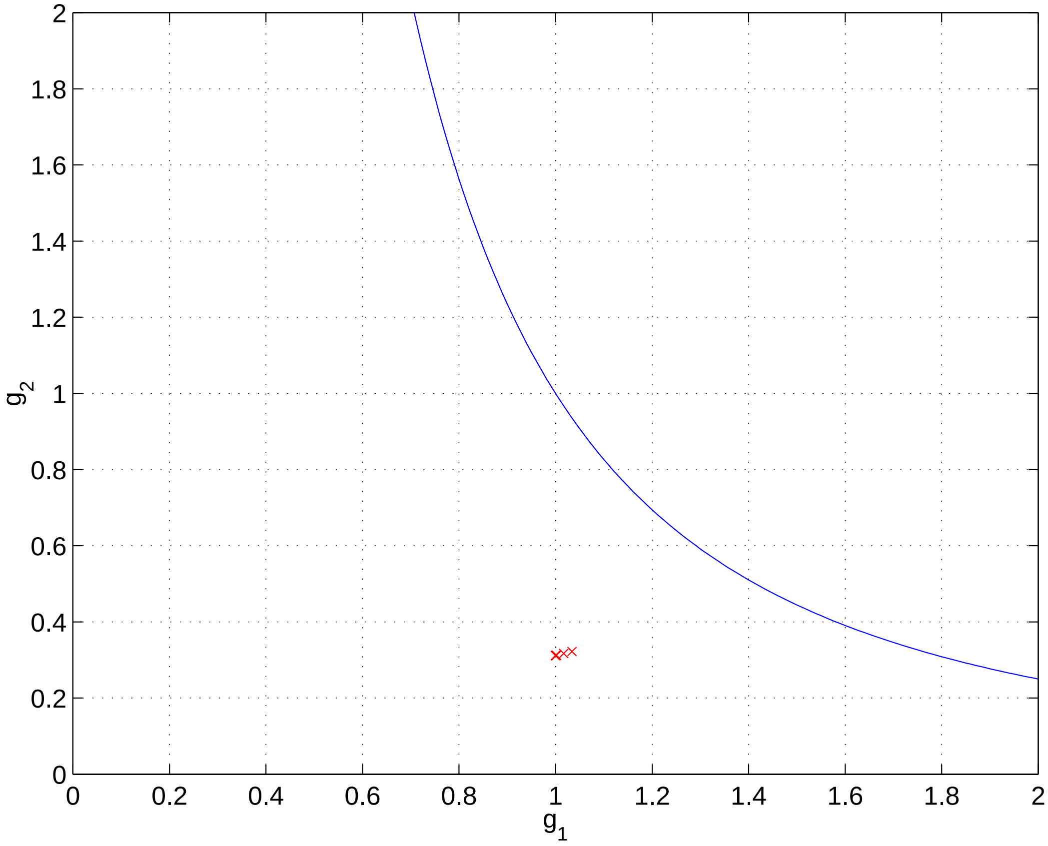
mode-stability is defined by cavity g-factors:

$$g_1 = 1 - \frac{L}{R_1}, \qquad g_2 = 1 - \frac{L}{R_2}.$$

the condition for stability is

$$0 < g_1^2 g_2 < 1.$$

The boundary of stability and the cavity state as function of the incident power.



Change of Beam Waist

axisymmetric beam propagation → resonant mode = (00).

beam waist-size (radius):

$$w = \sqrt{\frac{2z_R}{k}},$$

where z_R is Rayleigh length,

$$\begin{aligned} R_a &= z_a + \frac{z_R^2}{z_a}, \\ R_b &= z_b + \frac{z_R^2}{z_b}, \\ z_b - z_a &= L. \end{aligned}$$

Here z_a and z_b are the mirror coordinates with respect to (unknown) waist position.

numerical solution:

$$\begin{aligned} w_1 &= 2.027\,\text{mm}, \qquad &\text{(cold)} \\ w_2 &= 2.043\,\text{mm}, \qquad &\text{(hot).} \end{aligned}$$

(Heating is estimated for 100 W of incident power.)

Modematching

modematching coefficient:

$$m \equiv \frac{\langle E_1 E_2^* \rangle}{(|E_1|^2 |E_2|^2)^{\frac{1}{2}}}, \qquad M = |m|^2.$$

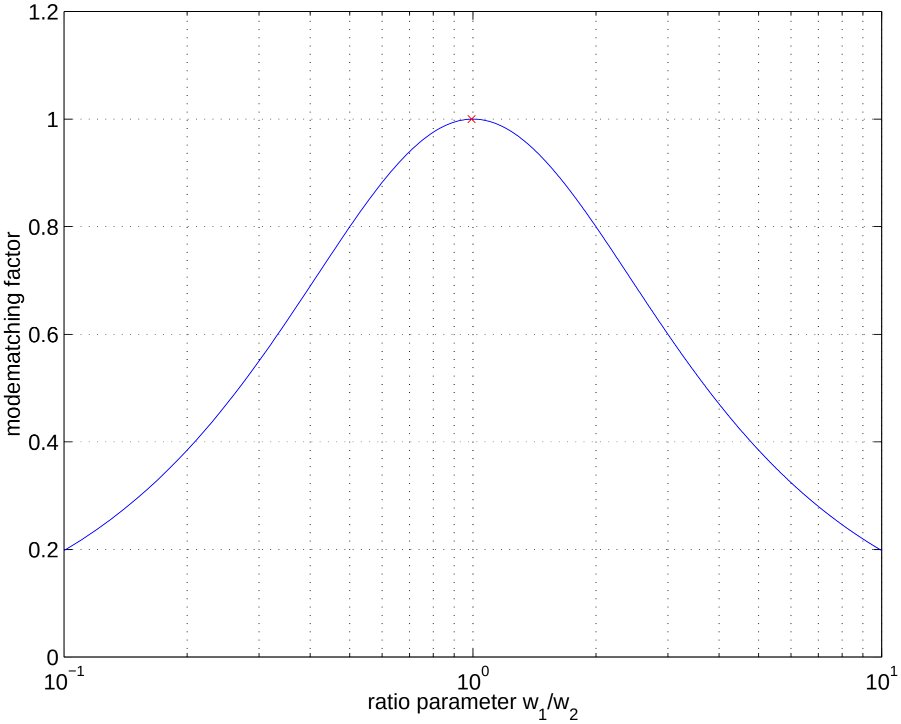
where E_1 and E_2 are complex amplitudes of the two modes.

axisymmetric heating: the spot is in the center of the mirror,
the fundamental (00) modes of cavity.

$$E_{1,2} = \exp\left\{-\frac{r^2}{w_{1,2}^2}\right\}, \quad \Rightarrow \quad m = \frac{2w_1 w_2}{w_1^2 + w_2^2}.$$

power coupling: $M = 0.99994$.

Modematching as a function of waist and the cavity state.



Time-Domain Modeling of Mirror Heating

propagation of heat through the substrate:

$$\tau \frac{\partial T}{\partial t} = \nabla'^2 T,$$

where ∇' is the gradient with respect to $x' = \frac{x}{w}$ and $y' = \frac{y}{w}$.

the characteristic time scale:

$$\tau = \frac{\rho C w^2}{K}.$$

the solutions are of the form:

$$\delta s(t) \propto e^{-t/\tau}.$$

frequency response - Laplace-domain transfer function:

$$H(s) = \frac{1}{s + \frac{1}{\tau}}.$$

time-domain evolution - the digital filter (time step dt):

$$y_n = b_0 x_n + b_1 x_{j-1} - a_1 y_{j-1}.$$

Tustin algorithm:

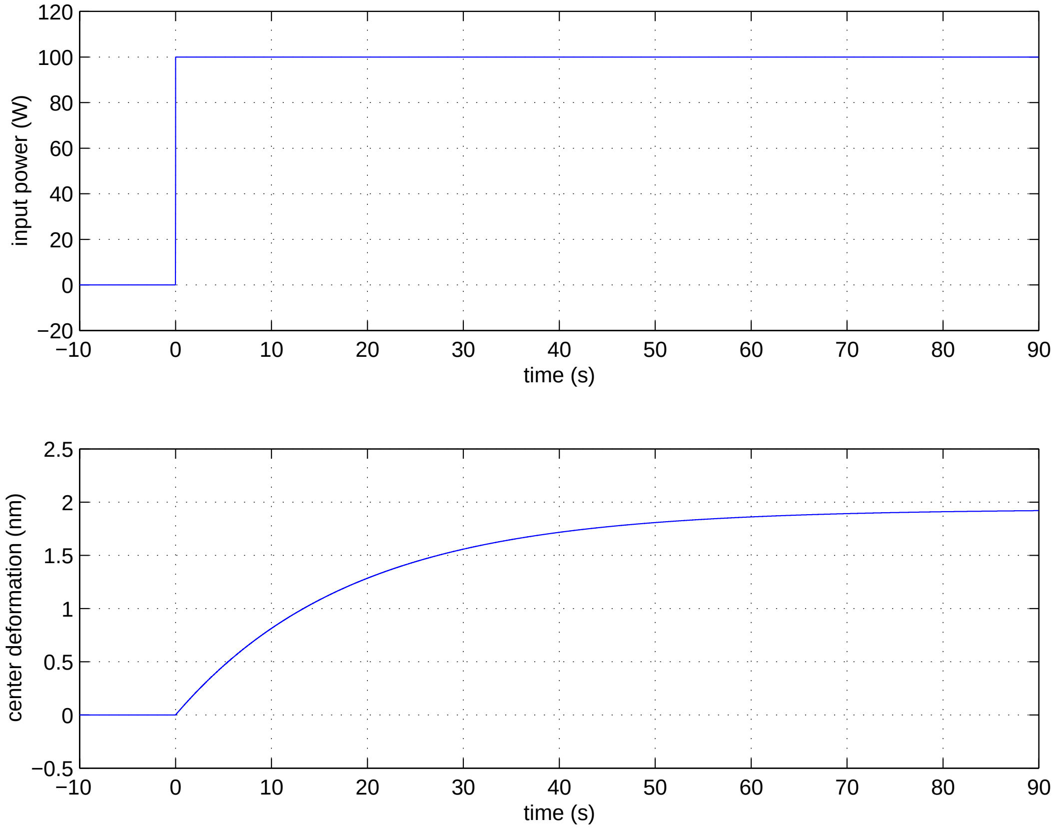
$$a_1 = \frac{dt - 2\tau}{dt + 2\tau}, \qquad b_0 = b_1 = \frac{dt}{dt + 2\tau}.$$

Step Response of Thermal Deformation

relaxation time

$$\tau = 18.32\,\mathrm{s}.$$

Deformation of the mirror with 100 W circulating power as a function of time.



Generation of Higher Order Modes

generation of higher order modes is characterized by the mixing matrix:

$$\langle mn|V|m'n'\rangle = \int\limits_S E^*_{mn}(\mathbf{r})V(\mathbf{r})E_{m'n'}(\mathbf{r})\, d^2\mathbf{r},$$

where $E_{mn}(\mathbf{r})$ are Hermit-Gaussian modes of MC cavity,

and $V(\mathbf{r})$ is the phase-shift operator:

$$V(x,y) = \exp\{-2ik\delta z(x,y)\}.$$

results of numerical calculations (y-shift $= 0.1w$):

$$\begin{aligned}
\langle 00|V|01\rangle &= -1.4\times 10^{-5} - i\,1.0\times 10^{-3},\\
\langle 00|V|10\rangle &= 0,\\
\langle 00|V|02\rangle &= -5.0\times 10^{-6} - i\,7.4\times 10^{-4},\\
\langle 00|V|20\rangle &= +4.9\times 10^{-6} + i\,2.5\times 10^{-4},\\
\langle 00|V|11\rangle &= 0.
\end{aligned}$$

The largest mixing occurs to (01)-mode,
of the order 0.001.

Conclusions

Implications for optics development:

- The effects of thermal deformations in 16m MC are not significant.

Implications for model development:

- effect of mirror deformations on the cavity field can be modeled using mode-decomposition approach
- time-domain evolution can be modeled using digital filters