Quantum Noise and Quantum Nondemolition in Gravitational-Wave Interferometers

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CaJAGWR Seminar

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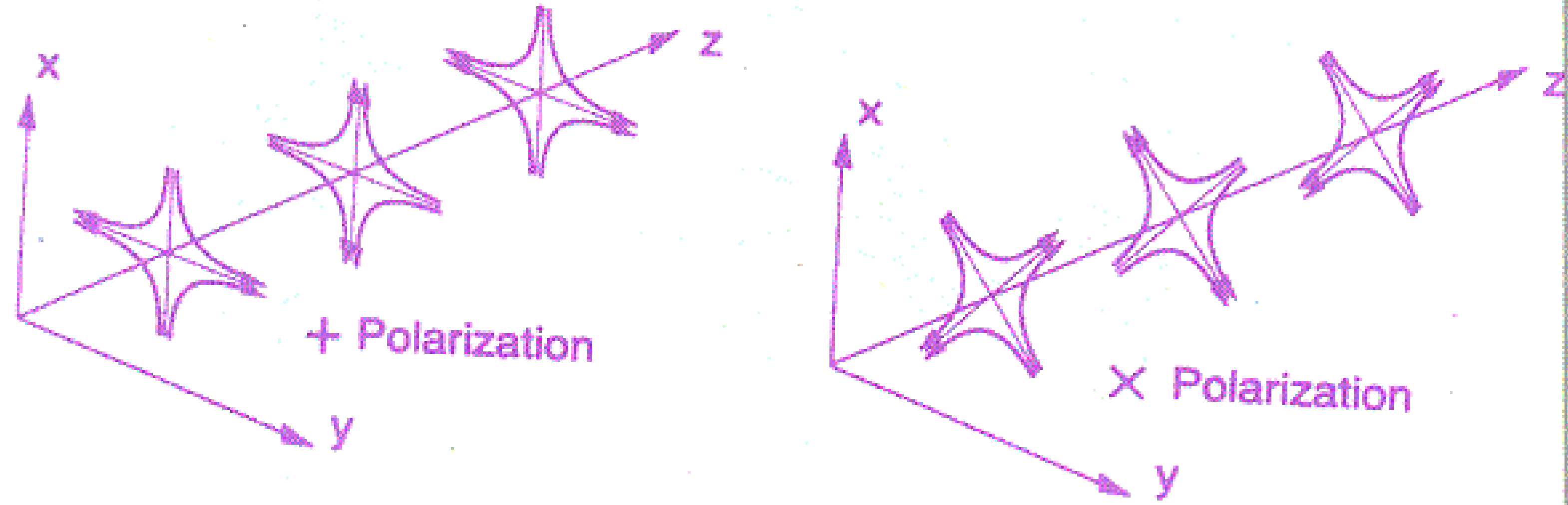
JPL

LIGO-G000327-00-M

GRAVITATIONAL WAVES

Ripples of warpage in the fabric of spacetime

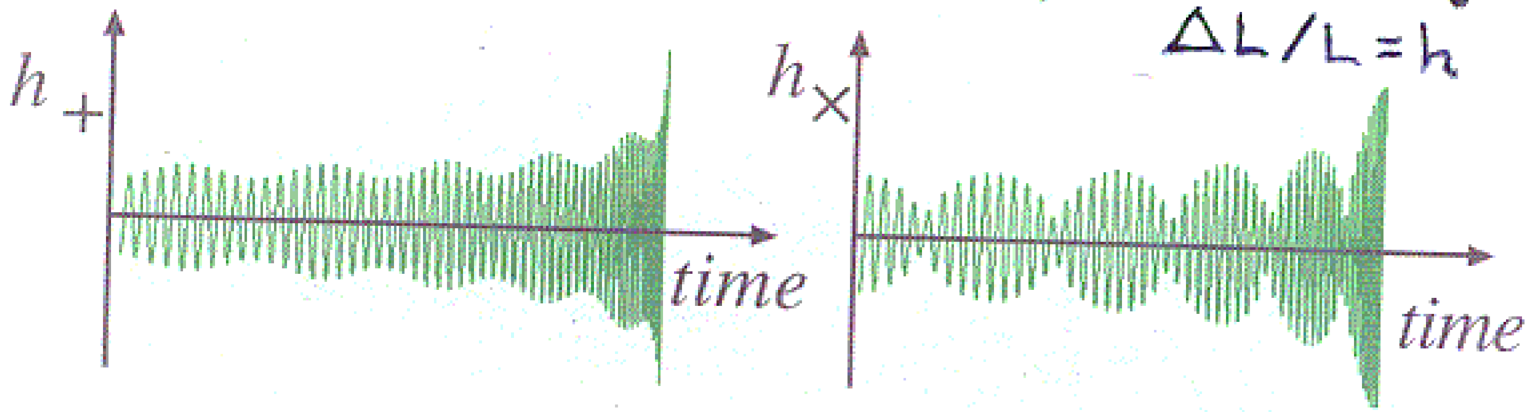
Forces Exerted by Waves



Waveforms (stereophonic)

ΔL

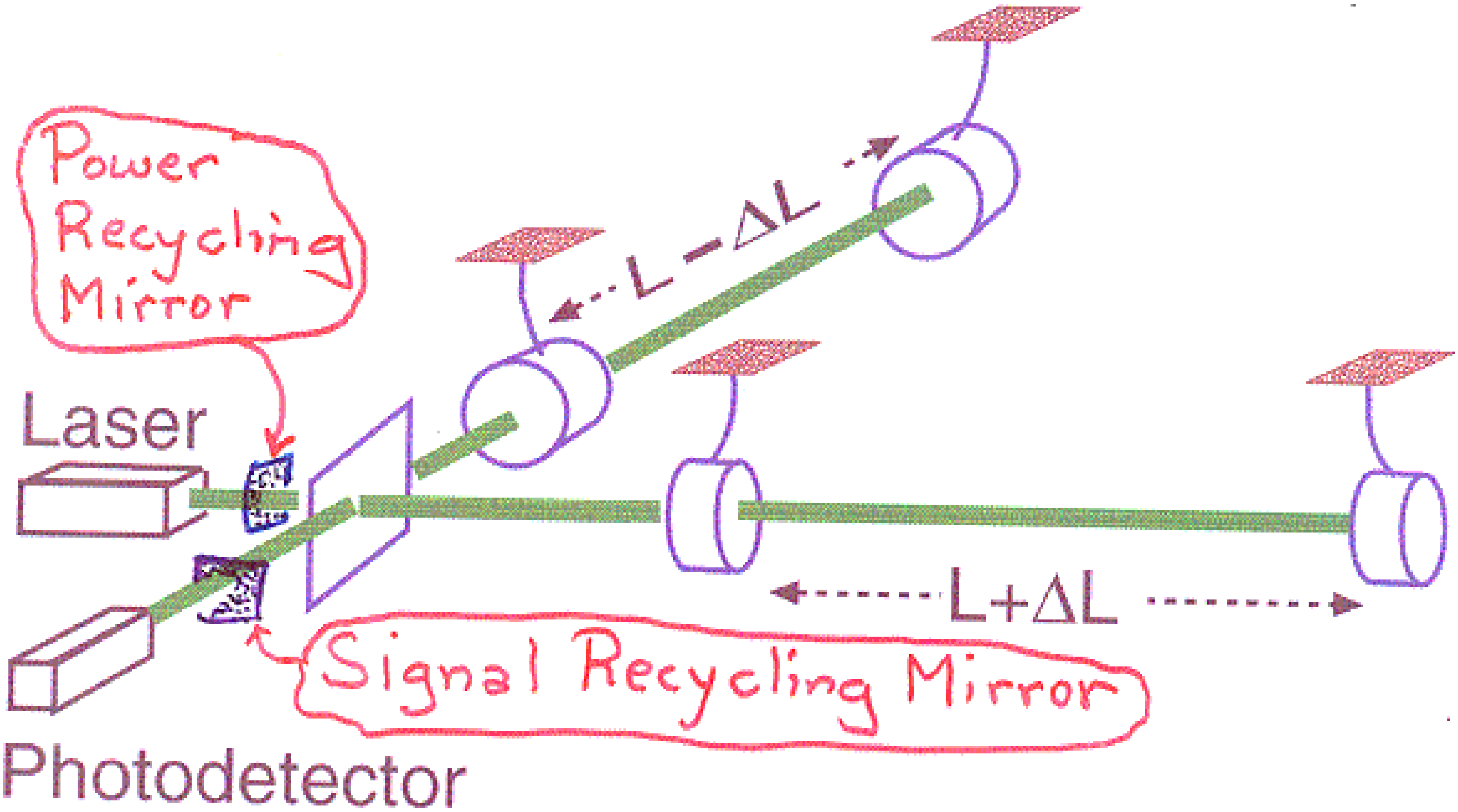
$\Delta L / L = h$



Overview of Bands, Detection Methods, & Sourc

Band f & λ	Detection Method	Sources					
ELF $f \sim 10^{-17}$ Hz $\lambda \sim 1$ Gpc	Anisotropy of Cosmic μWave Rad'n						
VLF $f \sim 10^{-9}$ Hz $\lambda \sim 10$ pc	Timing of millisecond pulsars						COSMIC STRINGS
LF $f \sim 10^{-4}$ to 10^{-1} Hz $\lambda \sim 0.01$ to 10 AU	LISA	Massive Holes (300 to 3×10^{7} solar masses) Soliton Stars Binary Stars	BLACK HOLES	NAKED SINGULARITIES	STRINGS + WALLS OR MONOPOLES	DOMAIN WALLS	
HF $f \sim 10$ to 10^{4} Hz $\lambda \sim 30$ to 3×10^{4} km	LIGO VIRGO GEO TAMA Resonant Masses	Ordinary Holes (3 to 300 $M_\odot$) Boson Stars Neutron Stars Supernovae					

GRAVITATIONAL-WAVE INTERFEROMETERS

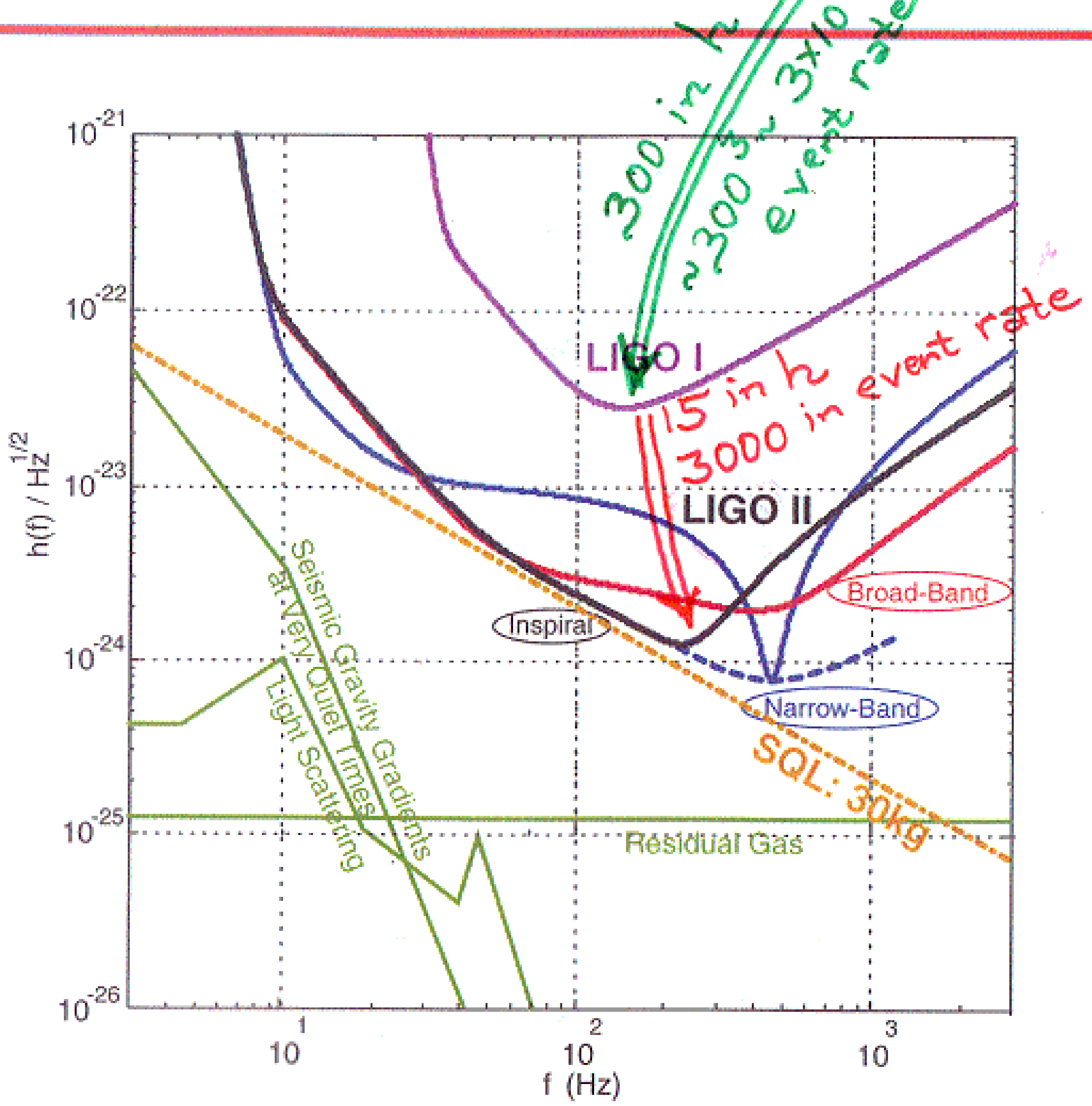


$$\Delta L = h\,L \lesssim 4\times10^{-16}\ \text{cm}$$

$h \lesssim 10^{-21}$

$L = 4$ km

LIGO Noise Curves

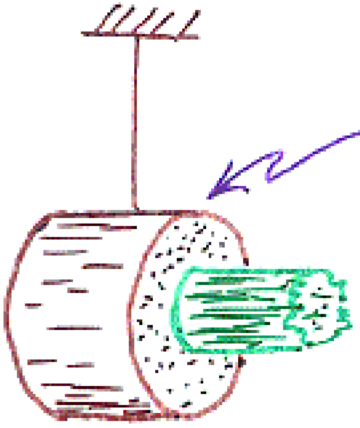


- Facilities permit factor 10 below LIGO II
- Standard Quantum Limit must be evaded

8) 5

STANDARD QUANTUM LIMIT (SQL

- QUICK OVERVIEW -



10^{27} atoms

3×10^{27} degrees of freedom

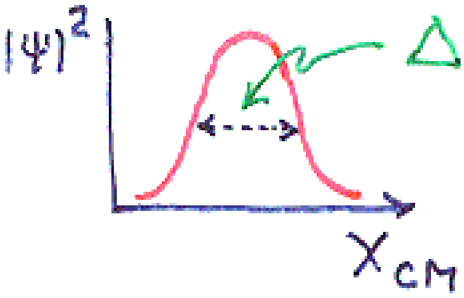
Interferometer is sensitive almost solely to one d.o.f.:

$\boxed{x_{cm} \equiv \text{center of mass position}}$

(slight sensitivity to others is "internal thermal noise")

I will ignore

- x_{CM} IS QUANTIZED!



0.01 sec

$$\Delta x_{CM} \gtrsim \sqrt{\frac{\hbar\tau}{m}} \sim 2\times10^{-17}\,\text{cm}$$

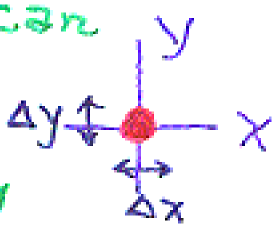
30kg

$$\boxed{\Delta h_{SQL} = \frac{\Delta x_{CM}}{L} \sim 5\times10^{-23}\ \text{rms}}$$

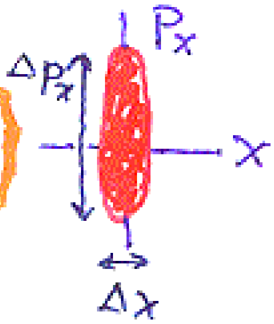
4km

ORIGIN OF THE STANDARD QUANTUM LIMIT

- Recall: $[\hat{x}, \hat{y}] = 0 \Rightarrow \hat{x}$ & $\hat{y}$ can be measured simultaneously with arbitrarily good accuracy



- $[\hat{x}, \hat{p}_x] = i\hbar \Rightarrow$ measurement accuracy is limited by $\boxed{\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}}$



IDEALIZED THOUGHT EXPERIMENT

- Measure $\hat{x}_0 \equiv \hat{x}(t=0)$ with accuracy Δx_0
- Wait time τ while gravitational wave acts

  In Heisenberg Picture: $\hat{x}_\tau = \hat{x}_0 + \frac{\hat{p}_0}{m}\tau + x_{GW}$
- Then measure $\hat{x}_\tau$ to discover effect of wave

  Accuracy is limited by:

$$[\hat{x}_0, \hat{x}_\tau] = [\hat{x}_0, \hat{x}_0 + \frac{\hat{p}_0}{m}\tau + x_{GW}] = [\hat{x}_0, \frac{\hat{p}_0}{m}\tau]$$

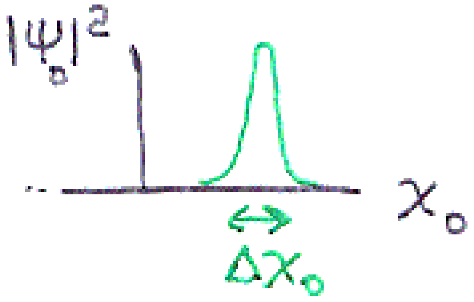
$$= \frac{i\hbar\tau}{m} \Rightarrow \boxed{\Delta x_0 \cdot \Delta x_\tau \geq \frac{\hbar\tau}{2m}}$$

- Best strategy: $\Delta x_0 = \Delta x_\tau = \sqrt{\frac{\hbar\tau}{2m}}$

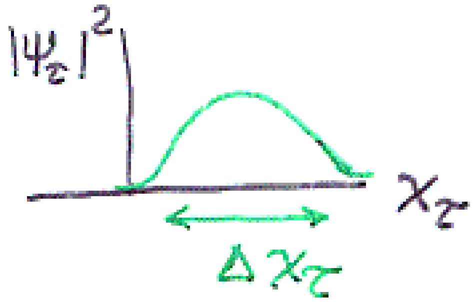
$$\Rightarrow \boxed{\begin{array}{l} \Delta x_{GW} \geq \sqrt{\frac{\hbar\tau}{m}} \\ \Delta h \geq \frac{\sqrt{\hbar\tau/m}}{L} \sim \frac{\sqrt{\hbar/m\omega}}{L} \end{array}} \quad \text{SQL}$$

In The Schrödinger Picture:

- First measurement collapses wave function



- If $\Delta x_0 < \sqrt{\hbar\tau/2m}$ then wave function <u>must</u> spread during time τ



- Noncommutation of $\hat{x}_0$ & $\hat{x}_\tau$ in Heisenberg Picture $\Rightarrow$ Measurement of $\hat{x}_0$ influences result of measuring $\hat{x}_\tau$ in an unpredictable way

- Apparent Conclusion: **20 YEAR MISUNDERSTANDING**
  - Position is a BAD variable to measure
  - Much better: Momentum

$$\hat{p}_\tau = \hat{p}_0 + P_{GW} \Rightarrow [\hat{p}_0, \hat{p}_\tau] = 0$$

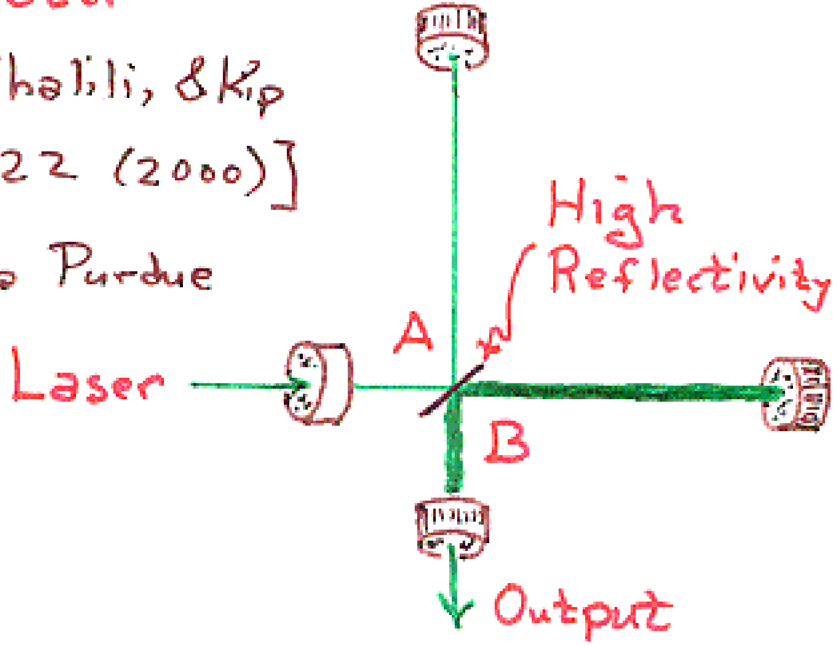
$\boxed{\text{Momentum is a Quantum NonDemolition Observable}}$

A GW Interferometer That Measures The Momenta (Speeds) of Test Masses

- Speed Meter -

[Braginsky, Gorodetsky, Khalili, & Kip
Phys Rev D 61, 044022 (2000)]

[detailed analysis: Patricia Purdue
...in progress]



- Two weakly coupled oscillators: A, B
- Light sloshes between A & B with period $P \sim 10^{-3}$ s
  e.g.
- Drive A with laser (carrier) light
  - B gets excited $E_0 \cos \omega_0 t$ ← 10^{15} Hz
- Mirror in B moves by $\hat{x}_0$ at $t=0$
  - phase shifts carrier light in B

$$E_0 \cos(\omega_0 t + 2k\hat{x}_0) = E_0 \cos \omega_0 t - \underbrace{2E_0 k \hat{x}_0 \sin \omega_0 t}_{\text{Signal}}$$

↑ wave number, ω_0/c

- Signal sloshes into A then back to B with sign change, and superposes on new signal to give

$$2E_0 k \sin \omega_0 t \cdot \underbrace{(-\hat{x}_0 + \hat{x}_P)}_{\simeq \boxed{d\hat{x}/dt} \cdot P}$$

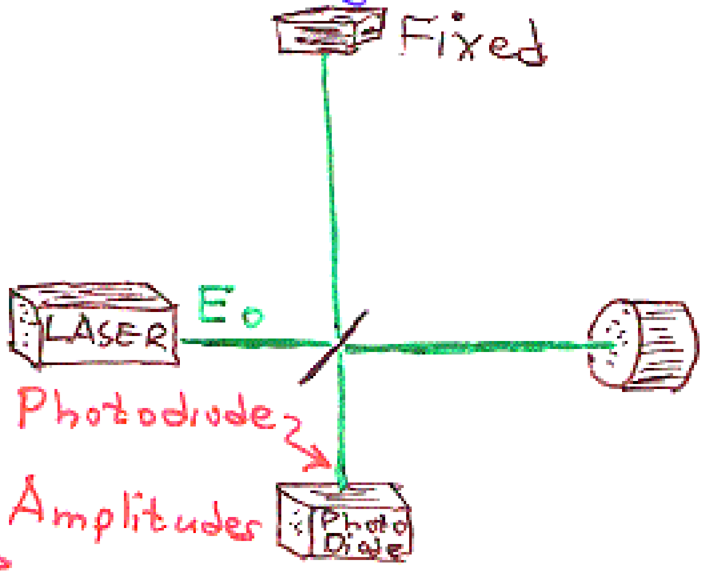
SPEED

- A Possibility for LIGO-III

Hint That Something Is Wrong With Anti-Position Viewpoint

[Braginsky, Gorodetsky, Khalili, Matsko, Vyatchanin & Kip – in preparation]

- Conventional, Position Measuring Interferometer



Electric Field Hitting Photodiode

Quadrature Amplitudes

$$\hat{E} = \hat{E}_1 \cos \omega_0 t + (\hat{E}_2 + \delta E_0) \sin \omega_0 t$$

10^{15} Hz

$$\hat{E}_2 = 2E_0 k \boxed{\hat{x}} + \boxed{\hat{\mathcal{E}}_2 \, \delta E_0}$$

Signal　　shot noise

$\hat{F}$ = Photon flux into photodetector $\propto \overline{\hat{E}^2} \simeq \hat{E}_2 \cdot \delta E_0 \propto \hat{E}_2$

- But: The EM commutation relations imply:

$$[\hat{E}_{2t}, \hat{E}_{2t'}] = 0 \quad \forall \ |t-t'| \gg \frac{1}{\omega_0} \sim 10^{-15} \text{ sec}$$

The photodetector & electronics integrate $\hat{F} \propto \hat{E}_2$ over $\Delta t \simeq 10^{-4}$ sec & digitize it into data samples

These Data Samples All Commute With Each Other

⇒ **Wave Function Collapse Cannot Influence The Data!**

How is this possible? Since $\hat{E}_2$ contains $\hat{x}$, which does not self commute.

$$\hat{E}_{2t} = E_0 k \underbrace{\hat{x}_t} + \hat{\mathcal{E}}_{2t}$$

$$\hat{\mathcal{E}}_{2t} \equiv E_0 k \hat{x}_t^{shot}$$

$$\hat{x}_t = \underbrace{\hat{x}_0 + \frac{\hat{p}_0 t}{m} + X_t^{GW}}_{\hat{x}_t^{old}} + \hat{x}_t^{R.P.}$$

← radiation pressure noise

$$\hat{E}_{2t} = E_0 k \left(\hat{x}_t^{old} + \hat{x}_t^{shot} + \hat{x}_t^{R.P.} \right)$$

$$\left.\begin{aligned} [\hat{x}_0^{old}, \hat{x}_t^{old}] &= \frac{i\hbar t}{m} \\ [\hat{x}_0^{shot}, \hat{x}_t^{RP}] &= -\frac{i\hbar t}{m} \end{aligned}\right\} \text{cancel to enforce } [\hat{E}_{20}, \hat{E}_{2t}] = 0$$

The light's noise (shot noise & radiation pressure noise) automatically protect the output data train from the influence of test-mass wave-function collapse

- This is true of any linear measuring system ... Braginsky & Khalili Quantum Measurement, chapter 6

LIGO DATA ANALYSIS

- DATA STREAM:

$$\hat{E}_{2t} = E_0 k \left(\hat{x}_0 + \frac{\hat{p}_0 t}{m} + X_t^{GW} + \hat{x}_t^{shot} + \hat{x}_t^{R.P.} \right)$$

- In data analysis, we filter out all signal & noise below 10 Hz

  i.e. we construct a discrete Fourier transform of the data train [construct linear combinations of the self commuting data samples].

- $\hat{x}_0 + \frac{\hat{p}_0}{m} t$ goes entirely into the near-zero-frequency data points, which get thrown away

- So our final data contain <u>no</u> information at all about $\hat{x}_0 + \frac{\hat{p}_0}{m} t$.

  They are not influenced at all by the quantum properties of the test masses.

- The quantum noise is entirely due to the light:

$$\hat{x}_t^{shot} + \hat{x}_t^{R.P.}$$

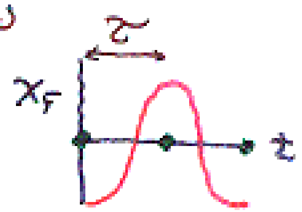
SIDE REMARK:

Khalili's Pedagogical Example [Von Neuman thought experiment]

a. Classical force $F_0 \cos\omega t$ acts on free mass m

$$\rightarrow \quad x_F = -\frac{F_0}{m\omega^2}\cos\omega t$$

b. 3 identical measuring devices make quick measurements of $\hat{x}$ at times $t = 0, \tau, 2\tau \ldots \quad \tau = \pi/\omega$



$$\hat{H} = \frac{\hat{p}^2}{2m} + \underbrace{K\delta(t-j\tau)\cdot(\hat{x}+x_F)\hat{\wp}}_{H_{\text{interaction}}} + H_{\hat{\wp}\hat{Q}}$$

Data are analyzed later to extract signal x_F

c. Measurement 0: $\frac{d\hat{Q}}{dt} = \frac{1}{i\hbar}[\hat{Q},\hat{H}] = K\delta(t)\cdot(\hat{x}+x_F)$

$$\Rightarrow \quad \hat{Q}_{after} = K(\hat{x}+x_F+\hat{Q}_0/K)$$

$\hat{Q}$ immediately before measurement

Readout with extremely high precision

Measured Quantity: $\boxed{\tilde{x}_0 = x_{F_0} + \hat{x}_0 + \hat{X}_0}$

measurement noise $\hat{Q}_0/K$

$\hat{x}$ @ time t_0

Back Action on test mass: $\frac{d\hat{p}}{dt} = \frac{1}{i\hbar}[\hat{p},\hat{H}] = -K\delta(t)\hat{\wp}$

$$\Rightarrow \quad \boxed{\hat{p}_{after_0} = m\dot{x}_{F_0} + \hat{p}_0 + \hat{P}_0}$$

$-K\hat{\wp}$ (Back Action)

$\hat{p}$ @ time 0

NOTE: $[\hat{X}_0, \hat{P}_0] = \left[\frac{\hat{Q}_0}{K}, -K\hat{\wp}_0\right] = -i\hbar$

measurement noise

Back Action

$[\hat{X}_0, \hat{P}_0] = -i\hbar$ measurement noise & BA $\qquad [\hat{x}_0, \hat{p}_0] = +i\hbar$ measured system

$[\tilde{x}_0, \hat{p}_{0\,after}] = 0$ readout quantity & momentum afterward

TRUE FOR LINEAR MEASUREMENTS

d. Measurement 1:

$$\boxed{\tilde{x}_1 = x_{F1} + \hat{x}_0 + \left(\frac{\hat{p}_0 + \hat{P}_0}{m}\right)\tau + \hat{X}_1}$$

Read out with extremely high accuracy

NOTE: $[\tilde{x}_0, \tilde{x}_1] = [x_{F0} + \hat{x}_0 + \hat{X}_0,\ x_{F1} + \hat{x}_0 + \left(\frac{\hat{p}_0 + \hat{P}_0}{m}\right)\tau + \hat{X}_1]$

$$= \underbrace{[\hat{x}_0, \frac{\hat{p}_0}{m}\tau]}_{i\hbar} + \underbrace{[\hat{X}_0 + \frac{\hat{P}_0}{m}\tau]}_{-i\hbar} = 0$$

& similarly for subsequent measurements:

$[\tilde{x}_i, \tilde{x}_j] = 0$ Readout quantities commute!

$\Rightarrow$ State reduction due to highly accurate measurement of $\tilde{x}_i$ will not influence result of a subsequent measurement of $\tilde{x}_j$

e. Measurement 2:

$$\boxed{\tilde{x}_2 = x_{F2} + \hat{x}_0 + \left(\frac{\hat{p}_0 + \hat{P}_0}{m}\right)2\tau + \frac{\hat{P}_1}{m}\tau + \hat{X}_2}$$

f. Later, in data analysis system, compute

$$\boxed{\tilde{x}_0 - 2\tilde{x}_1 + \tilde{x}_2 = -4\frac{F_0}{m\omega^2} + \hat{X}_0 + \hat{X}_2 + \left(\frac{\hat{P}_1\tau}{m} - 2\hat{X}_1\right)}$$

Evaluate expectation value & variance for this quantity in Initial States of the 3 measuring devices (because state reductions of $\tilde{x}_j$ have not affected subsequent $\tilde{x}_k$ readouts)

- $\hat{x}_0$ & $\hat{p}_0$ do not influence the measurement accuracy
- standard procedure: $\Delta X_j = \Delta P_j \tau/m = \sqrt{\frac{\hbar\tau}{m}} \Rightarrow \Delta F = \Delta F_{SQL}$
- to beat SQL for F: arrange initial states so $\Delta X_0 \simeq 0$ (huge backaction), $\Delta X_2 \simeq 0$, $\Delta\left(\frac{P_1\tau}{m} - 2X_1\right) = 0$

Our Goal:

- To pass the highly classical GW signal through a quantum detector (test masses) without the detector's quantum properties influencing ("demolishing") the signal

  Quantum Non Demolition (QND)

- By encoding the GW signal in modulations of the laser light & then filtering out frequencies $\lesssim 1\,\mathrm{Hz}$, we automatically succeed!

- Then where does the SQL come from?

  Not from test-mass quantization.

  Rather: Wholly & solely from the light:

$$\Delta X_{shot} = \Delta X_{SQL} \sqrt{I_0 / 2 I_{SQL}}$$

a critical power (I_{SQL})

laser power (I_0)

$$\Delta X_{RP} = \Delta X_{SQL} \sqrt{I_{SQL} / 2 I_0}$$

"Heisenberg microscope"

IF THE SHOT NOISE & RADIATION PRESSURE NOISE ARE UNCORRELATED, THEN

$$\Delta X_{light} = \sqrt{(\Delta X_{shot})^2 + (\Delta X_{RP})^2} \geq \Delta X_{SQL}$$

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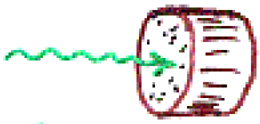
The Nature of Radiation Pressure Noise

– Ponderomotive Squeezing –

- **Light Impinging on Test Mass:**



$$\hat{E} = (E_0 + \hat{E}_{1t}) \cos\omega_0 t + \hat{E}_{2t} \sin\omega_0 t$$

Quantum Noise; varies on time $\sim \frac{1}{\Omega} \sim 10^{-2}$ s

- **Light Pressure:** $\hat{P}_t \propto \overline{\hat{E}^2} \simeq E_0 \hat{E}_{1t} \propto \sqrt{I_0}\, \hat{E}_{1t}$

Component @ frequency Ω ... Fourier transform: $P_\Omega \propto \sqrt{I_0}\, \hat{E}_{1\Omega}$

Drives mirror displacement $\hat{x}_\Omega^{R.P.} \propto \frac{\sqrt{I_0}\, \hat{E}_{1\Omega}}{m\Omega^2}$

- **Mirror Motion Puts Phase Shift on Reflected Light**

$$E_0 \cos(\omega_0 t + k\hat{x}_t) = E_0 \cos\omega_0 t - E_0 k \hat{x}_t \sin\omega_0 t$$

- **Reflected Light**

$$\hat{E}^{out} = (E_0 + \underbrace{\hat{E}_{1t}}_{\hat{E}_{1t}^{out}}) \cos\omega_0 t + \underbrace{(\hat{E}_{2t} - E_0 k \hat{x}_t)}_{\hat{E}_{2t}^{out}} \sin\omega_0 t$$

- **Quantum Noise in Reflected Light, at frequency Ω**

$$\hat{E}_{1\Omega}^{out} = \hat{E}_{1\Omega}, \quad \hat{E}_{2\Omega}^{out} = \hat{E}_{2\Omega} + \alpha \frac{\sqrt{I_0}}{m\Omega^2} \hat{E}_{1\Omega} + \text{GW signal}$$

Ponderomotive Squeeze

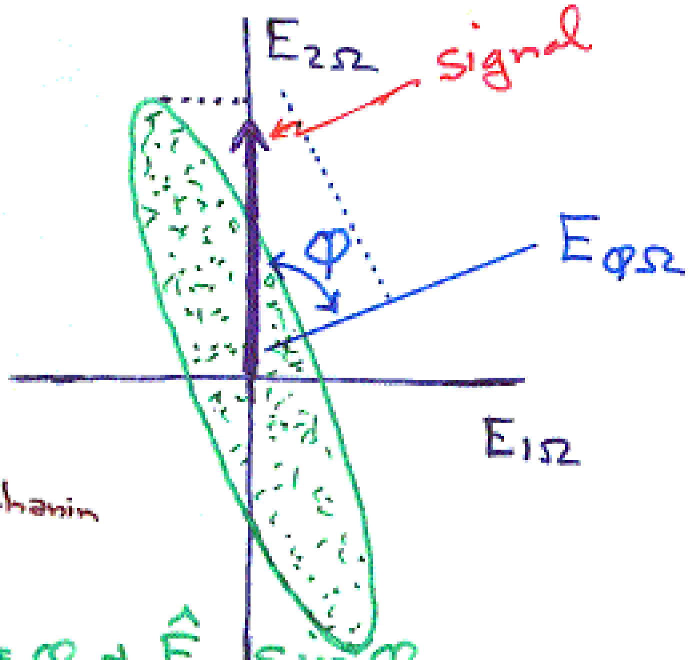
The squeeze is stronger for: Larger I_0 = laser power
Lower Ω = GW frequency

[Braginsky; Matsko & Vyatchanin]

Interferometer Readouts

- Conventional (LIGO-I) interferometer measures $\hat{E}_2$ via photodetection

  $\dfrac{\text{Signal}}{\text{noise}} < 1$



- Simple Idea [Matsko, Vyatchanin & Zubova]

  measure $\hat{E}_\varphi = \hat{E}_2 \cos\varphi + \hat{E}_1 \sin\varphi$

  - via homodyne detection -

  $\dfrac{\text{signal}}{\text{noise}} > 1$ $\boxed{\text{Beats the SQL!}}$

- Problem: $\tan\varphi \propto \frac{1}{\Omega^2}$

  for broad-band QND need frequency-dependent Homodyne phase.

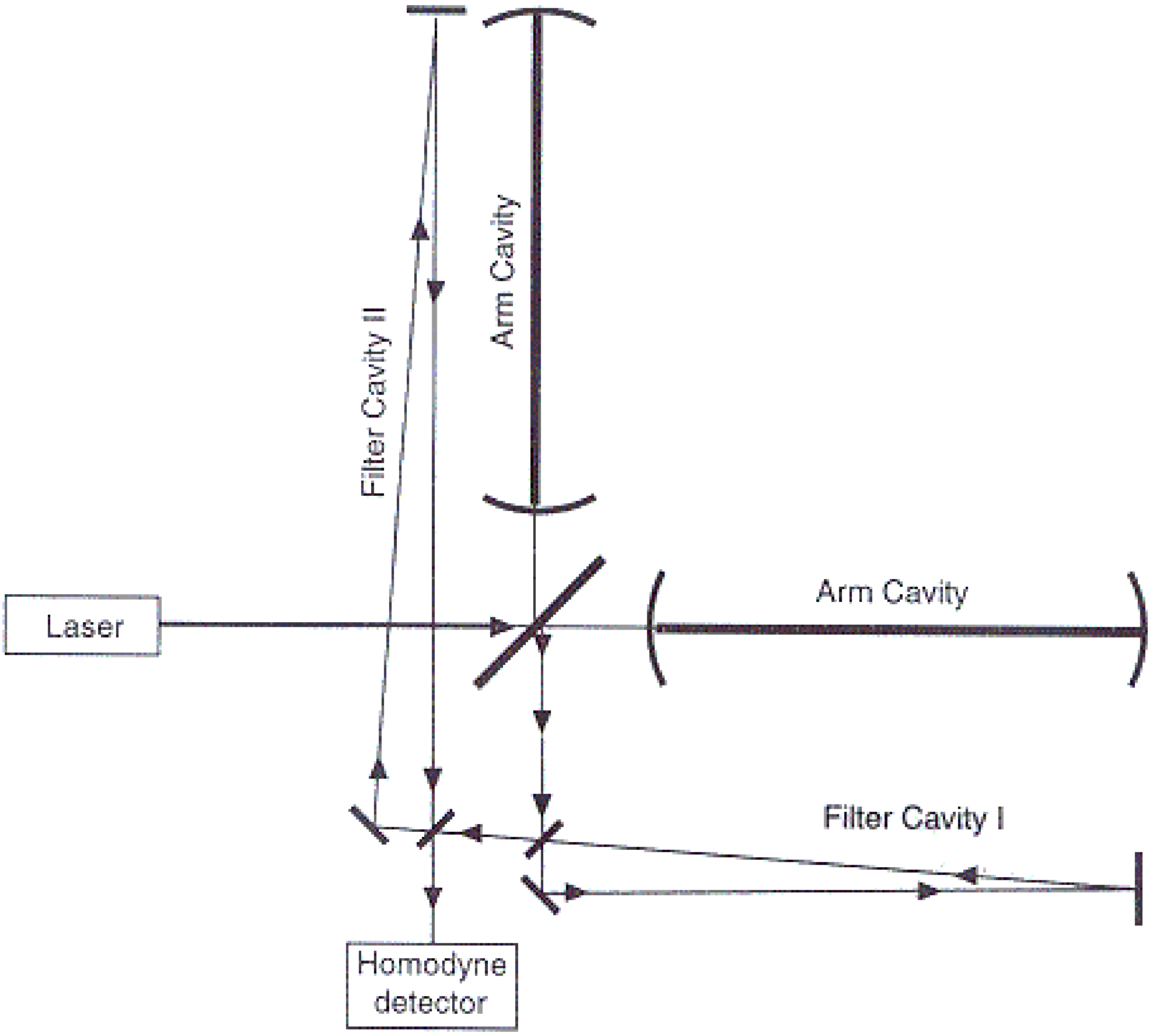
- Solution: Filter the light before detection

  - filtering produces $\varphi(\Omega)$

  [Kimble, Levin, Matsko, Vyatchanin & Kip Phys Rev D, submitted; gr-qc/0008026]

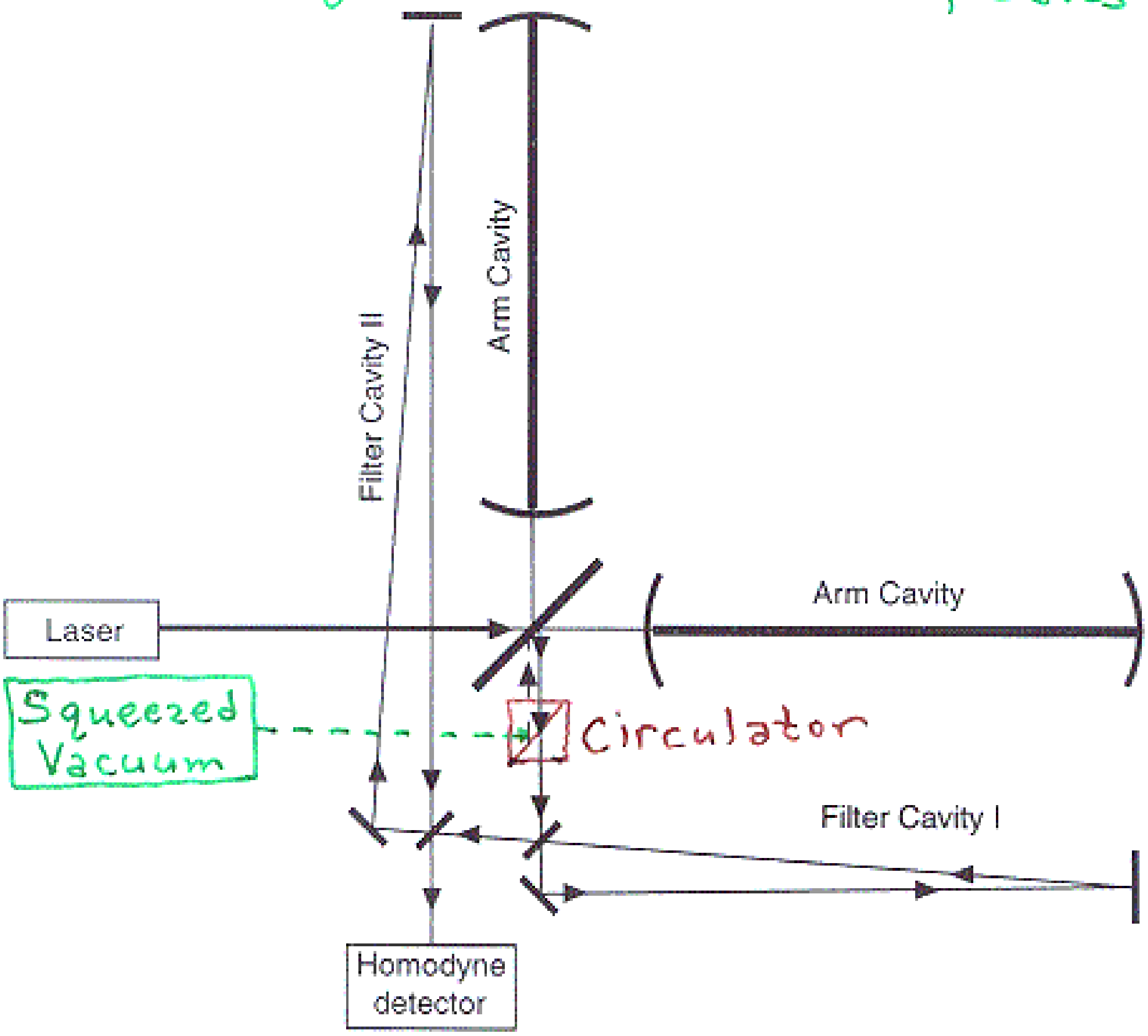
  ... conceptual designs that may be practical for LIGO-III.

Variational Output Interferometer



SQUEEZED VARIATIONAL INTERFEROMETER

SQUEEZED VACUUM Input
(radiation pressure ponderomotively unsqueezes it ... Unruh, Caves - mid 80's)

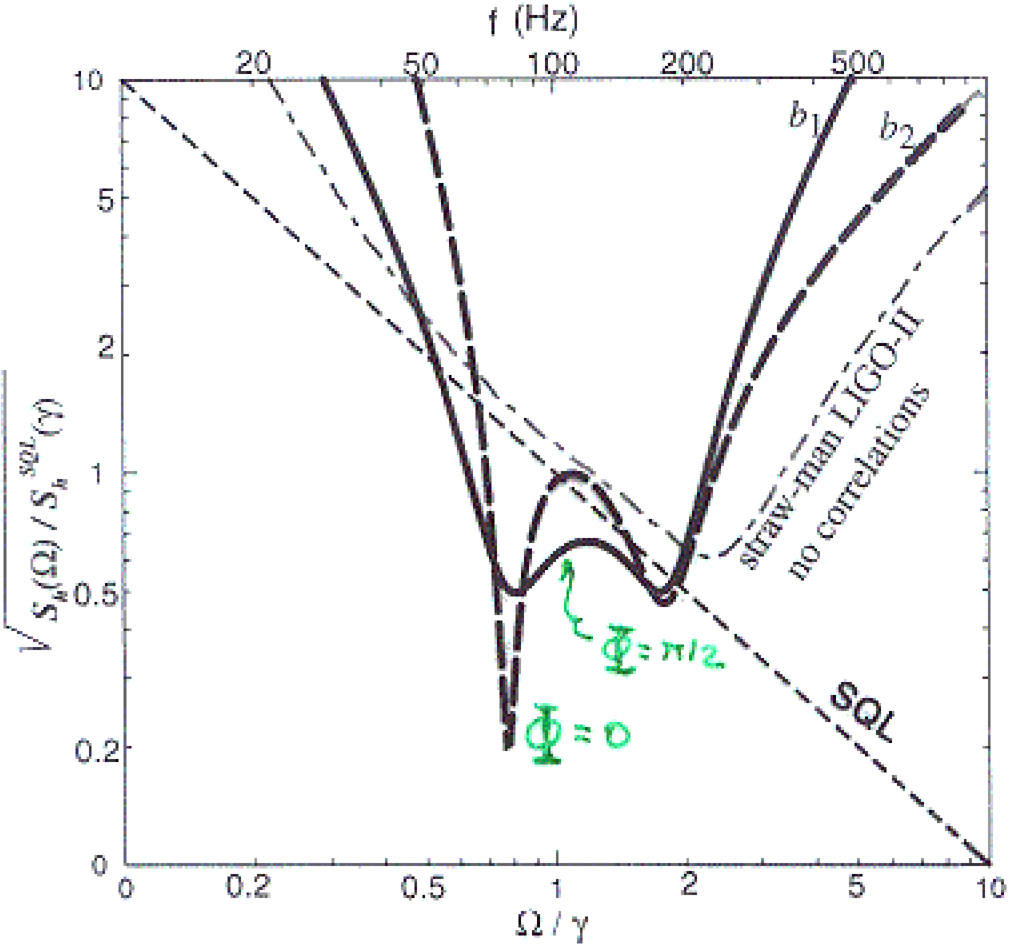


Bottom Line: With vigorous R&D may beat SQL by factor ~ 4 or 5 in LIGO-III using ~same laser power as in LIGO-II.

LIGO-II

[Buonanno & Chen, Phys. Rev. Lett. submitted; gr-qc/0010011]

- The signal recycling mirror filters the interferometer output! [not optimally, but pretty good]
- Produces a measurement of $E_{\Phi\Omega} = E_{2\Omega}\cos\Phi + E_{1\Omega}\sin\Phi$ where $\Phi(\Omega)$ is adjusted by varying reflectivities of mirrors, position of SR mirror, readout parameters



The SQL is not such a big deal after all

– But to beat the SQL, we must also reduce all thermal noises below it [& do the optics very well]

Open Issues – Many!

- **Optimization of LIGO II**
- **LIGO III Designs with Signal Recycling**
- **Speedmeter Performance**
- **QND Interferometer Designs with Low Optical Power (radical design changes)**
  -- Braginsky pushing this hard
- **R&D… achieving low levels of losses**

. . .

see last section of Kimble et al.
gr-qc/0008026