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David Blair

LIGO Hanford Observatory
P. O. Box 1970; Mail Stop S9-02
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (504) 686-3100
Fax (504) 686-7189
E-mail: info@ligo.caltech.edu

California Institute of Technology
LIGO Project - MS 18-34
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

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Abstract

This report reviews recent estimates of high frequency stochastic background signals from the era of early star formation. Independent estimates of this background are summarised. The possibility of detection of this background through cross correlation between LLO and ALLEGRO is discussed. The LLO-ALLEGRO pair has the unique capability of being a stochastic background detector for which the signal can be switched on and off by 45 degree rotation of ALLEGRO. It is shown that advanced LIGO sensitivity combined with planned improvements in ALLEGRO can reach astrophysically significant levels. Advanced resonant mass technology combined with narrow band operation of LIGO could allow the predicted background to be detected with a high signal to noise ratio depending on the birth spin rate of neutron stars.

1. Introduction.

Stochastic gravitational waves can be detected by cross correlation of nearby detectors. To distinguish a real background one has to ensure that no local sources of correlation exist that could mimic an extraterrestrial source. The idea is that detector noise in each detector is uncorrelated, but the signals are correlated, so that a long term integration of the product signal from the two detectors will have a signal component that increases linearly with time, while the noise component will increase only with the half power of integration time.

In section 2 of this report, estimates of stochastic gravitational waves from the era of early star formation are reviewed. In spite of rather poor knowledge of gravitational wave emission from individual sources, it is shown that recent independent estimates agree that this background exists and is potentially detectable with energy density Ω_g (as a fraction of closure density) $\sim 2.10^{-8}$ with frequency in the 100Hz-1.5kHz range. The background has distinctive spectral and statistical properties. Section 3 reviews the sensitivity of cross correlation detection, and its dependence on the overlap function for the two detectors. In section 4 the choices of orientation of ALLEGRO are discussed, including the unique capability that exists for the stochastic signal to be switched off by simple 45 degree rotation of ALLEGRO. This is possible because the detectors are essentially coplanar being locally horizontal and close together. This capability allows local correlations to be measured independently. Section 5 considers the specific case of the ALLEGRO-LLO pair of detectors. It is shown that a significant signal to noise ratio for the predicted stochastic background will be eventually achievable if both detectors are improved as planned. The report concludes with a set of recommendations.

2. Estimates of ESF Stochastic Background

Most estimates of stochastic gravitational waves from the *early universe* indicate a gravitational wave amplitude below the reach of planned terrestrial gravitational wave detectors. However stochastic signals from the *era of early star formation* (ESF era) present a different picture. The ESF stochastic background is estimated by integrating single source estimates (event rate, amplitude and spectrum) to cosmological distances. The first preliminary estimate (Blair and Ju 1996, hereafter BJ96) considered gravitational radiation from supernovae and/or the rapid spin down of new born pulsars due to mass quadrupole instabilities.(see for example Houser et al 1994). This has now been followed up with further estimates (Ferarri et al 1999, Schneider et al 1999(hereafter SFM99) ,Ferrari et al 1998(hereafter FMS98), Burman et al 1999, Owen et al1998), all of which confirm the existence of a potentially detectable ESF stochastic background spectrum as discussed below.

The ESF background estimates depend critically on two factors. The first is the star formation rate density ρ_* in the ESF era. BJ96 assumed that ρ_* increased by a factor of 100 at redshift z~1-2, while recent data indicates that ρ_* increases by a factor ~ 15-20. The second is the total efficiency for gravitational wave production. BJ96 assumed conversion efficiency~10^{-3} consistent with predicted radiation due to mass quadrupole deformations of rapidly rotating neutron stars (such as predictions by Houser). Owen et al and FMS98 considered only unstable mass current modes (r-modes) which according to recent analysis are inevitable in all rotating stars (Friedman and Morsink (1997). Owen et al (1998) showed that r-modes cause newborn neutron stars to spin down on a timescale of hundreds of seconds, thereby converting ~1% of rest mass to gravitational waves. Mass quadrupole instabilities can cause more rapid spin down. SFM99 separately considered the gravitational waves emitted from black hole forming collapses of massive stars, using predictions for axisymmetric collapse by Stark and Piran(1985).

Owen et al (1998) obtained a "rough first cut" estimate of the ESF background assuming a constant star formation rate up to z=4. They analysed particularly the low frequency most highly redshifted component which might be detectable by the two LIGO observatories. (Non-redshifted components have frequency too high for cross correlation between LLO and LHO.) FMS98 repeated Owen

et al's analysis using latest star formation rate data and various cosmological models.
The statistical properties of the background depend on the emission mechanism. Supernovae create short duration events (milliseconds) concentrated at higher frequency. R-modes create long duration events. R-modes are likely to coexist with mass quadrupole deformations, which will act to shorten the spin-down time. The short duration events create a *popcorn noise* component unless the event rate exceeds a few hundred per second, while long duration events merge into a continuous background except for nearer but much less frequent events.
This property of the spectrum -nearer sources are stronger and less frequent while the further sources merge into the Gaussian background- is characteristic of popcorn noise. The spectrum has a power spectral density proportional to f^{α}, $\alpha>1$. Processes of this type are well known to electrical engineers. See for example Motchenbacher and Fitchen(1973)) The popcorn component will be stronger if the spectrum is dominated by shorter more intense emission processes.

The major predictions for the ESF background are the following:

1) Bursts from black hole and neutron star formation occur at a rate of about 20 times per second, with about 25% being massive systems which give rise to black holes. This result will change as the high redshift star formation rate becomes better known.

2) Bursts from black hole formation make a small contribution to the total background if the axisymmetric collapse calculations are correct. They produce a popcorn noise component in the background spectrum.

3) Neutron star formation and rapid evolution by mechanisms such as bar mode instability, or r-modes produces a generally continuous background component to the spectrum.The individual signals are of longer duration and they overlap although nearer stronger components still contribute to making the spectrum ultimately non-Gaussian.

4) The spectrum should be modelled as one containing both a *popcorn* component and a Gaussian component. This signature should be used to develop efficient search algorithms.

5) The ESF stochastic background intensity and frequency distribution functions can be related to the star formation history of the universe, and hence represents a powerful probe of the era of star formation.

6) The spectral energy density of the ESF stochastic background, expressed in the units of closure density, is ~ 1-3 x 10^{-8}, dominated by the neutron star

component. For bandwidth reasons it is likely to be most detectable at high frequency 400Hz to 1kHz using close spaced pairs of detectors.

Figure 1 presents the stochastic background spectrum predicted by Ferrari et al (top curve, derived from SFM99), Blair et al, (middle curve) and a generic stochastic background from the early universe with $\Omega = 10^{-8}$ (straight line). The spectrum is dominated by the longer duration events associated with neutron star formation and evolution, but there can be little doubt that even this component will contain a popcorn noise component due to less frequent closer events. Figure 2 gives the r-mode ESF spectrum, expressed in terms of closure density,(as defined in Allen (1996) eq 2.8 and 2.9) from FMS98.

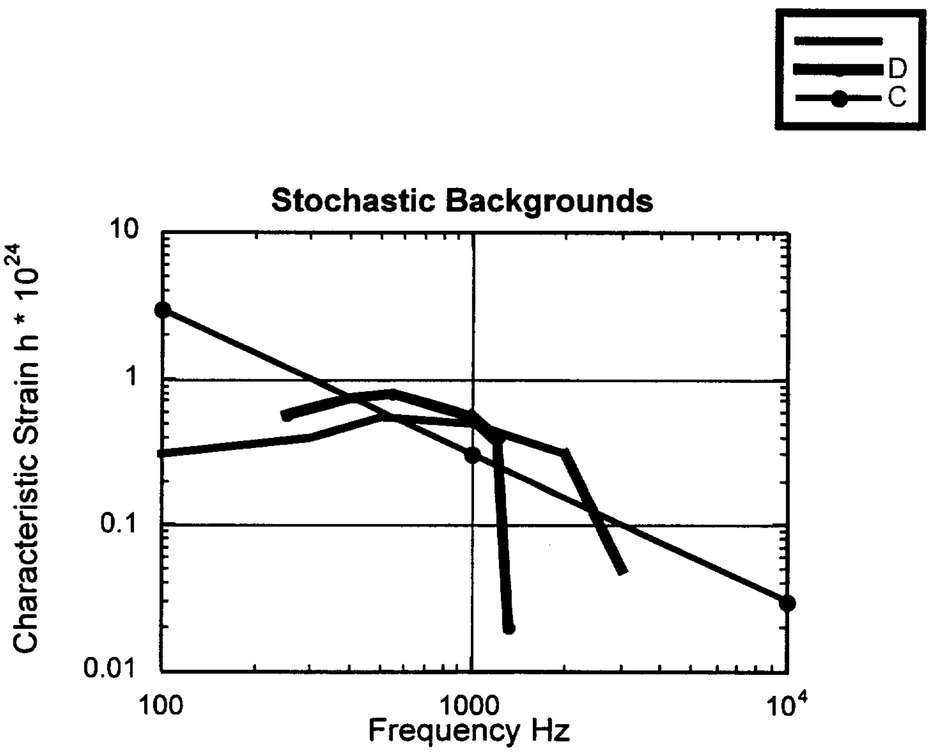

Figure 1. Comparison of two estimates of the ESF stochastic background spectrum (curves B and D), compared with a stochastic background from the early universe with $\Omega_g = 10^{-8}$(Curve C). The characteristic amplitude is the rms amplitude for a bandwidth equal to the observation frequency. The spectral strain per root Hz is obtained by dividing the characteristic strain by $f^{1/2}$.

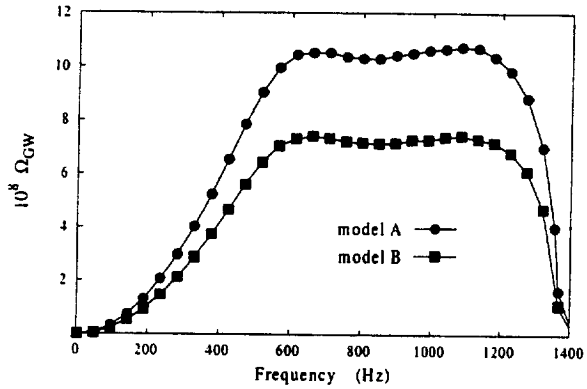

Figure 2 : The cosmological energy density of the r-mode component of the ESF background, from FMS98. Curve A: Ω=1.0,H=50, Curve B: Ω=0.3,H=60.

The cosmological background energy density can be expressed (BJ98) as

$$\Omega_g=\Omega_b.f^*.f_{gc}.\varepsilon$$

where Ω_b is the fraction of closure density in the form of baryons, f* is the fraction of baryons which form stars, f_{gc} is the fraction of stars which undergo gravitational collapse in a Hubble time, and ε is the mean gravitational wave conversion efficiency. Taking Ω_b=0.05, $f^*=10^{-1}$, $f_{gc}=10^{-2}$, then we require $\varepsilon=2x10^{-4}$ to achieve $\Omega_g \sim 10^{-8}$. This is not unreasonable when one considers the angular momentum which must be lost as stars collapse to form neutron stars.
It needs to be remembered that all the above estimates contain significant uncertainties. The amplitude could reduce due to non-gravitational mechanisms for energy loss of newborn neutron stars. The event rates depend on data obtained at the limits of astronomical observation where dust obscuration means that the star formation rates used here are likely to be lower limits. Gravitational wave emission from supernovae is certain to be revised when fully relativistic 3D hydrodynamic codes become effective.

3. Cross Correlation detection

Stochastic background signals can be detected by cross correlation of nearby detectors. (see for example Allen 1996) The signal appears as a correlated component of the noise, while the independent instrument noises are uncorrelated, and hence their noise contribution integrates towards zero. If the antennas are co-located then the stochastic signal can be obtained at any frequency by multiplying the two detector outputs and integrating the product over a long time series. In the case of co-located detectors the signal is always correlated for all frequencies. However if the detectors are not colocated there is a maximum frequency (set roughly by the reduced wavelength) for which a stochastic background will be correlated. This sets an upper limit to the frequency for cross correlation of the LIGO detectors at Hanford and Livingstone of substantially less than 60Hz. (See Allen (1996) for details)

Formally the product signal has a magnitude proportional to $h^2(f).B\tau.\gamma(d,f,\phi)$, where h(f) is the spectral strain, B is the bandwidth, τ is the integration time and $\gamma(d,f,\phi)$ is the overlap function (Flanagan (1993)) which depends on the detector spacing d, detection frequency f, and detector orientation ϕ. The overlap function has a value of unity for colocation (d=0) with ϕ = 0,90,180 degrees, but it falls rapidly to zero when the spacing d equals half a wavelength. The overlap function has a secondary maximum at about λ separation, with a value of about 0.1, and additional exponentially reducing peaks at 1.5λ and 2λ etc. The secondary peaks have been used for stochastic background searches between NAUTILUS and EXPLORER.(Vitale 1997). In the important case of ϕ=45,135...degrees, the overlap function $\gamma(d,f,\phi)$ is equal to zero. This represents orthogonal detectors with each sensitive to the opposite polarisation. This means for example that a circularly polarised signal would appear in quadrature between each detector, and the sum of a large number of such signals (with random angular momentum) will integrate in the same way as does instrument noise.
The uncorrelated noise product signal increases as $(S_{n1}.S_{n2})^{1/2}.B^{1/2}.\tau^{1/2}$, so the minimum detectable signal amplitude is given by

$$h_{min} = \gamma^{-1/2}(d,f,\phi)[(S_{n1}.S_{n2})/B\tau]^{1/4} \qquad (1)$$

The two LIGO detectors could only be used to detect the low frequency tail of the r-mode ESF background. However in the case of LLO and ALLEGRO the distance (42km) sets a maximum frequency well in excess of 1kHz. (The first zero in the overlap function occurs at 4.6kHz.) However because the interferometer detects double the strain of a bar the overlap function has a maximum absolute value of less than unity. It is negative because of the relative orientations of LLO and ALLEGRO.

4. Orientation of ALLEGRO

Currently ALLEGRO is optimally aligned with other bars to optimise the probability of burst detection coincidences. Its long axis orientation is 40.6 degrees W of N. LLO is oriented 108 deg W of N. The polarisation sensitivity of both antennas depends on cos2ϕ where ϕ is the polarisation angle relative to the axis of the bar or an interferometer arm. Thus the misalignment would be worst for an angle of 45n degrees where n=0,1,2.... Thus there is a significant loss of signal since ϕmod45=22.4 deg. The LSU group have calculated the overlap function to have magnitude -0.25 at 900Hz with the present orientation, compared with -0.37 if ALLEGRO was coaligned. (Hamilton et al (1999))

5. Strain Sensitivity of the ALLEGRO-LLO Pair

We now consider the specific case of the noise projected for advanced LIGO and the projected noise and bandwidth of ALLEGRO. The bandwidth is greatly increased by using a 3-mode transducer. Further improvements are achieved as the SQUID noise is improved from $4000\hbar$ (next upgrade) to $200\hbar$ (double DC SQUID) towards $1\hbar$ (the quantum limit).

The ALLEGRO projected noise performance when the 3-mode transducer is installed with a $4000\hbar$ SQUID is shown in Figure 2. The detector achieves a bandwidth of 50Hz and a spectral strain sensitivity about 2.10^{-21}. If the SQUID is improved to $200\hbar$ ($30\hbar$ SQUIDs have been demonstrated) the spectral strain sensitivity falls to 10^{-21} between 890Hz and 940Hz, and in most of the band is~8.10^{-22}. This sets the cross correlation bandwidth. In the same band we will assume LIGO 2 sensitivity of 6.10^{-24} for broadband operation. We will assume 3 years of integration time. Using equation 1 we obtain $h_{min}\sim 4.9.10^{-25}$ using the optimum γ-value discussed below. The value of h_{min} compares with the predicted signal amplitude of ~5. 10^{-25}, implying an energy SNR~ 1.

The above result merely demonstrates that astrophysically interesting sensitivity can be achieved by the ALLEGRO-LLO combination of detectors. We note that future improvements are possible in both detectors. For ALLEGRO the use of ultralow temperature cooling can allow it to approach the quantum limit (10-fold improvement in spectral strain sensitivity), while for advanced LIGO narrow band operation of the detector can allow significant additional sensitivity in the region of 1kHz. Assuming only improvement to ALLEGRO a 3-year noise floor of $1.3.10^{-25}$ is achieved, which represents an energy SNR~7. If we assume narrow band operation of LLO, with a noise floor of 1.10^{-24} and 100Hz bandwidth, the 3 year noise floor improves to $4.1.10^{-26}$ and the ESF stochastic background SNR exceeds 100.

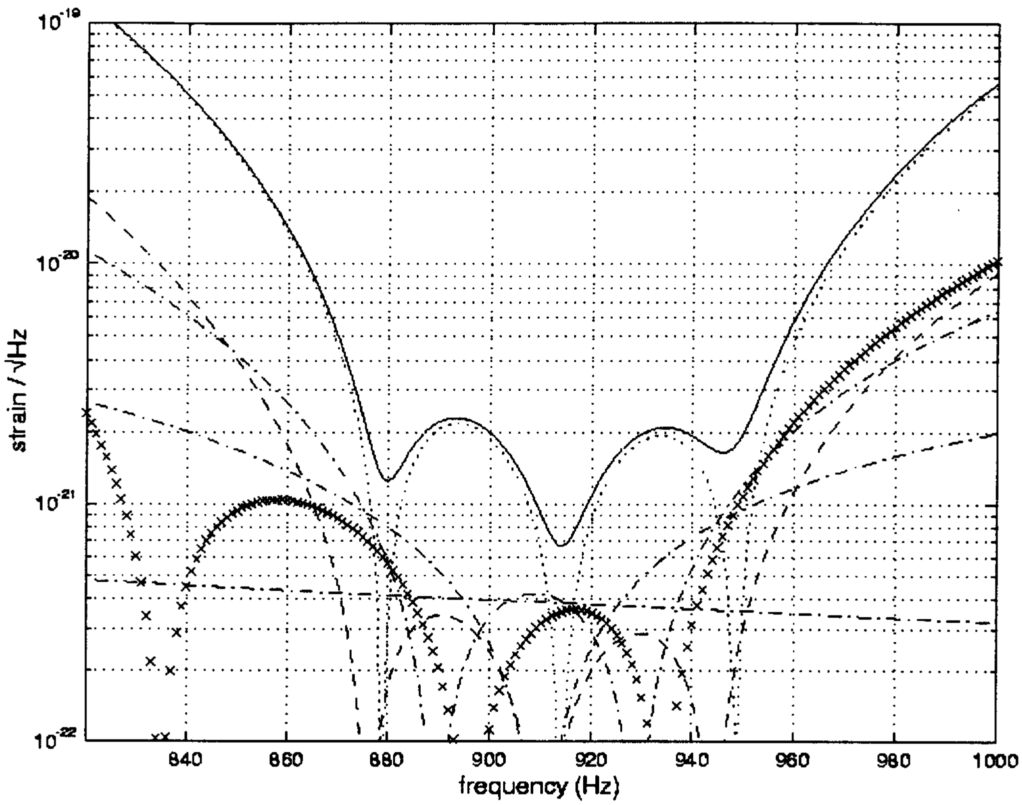

Figure 2. Strain spectral density of ALLEGRO with a 3-mode transducer and 4000ħ SQUID. The bandwidth is about 50Hz. The spectral strain sensitivity improves by a factor ~2 when a 200ħ SQUID is installed.

Results are summarised in Table 1 for two separate integration times 10^7 s (~4 months) and 10^8 s (~3 years). The noise of the detectors is assumed to be flat over the specified bandwidth. The various combinations compared are: ALLEGRO now (1Hz bandwidth), ALLEGRO with a 200ħ 3 mode transducer and ALLEGRO Quantum Limited, with LLO at LIGO 1 (L1), LIGO 2 (L2) and narrow band LIGO 2 (L2NB) sensitivity. Finally, for comparison, Table 1 shows the sensitivity achievable by a pair of narrow banded interferometers spaced within 100km of each other.

Table 1.

Detectors	LLO 900Hz	ALLEGRO	Bwidth Hz	Noise h_{rms} $\tau = 10^7$ s	Noise h_{rms} $\tau = 10^8$s	SNR $\tau = 10^8$s
L1-A1999	10^{-22}	10^{-21}	1	$7.3.10^{-24}$	$4.1.10^{-24}$	$1.5.10^{-4}$
L2-A 3-mode	10^{-23}	10^{-21}	50	$8.6.10^{-25}$	$4.9.10^{-25}$	0.1
L2-A QL	10^{-23}	10^{-22}	100	$2.3.10^{-25}$	$1.3.10^{-25}$	1.5
L2NB-AQL	10^{-24}	10^{-22}	100	$7.3.10^{-26}$	$4.1.10^{-26}$	15
L2NB-L2NB	10^{-24}	10^{-24}	100	$7.3.10^{-27}$	$4.1.10^{-27}$	1500

The last colum shows the signal to noise ratio for a stochastic background with characteristic amplitude of 5.10^{-25}. The SNR is very poor in the narrow band case. The signal to noise ratios for 3 year integration exceeds unity only if the ALLEGRO antenna is substantially improved, including a quantum limited transducer system. A more promising long term option would be to replace ALLEGRO with a 3m diameter spherical detector operating with a transducer noise ~ $100\hbar$. Such a system would be equivalent to the existing bar at the quantum limit, and would allow both polarisations to be correlated simultaneously.
Bearing in mind the uncertainties in the ESF stochastic background, the second line in Table 1 represents a goal of significant astrophysical merit. A quantum limited bar, or $100\hbar$ sphere would achieve a substantial SNR. On the other hand a pair of narrow banded interferometers represent the best future option, but unless they are separate and independent systems it would be difficult to be confident of the results.

6. Conclusions, Practical Concerns and Recommendations

Improvement of ALLEGRO and cross correlation with advanced narrow band LIGO LLO achieves astrophysically significant sensitivity and provides a unique probe into the era of early star formation. However, based on present (rather uncertain) estimates the probability of detection will be low until both detectors achieve nearly ideal performance.

The advantage of cross correlating ALLEGRO and LLO is that they are not co-located and the differences in technology mean that the significance of an observed correlation is less open to question. In addition, as discussed below, the signal can be switched off.

ALLEGRO and LLO are far enough apart that seismic signals are uncorrelated. However tests should be undertaken to determine the level of common sensitivity to electricity grid harmonics and transients, and also electrical pulses due to lightning etc.(See for example Allen 1997.) Proceedures need to be tested for vetoing parts of the data, both temporally and spectrally to reduce local correlations.

When ALLEGRO is relocated after building work, it should be set up so that it can be rotated 45 degrees between optimal alignment with LLO (18 deg W of N) and null alignment (63 deg W of N). In the short term the chance of a positive detection is small, but when sensitivity is high enough in both detectors, ALLEGRO could be aligned in a compromise position where burst detection is not sacrificed. For example it could be rotated 10 degrees to reduce the ALLEGRO-LLO misalignment to reasonably negligible 12.4 degrees.

LIGO and ALLEGRO should plan a long term strategy to ensure that both detectors achieve adequate performance concurrently and that the data is aquired in an appropriate format.

Lazzarini has suggested (Hamilton1999) that an ideal cross correlation experiment would involve alternating periods (say 3 month cycles) with ALLEGRO and LLO coaligned and orthogonal to create a square wave modulation of the cross correlation signal, allowing discrimination between local sources of correlation and a stochastic background. Such an experiment should be undertaken during the initial operation of LLO to test the concept.

A spherical detector at LSU would allow the above experiment to be performed by simply comparing the cross correlations in the two orthogonal polarisations in the plane of LLO, one of which should yield a signal while the other will give a null output and allow local correlations to be calibrated out.

Effort is required to better characterise the ESF stochastic background to define improved methods for detecting the clearly non-Gaussian spectrum.

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