Developing an Earth-Tides Model for LIGO Interferometers, Abstract

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The gravitational pull of the Sun and the Moon causes tidal strains on the LIGO interferometers which changes the distance between the mirrors and the largest background effect removed by LIGO's servo mechanisms. I am incorporating a theoretical model in Paul Melchior's "The Tides of the Planet Earth," a set JPL planetary ephemerides and the US Naval Observatory's NOVAS program (which can find the position of the Sun and the Moon relative to the Earth) into a C-based program that will calculate Earth tides.

The program is currently producing what is believed to be valid theoretical data (although there is as of yet no experimental data to compare it to). According this data, the arms of the interferometers should change length by about two hundred sixty microns in normal use. I am currently waiting for the acquisition of real data to analyze with a linear fitting routine.

Introduction

Earth tides are essentially the shifting and stretching of the Earth as a result of the gravitational pull of the Moon , the Sun or other celestial bodies. This stretching of the Earth will cause the ground underneath an interferometer like LIGO to expand and contract during the course of a day. This movement in turn changes the distance between the mirrors of the interferometer. If this effect is not dealt with, it will be much greater than the effects on the order of 10^{-18} meters that LIGO is looking for. Actuators will be used to take out the effects of Earth tides. We need to have a model of Earth tides so that we can distinguish them from other low frequency effects and so that we know where to preset the actuators so that the are not pushed out of their dynamic range by the tides.

The gravitational field exerted by an external body of mass M at a distance d from the Earth is of course:

$$g = G M / d^2$$

The field is not uniform. The force it causes changes with d, the distance. The gravitational force at the center of the Earth (caused by the Moon, for instance) balances with the centrifugal force and the acceleration of the orbit. But the side of the Earth closer to the external body is pulled more strongly by the external body so that the gravitational force is not cancelled out by the acceleration. This causes the Earth to be stretched towards the external body on the side that is closest to it.

Similarly, the side of the Earth that is opposite the external body feels a weaker gravitational force and again the gravity and acceleration do not balance out. Since the gravity is weaker here than at the center of mass of the Earth, it is overcome by the acceleration, and this side of the Earth "falls away" from the external body. Thus, the side of the Earth opposite the external body stretches away from the external body.

Since this force, called the tidal force, is a result of a difference in the gravitational field which is in turn caused by a difference in distance, and since this distance is equal to the radius of the Earth, the force is proportional to the derivative of the gravitational field with respect to distance times the radius of the Earth, r.

$$F_{tidal} \alpha\, r \ (dg/dd) = - 2 G M r / d^3$$

While the basic idea of Earth tides is fairly simple, we are interested in calculating the tidal strain on an interferometer arm at LIGO. This quantity is a good deal more complicated than the above equation because one must take into account the coordinates of the site, the coordinates of the external body, the direction in which one wants to calculate the strain and the stiffness of the Earth.

Paul Melchior, Director Emeritus of the International Center for Earth Tides, has made a theoretical model of Earth tides[1] which provides the model for tidal strain that we are using at LIGO. It breaks the effects down into three different waves, the tesseral (with approximately a twenty four hour cycle), the sectorial (twelve hour cycle) and the zonal

[1] Published in his book, *The Tides of Planet Earth*, Pergamon Press, Oxford, (1978).

(month cycle for the Moon and year cycle for the Sun). These three components can be simply added to obtain the total tidal effect. They are equal to the following expressions:

Tesseral component =

$$\{[(h - 4l)\cos^2\beta + (h - 2l)\sin^2\beta]\cos H + [2l \sin\beta \cos\beta/\cos\theta]\sin H\} * D\,(c/d)^3\,(r/a)^2\,M_{body}\sin 2\theta \sin 2\delta / a\,g\,M_{lunar}$$

Sectorial component =

$$[\{[h \sin^2\theta + 2l(1 - 2\sin^2\theta)]\cos^2\beta + [h \sin^2\theta - 2l(1 + 2\sin^2\theta)]\sin^2\beta\}\cos 2H + \{(4l \cos\theta)\sin\beta\cos\beta\}\sin 2H] * D\,(c/d)^3\,(r/a)^2\,M_{body}\cos^2\delta / a\,g\,M_{lunar}$$

Zonal component =

$$[\{h(\cos^2\theta - 1/3) - 2\,l\cos 2\theta\}\cos^2\beta + \{h(\cos^2\theta - 1/3) - 2\,l\cos^2\theta\}\sin^2\beta] * D\,(c/d)^3\,(r/a)^2\,M_{body}(\sin^2\theta - 1/3) / a\,g\,M_{lunar}$$

Explanation of terms:

Astronomical constants:
a = the radius of a sphere with the same volume as the Earth = 6,371,023.6 meters
c = mean distance from the Earth to the Moon = 384,000,000 meters
D = Doodson's Constant = $3GM_{lunar}a^2/4c^3$

Site coordinates:
λ = longitude
ϕ = latitude
θ = colatitude = $\pi/2 - \phi$
r = radius at site

Sidereal variables:
H = $\omega t' - \alpha - \lambda$
t' = sidereal time
ω = the Earth's sidereal angular velocity

Celestial body coordinates:
α = right ascension
δ = declination
d = distance from Earth

β and its components:
$\cos\beta = \sin z \cos\psi$
z = zenith angle = $\delta - \lambda$
ψ = azimuth angle = direction in which strain is being calculated, the direction of the beam tube (0 = due north)

Love numbers, h and l:
Measurements of how stretchable the Earth is. They vary by site and must be measured, but are generally equal to about .6 and .08, respectively.

The terms containing the angle H are particularly important. H is essentially a local hour angle that takes into account the right ascension of the external body. It varies with the sidereal velocity (rotation) or the Earth. It is an angle that cycles once per sidereal day. A sidereal day differs from a normal, solar day in that a sidereal day is one rotation of the Earth with respect to the stars while a solar day is one rotation with respect to the Sun and is thus affected by the orbit of the Earth around the Sun. H changes much quicker than the angles associated with the external bodies, δ , α, z and β (the latter two vary with δ) which are the only other angles that vary with time. This means that over short time spans (such as a day) H will be the only angle, and indeed the only variable, that changes quickly. The rest of the expression will remain relatively constant.

The tesseral term, which contains the terms cos H and sin H will appear to be essentially a simple sinusoidal function with a period of one sidereal day. There will be long term changes in the amplitude of this wave, but over the course of a day its amplitude will remain relatively constant.

Similarly, the sectorial term, which has the terms cos 2H and sin 2H, will manifest itself as a sinusoidal function with a period of half a sidereal day. The zonal term, which has no H term, will remain essentially constant during a day and will only slowly change as the declination of the external body changes.

Using Melchior's model of Earth tides, my work has been to create a program using the C programming language, which can predict Earth tides and also fit the model to real data. Most of the uncertainty that must be fit to comes in the Love numbers (geophysical measurements of the rigidity of the Earth), so these are the parameters that are adjusted in the fit.

The program predicts that the distance between the LIGO mirrors will fluctuate within a range of about 260 microns (peak to peak), making it one of the largest background signals. It also makes theoretical predictions about how large the Earth tide effect will be at any given time. There is no experimental data to confirm or refute the output of the program, yet. In the future, we would like to use the fitting routine embedded in the program to fit real data to Melchior's theoretical model so that the Earth tide strain on a LIGO interferometer can be predicted accurately.

Materials and Methods

The program that I have made is called "Paladin." This program takes in a time (measured either in Julian dates or in the more standard year, month, day, hour times) and data for the site. Since this program was designed for the LIGO interferometers, it calculates the effects of the Earth tides as a strain in a particular direction, included in the site input.

A major difficulty in producing this program was finding a way to obtain the coordinates of the Moon and the Sun (with respect to the Earth) at any given time. This was eventually accomplished with the aid of a program made by the US Naval Observatory (USNO) called NOVAS and solar system ephemerides files containing position data for our solar system produced by the Jet Propulsion Laboratory (JPL).

Paladin uses the US Naval Observatory's NOVAS program, C version[2]. Specifically, Paladin uses the subroutine app_planet to calculate the position of the body causing the tidal strain. Most likely the user will be concerned with the Sun and the Moon). Paladin uses the subroutine julian_date to find Julian dates using the year, month, the day and the hour. It uses the subroutine sidereal_time to find the sidereal time of the Earth. It uses the subroutine Earthtilt to find the equation of the equinoxes which is used in sidereal_time. The files novas.c, novas.h, novascon.c, novascon.h, solsys2.c, solarsystem.h, and reaeph0.c must be compiled and linked with Paladin for it to run.

The NOVAS program requires ephemerides files, which contain the positions of the nine planets, the Moon, and the Sun. We make use of the JPL ephemerides unxp1950.405, unxp2000.405, unxp2050.405 and unxp2100.405[3]. These files contain the position of the desired objects from the years 1950 to 2150. These ephemerides are read by the JPL FORTRAN subroutine PLEPH found in testeph.f, a program made by JPL. The NOVAS program jplint.f acts as a link between the JPL FORTRAN and the NOVAS C.

Paladin takes in a date and time from the user, finds the position of the Sun and the Moon using the JPL ephemerides and NOVAS and using this information, site information and Melchior's model, calculates data associated with the tidal strain caused by the Sun and the Moon (the tidal strain caused by other bodies is insignificant).

This data can be expressed in several forms. The program can output the total strain felt by one arm of the interferometer. It can output the common mode, the average of the tidal strain felt by the two arms at one site (the tidal strain will differ arm by arm because of differences in the bearing of each arm). It can output the differential mode, the difference between the tidal strain felt by each arm. It can also output the individual components (tesseral, sectorial or zonal) of the wave. It can further break the tesseral or sectorial components down into their "sine component" (the part that is due to the sin H

[2] NOVAS was produced by George H. Kaplan and others at the United States Naval Observatory. The C version of NOVAS (used in Paladin) was produced by John A. Bangert. Information on the program can be found at http://aa.usno.navy.mil/AA/software/novas/novas_info.html.

[3] The documentation for the JPL ephemerides was written by Dr. E. Myles Standish at the Jet Propulsion Laboratories. Information on the ephemerides can be found at http://www.willbell.com/software/jpl.htm.

or sin 2H terms) and their "cosine component" (the part that is due to the cos H or cos 2H terms).

The most detailed output that the program can produce is a set of twelve components per external body, each of which is linearly dependent on the Love numbers. This last form of data output is important because it will allow Paladin to fit these the Love numbers to real data using a linear fitting routine. The linear fitting routine used in Paladin is called lfit and it was taken from "Numerical Recipes in C.[4]"

[4] Paladin makes use of the C code function lfit, from page 674 of "Numerical Recipes in C," second edition. In addition, lfit makes use of the functions gaussj (page 39) and cosvrt (page 675).

Results

Before any programming was begun, several important preliminary calculations were performed. The first was to determine the relative strengths of the tides caused by different external bodies. All tidal effects are proportional to the mass of the external body divided by the cube of the distance to that body.

If one calculates M / d^3 for the bodies in our solar system, the Moon produces the largest number ($1.3 * 10^{-3}$), followed by the Sun ($5.9 * 10^{-4}$), and Venus is a distant third (about 10-^{7} at minimum distance). So we need only worry about the Sun and the Moon.

F. Raab and M. Fine also made some useful approximations of what the Earth tides experienced by the LIGO interferometers should be in their paper entitled "The Effect of Earth Tides on LIGO Interferometers.[5]" They estimated that the following approximate peak to peak amplitudes in the tidal oscillations:

Estimates for the Maximum Range of Earth Tides Made by Fine and Raab

	Hanford	Livingston
Tesseral Common Mode	113.6 microns	99.4 microns
Tesseral Differential Mode	22.0 microns	20.0 microns
Sectorial Common Mode	71.0 microns	110.7 microns
Sectorial Differential Mode	31.1 microns	30.4 microns

Paladin has also made theoretical predictions for the total common and differential modes, which are larger than the sum of the tesseral and sectorial common and differential modes estimated by Fine and Raab. The maximum range (peak to peak) of Earth tide displacement observed from Paladin calculations are as follows:

The Largest Range of Earth Tides Outputted by Paladin

	Hanford	Livingston
Total Common Mode	213.6 microns	219.6 microns
Total Differential Mode	161.6 microns	86.8 microns

The fact that Paladin found common mode and differential modes greater than the sum of the tesseral and sectorial common and differential modes estimated by Fine and Raab is largely due to the fact that Paladin included the zonal term and accounted for slow changes in declination of the external body. These are long term effects that will not appear during the course of a day. During the course of one day, Paladin predicts the common and differential modes at Hanford and Livingston to have a significantly smaller peak to peak range:

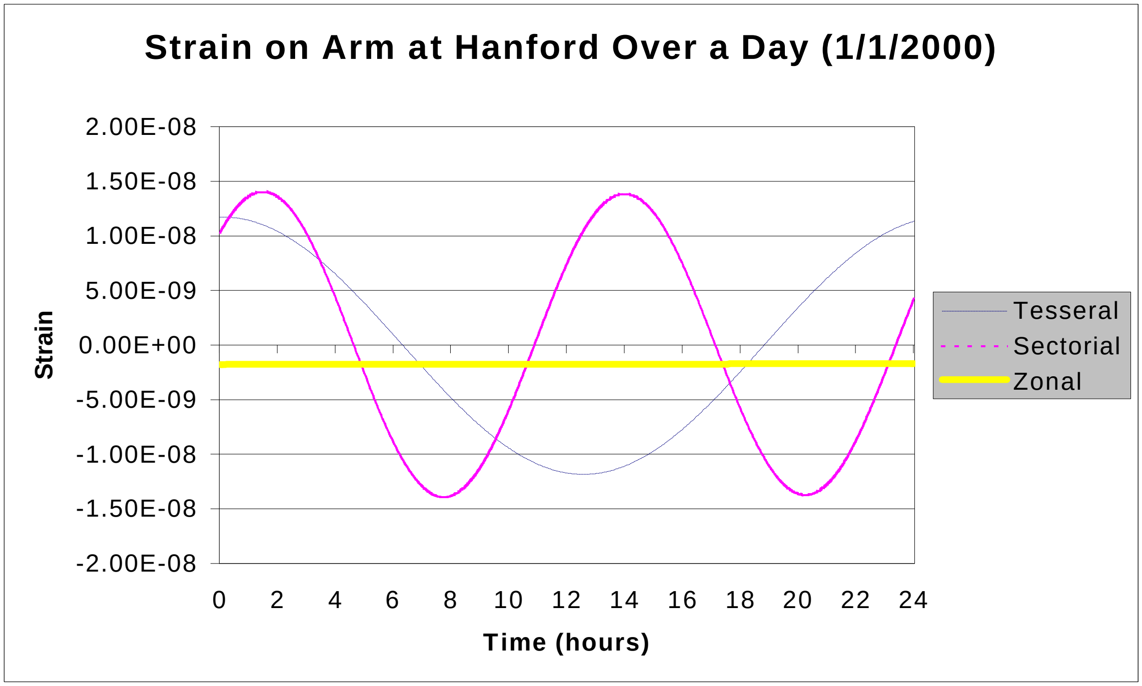
Expected Daily Ranges of Earth Tides Predicted by Paladin

	Hanford	Livingston
Total Common Mode	80-180 microns	80-180 microns
Total Differential Mode	30-90 microns	25-60 microns

[5] F. Raab and M. Fine, *The Effect of Earth Tides on LIGO Interferometers*, LIGO-T970059-01-D, http://www.ligo-wa.caltech.edu/earth_tides.pdf, (1997).

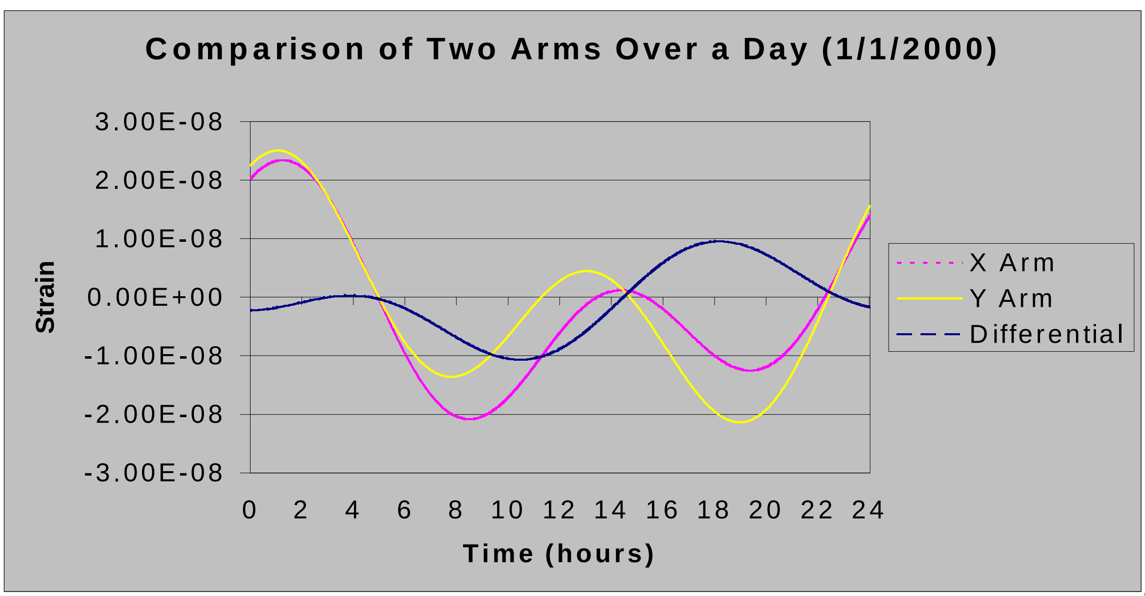
Analysis

Data produced by the Paladin program has facilitated analysis of the general behavior of Earth tides. The following is a graph of the tidal effects caused by the Moon over a day. The three tidal components are plotted separately. Note that the y axis values are strain and not displacement (one must multiply by 4000 meters to convert them):



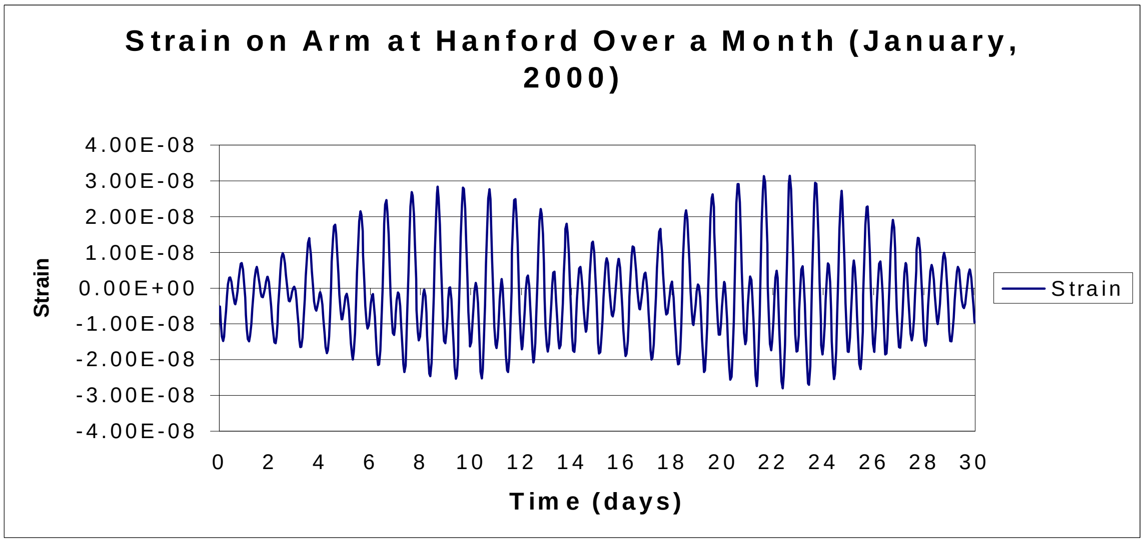
As expected, the tesseral component is a simple sine wave with a period of one day, the sectorial component is a sine wave with a period of half a day and the zonal component remains essentially constant.

We also make observations on the amplitude of the total wave (caused by all three components of the tides of both the Sun and the Moon) affecting each arm and the difference in amplitude between these two waves:



In this graph we see two important behaviors. First, the total strain on each arm tends to undergo one oscillation at high amplitude followed by an oscillation of lower amplitude. This stems from the fact that the sectorial wave (with a twelve hour cycle) tends to interfere constructively with the tesseral wave (with a twenty four hour cycle) for one oscillation, creating a large amplitude and then interfere destructively for another oscillation, creating a small amplitude. Physically, this phenomenon comes from the fact that there are two tides, one facing the external body and one opposing it. Unless the external body is directly over the equator, these two tides will have maxima at different latitudes so that one will be closer to the site, creating a large tide, and one will be father from the site, creating a small tide. We also see that the differential mode has a smaller amplitude than the total strains. This will be important when we devise methods for removing the effects of the tides (discussed later).

Paladin has also produced long term data that shows several more interesting effects:



First, the alternating between a high amplitude oscillation and a low amplitude oscillation mentioned earlier is evident if one looks at an individual day on the graph. Second, there is a general expansion and contraction of the envelope of the oscillations with a period of about half a month. This is due to the revolution of the Moon around the Earth. When the Moon is either directly in line with the Sun, as it is during a new Moon, or directly opposing it, as it is during a full Moon, the tides caused by the two will tend to interfere constructively. While it may seem counterintuitive that the same effect is produced when the Moon is in the same direction as the as when the Moon is in the opposite direction, one must remember that if the Moon is opposite the Sun, the land opposing the Moon will tend to "fall" away from the Moon and towards the Sun and when they are in the same direction, both the Sun and the Moon will be pulling in the same direction. When the Moon is perpendicular to the Sun from the Earth, as in a half Moon, the two will interfere destructively. Finally, even the amplitude of the maximum wave at constructive interference changes because the declination of the Moon and Sun slowly changes.

Conclusion

Possibly the most important question that one must ask when confronting the issue of Earth tides is whether or not their large amplitude effects can be dealt with. The actuators currently being used at LIGO each have a maximum range of about 140 microns. Paladin predicts that each arm at the LIGO Hanford site should expand and contract within a range of about 220 microns. The daily oscillations should be significantly less, but are predicted to be as high as 180 microns which is well beyond the range of a single LIGO actuator.

Fortunately, by modulating the wavelength of the laser being used in the interferometer one can maintain the lock on the interferometers and let the mirrors drift as long as both arm lengths change by the same amount. This change in wavelength compensates for the common mode of the tides, leaving the actuators to deal with only the differential mode. This differential mode has a predicted maximum peak to peak amplitude of 161.6 microns at Hanford and 86.8 microns at Livingston. Since the differential mode causes a difference in movement between LIGO's two end mirrors, one can drive one mirror move one way (towards the vertex of the interferometer) and drive the other end mirror the opposite direction (away from the vertex of the interferometer) so each mirror need only move by half the differential mode amplitude. These experimental techniques will theoretically limit the range of required motion to 45 microns in a day, well within the maximum range of the actuators.

While Melchior's model has been validated experimentally, there is currently no experimental data to check the model as applied to the LIGO sites at Hanford or Livingston. The Love numbers have an uncertainty of about ten percent. Since they are linear coefficients in the Earth tides model, there is an uncertainty of approximately ten percent in any calculations made with the model. Local topography such as a large ridge at the Hanford site may cause significant deviations away from the ideal. There is also uncertainty in how ideally the desert terrain at Hanford or the forested terrain at Livingston will be. It is believed that with the linear fitting, these factors should be mostly accounted for, and the model should prove to be at the very least a good estimate of actual tides. Experimental data will be needed to firmly establish the validity of the model's predictions for Earth tides at the LIGO sites.

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