CARDIFF UNIVERSITY

Optimal choice of templates for gravitational waves from inspiralling compact binaries

R.P. Croce[1], Th. Demma[1], V. Pierro[2], I.M. Pinto[2], D. Churches[3] and B.S. Sathyaprakash[3]

B.S. Sathyaprakash
Department of Physics and Astronomy
Cardiff University

LSC@LHO, 15-17 August 00
LIGO-G000219-00-D

B.Sathyaprakash@astro.cf.ac.uk

http://www.astro.cf.ac.uk/pub/B.Sathyaprakash

GEO 600

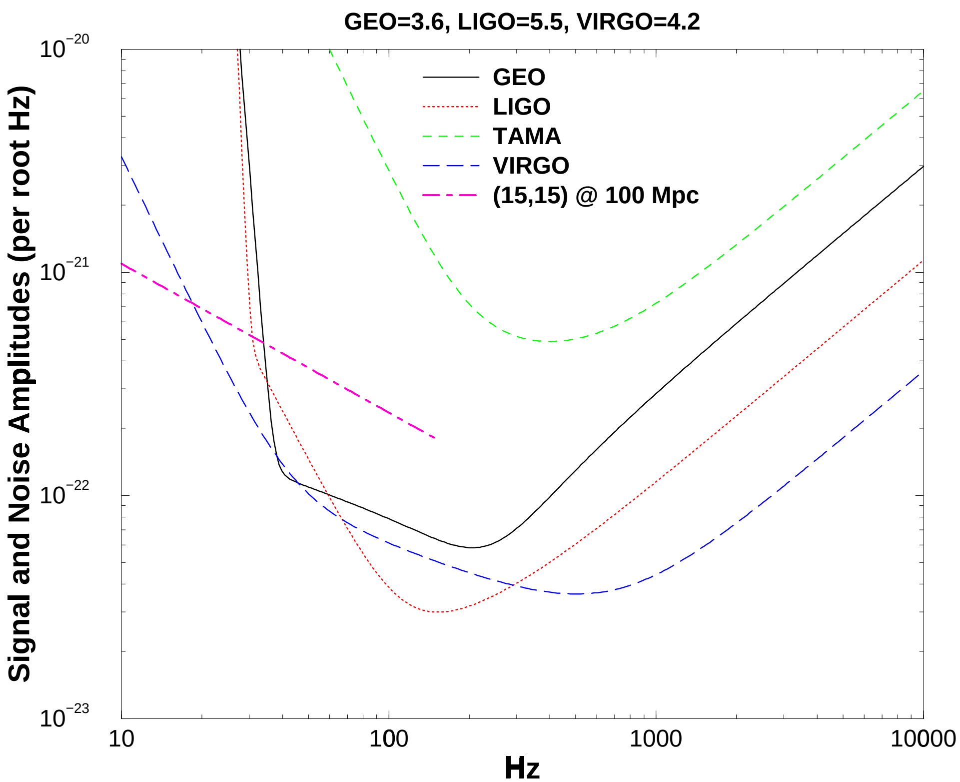
PLAN OF THE TALK

- Prospects for detecting binary inspirals
- Chirp parameters
- Fourier-domain representation of inspiral signals
- Overlaps of exact and approximate waveforms - The match
- Template density
- The interpolation method for Newtonian signals
- The interpolation method for post-Newtonian signals
- Conclusions

B.Sathyaprakash@astro.cf.ac.uk

http://www.astro.cf.ac.uk/pub/B.Sathyaprakash

Noise performance of large interferometers and binary inspiral

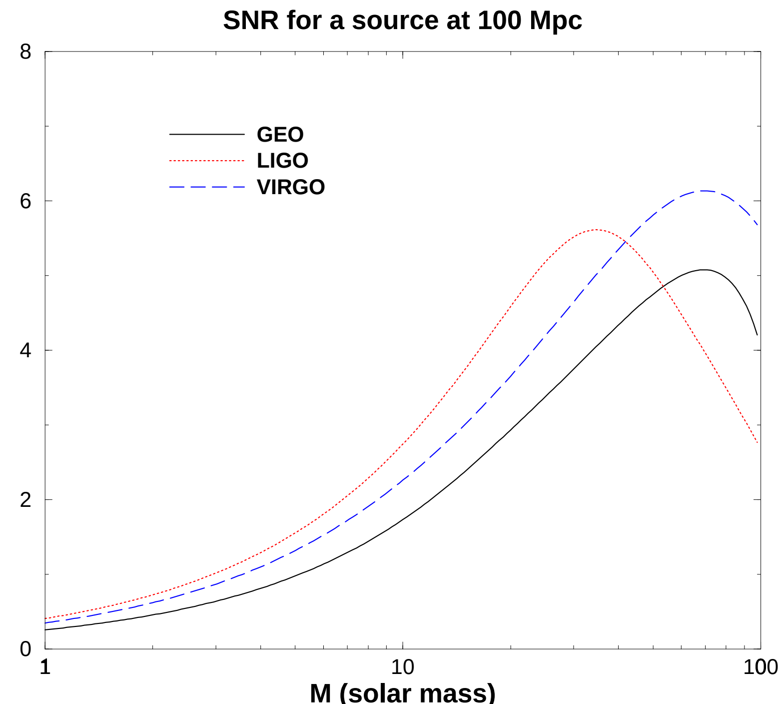


B.Sathyaprakash@astro.cf.ac.uk

http://www.astro.cf.ac.uk/pub/B.Sathyaprakash

SNR for binary inspirals at 100 Mpc

Damour, Iyer & Sathyaprakash, 2000



For unequal binaries SNR goes down by a factor $\eta^{1/2}$ and span is $\eta^{3/2}$ lower.

B.Sathyaprakash@astro.cf.ac.uk

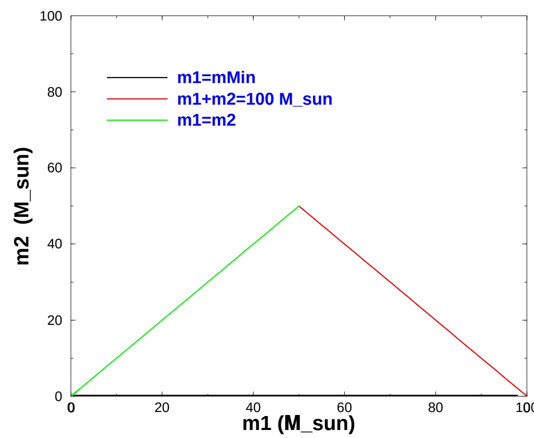
http://www.astro.cf.ac.uk/pub/B.Sathyaprakash

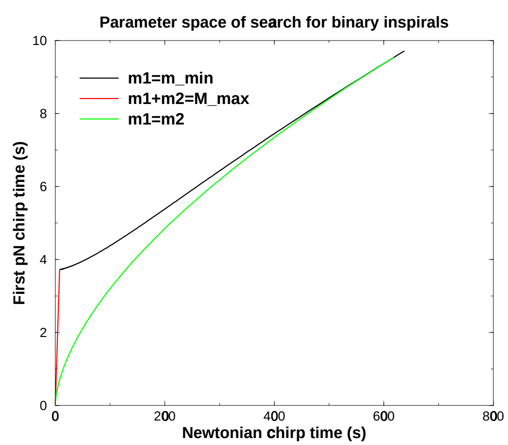
Chirp parameters (formerly *chirp times*)

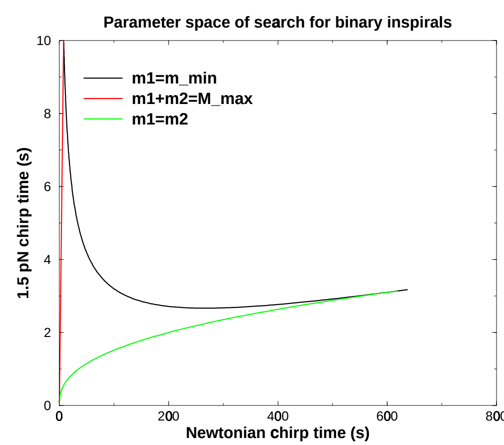
Sathyaprakash 1995

$$\tau^0 = \frac{5}{256\eta v_0^5}, \quad \tau^2 = \frac{5}{192\eta v_0^3}\left(\frac{743}{336} + \frac{11}{4}\eta\right), \quad \tau^3 = \frac{\pi}{8\eta v_0^2},$$

$$\tau^4 = \frac{5}{128\eta v_0}\left(\frac{3058673}{1016064} + \frac{5429}{1008}\eta + \frac{617}{144}\eta^2\right), \quad v_0 = (\pi m f_0)^{1/3}$$







B.Sathyaprakash@astro.cf.ac.uk

Frequency-domain phasing formula

Thorne 1986; Dhurandhar, Schutz & Watkins 1989

The usual stationary phase approximation to the FT of an inspiral signal is

$$\tilde{h}(f) = \frac{C}{2r}\frac{v(f)^2}{\sqrt{\ddot{\varphi}(t_f)}} e^{i[\psi_f(t_f) - \pi/4]}$$

where t_f is the stationary point defined by: $2\pi f \equiv \dot{\varphi}(t_f)$. In terms of the chirp parameters τ^k, the phase takes on a particularly simple form:

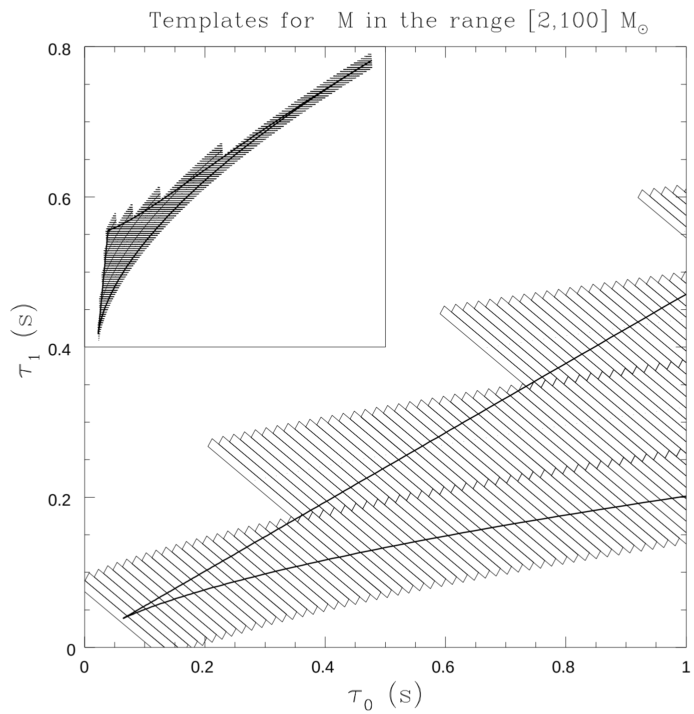
$$\psi_f(t_f) = 2\pi f t_C - \varphi_C + \frac{6}{5\nu^{5/3}} \sum_0^4 \Psi_k(f)\tau^k$$

$$\Psi_0 = 1, \quad \Psi_1 = 0, \quad \Psi_2 = \frac{5}{3}\nu^{2/3}, \quad \Psi_3 = -\frac{5}{2}\nu, \ \Psi_4 = 5\nu^{4/3}, \quad \text{where,} \quad \nu = \frac{f}{f_0}.$$

B.Sathyaprakash@astro.cf.ac.uk

Template density

Sathyaprakash & Dhurandhar 1991, 1994; Owen & Sathyaprakash 1999



To search for $m_{\rm min} \geq 0.2 M_\odot$ we would need $\sim 10^5$ templates. $n_{\rm filters} \propto m_{\rm min}^{-8/3}$.

B.Sathyaprakash@astro.cf.ac.uk

http://www.astro.cf.ac.uk/pub/B.Sathyaprakash

Overlap of waveforms with different parameters

Sathyaprakash & Dhurandhar 1991

Consider two waveforms of slightly different chirp parameters: τ^k and $\tau^k + \Delta\tau^k$. Their overlap, in the stationary phase approximation, is:

$$\mathcal{O}(\Delta\tau^k) = \frac{1}{||h||^2} \max_{\Delta T_c} \left| \int_{f_\mathrm{i}}^{f_\mathrm{s}} \frac{df \;\, f^{-7/3}}{S_n(f)} \exp\left[2\pi i f \Delta T_c + i\Psi_k(f)\Delta\tau^k\right] \right|$$

B.Sathyaprakash@astro.cf.ac.uk

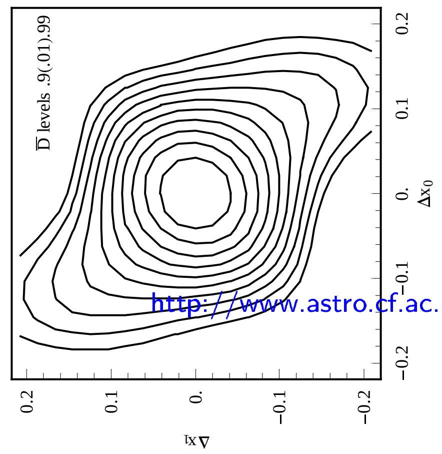


Fig. 2 Contour levels of $\bar{D}(\Delta x_0, \Delta x_1)$.

Interpolation method for Newtonian signals

R.P. Croce, Th. Demma, V. Pierro & I.M. Pinto 2000

A function $h(t)$ with a support $[f_0, f_0 + \Delta f]$ in the frequency-domain is fully represented by its samples $h_k \equiv h(t_k)$, $t_k \equiv k/(2\Delta f)$,

$$h(t) = \sum_{k=-\infty}^{\infty} h_k \text{sinc}(\pi \Delta f t_k)$$

Treat the overlap of a signal with a template as a function of the difference in in their parameter values $\mathcal{O}(\Delta\tau^0)$. Applying the interpolation formula in this case,

$$\mathcal{O}(\Delta\tau^0) = \sum_{m=-\infty}^{\infty} \mathcal{O}(\Delta\tau^0_m)\text{sinc}\left[(\delta\tau^0)^{-1}\pi\left(\Delta\tau^0 - \Delta\tau^0_m\right)\right].$$

Compute overlaps at template bank points using overlaps at interpolating points. At a minimal match of 0.95 this leads to a drop in number of templates by 1.4.

B.Sathyaprakash@astro.cf.ac.uk
http://www.astro.cf.ac.uk/pub/B.Sathyaprakash

Interpolation method for post-Newtonian signals

Generalisation of the method for post-Newtonian chirps is obvious: Simply use the two-dimensional interpolation formula:

$$\mathcal{O}(\Delta x^0, \Delta x^2) = \sum_{m,n=-\infty}^{\infty} \mathcal{O}(\Delta x^0_m, \Delta x^2_n) \ \mathrm{sinc}\left[\frac{\pi\left(\Delta x^0 - \Delta x^0_m\right)}{\delta x^0}\right] \mathrm{sinc}\left[\frac{\pi\left(\Delta x^2 - \Delta x^2_n\right)}{\delta x^2}\right]$$

where $\Delta x^0_{m+1} - \Delta x^0_m = \delta x^0$, $\Delta x^2_{n+1} - \Delta x^2_n = \delta x^2$ and (x^0, x^1) are isotropic coordinates

$$\Delta\tau^0 = \Delta x^0 \cos\vartheta - \Lambda \Delta x^1 \sin\vartheta,$$
$$\Delta\tau^1 = \Delta x^0 \sin\vartheta + \Lambda \Delta x^1 \cos\vartheta.$$

At a minimal matches ≥ 0.95 the number of interpolating templates are only 25% of all lattice templates.

B.Sathyaprakash@astro.cf.ac.uk

B.Sathyaprakash@astro.cf.ac.uk

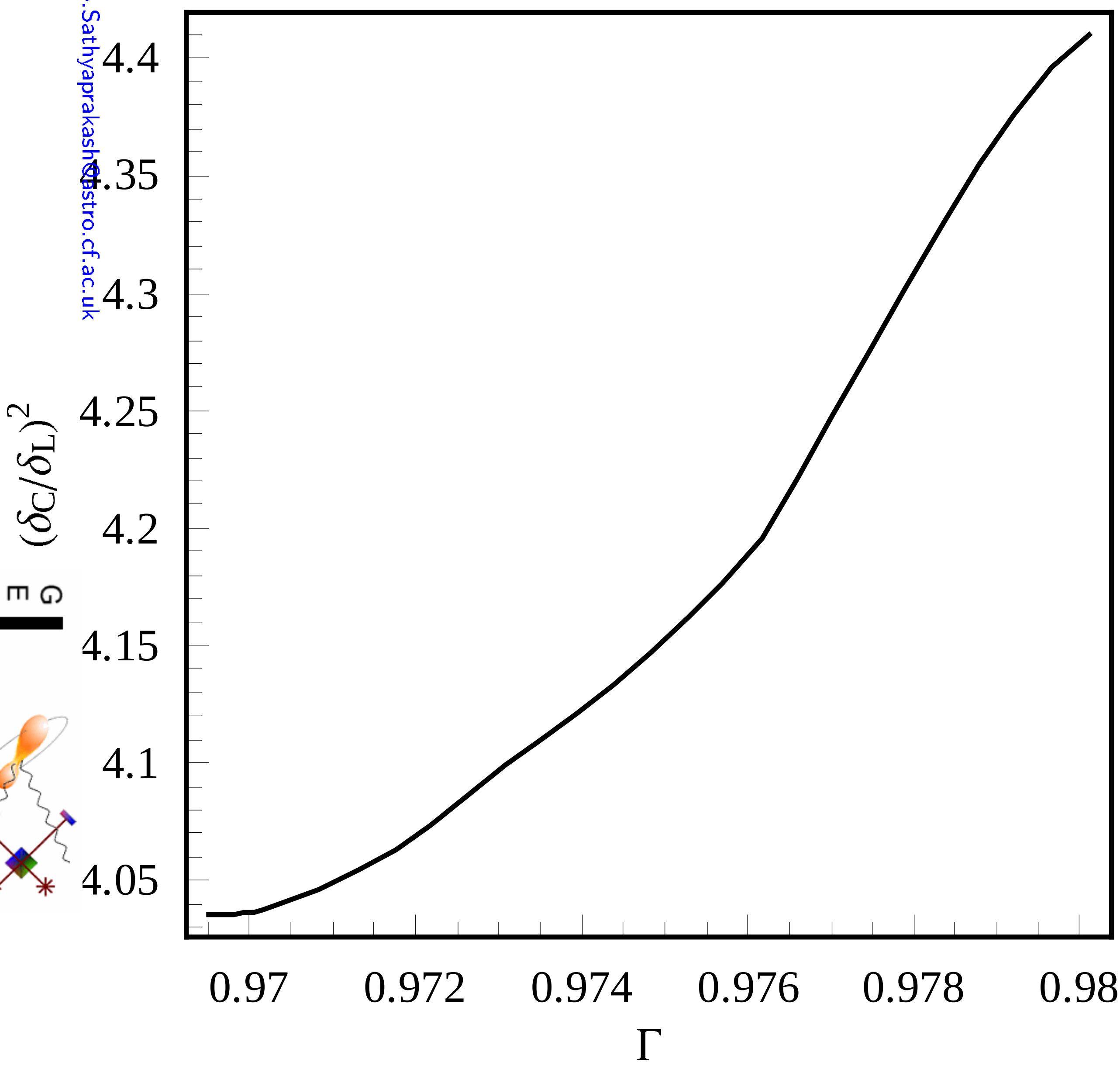


Fig. 8 - 2D template density reduction vs. Γ.

Conclusions

- For minimal matches > 0.95 interpolation method saves on computational costs by a factor 4.

Future work needed

- Theory for minimal matches < 0.90 likely to be complicated
- Simulation in the presence of noise (simulated (Gaussian) as well as real)
- Search software in LAL standard (possible next milestone for Cardiff)