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Generation of Diagnostic Magnetic Fields for Test Mass Chambers
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1 Overview

We propose a magnetic field generation system for the LIGO PEM. It is to consist of two coils of 1 meter diameter. Depending upon the relative currents delivered to the coils, the field in the region between the coils can range from being quite uniform to having a large spatial gradient. The frequency can vary from DC to greater than 1 kHz. The maximum field strength, depending upon details of the setup, will be approximately 10^{-4} T at low frequency and $\approx 10^{-5}$ T at 1 kHz.

We propose to fabricate and test the coils and their mounting stands at the U. of Oregon and then transport them to the LHO. Fabrication can begin the week of Sept. 7. Oregon proposes to donate shop time and small costs for fabrication. The only identified cost to be borne by MIT is for a power amplifier. This is detailed in Section 3.

Below, we review the requirements, discuss the proposed solution, give results of some field calculations, followed by some technical detail and specification and cost of components.

1.1 Requirements

The following requirements for a magnetic field generator are taken from the PEM Design Requirements Document (LIGO-T960127-02-D).

- field magnitude: $10^{-13} < B < 10^{-5}$ T
- frequency range (sinusoidal): $1 < f < 1000$ Hz
- bursts duration (unspecified waveform): 10 to 300 μs

1.2 Additional requirements

We have assumed that the system also have the following properties:

- Provide the capability to apply a broad range of field gradients, from uniform to highly varying, to allow a greater degree of experimental control (see below)

- Any external field configuration should be reproducible at different times for a given test chamber and from chamber to chamber at the level of systematic measurement and calibration uncertainties. This level presently is assumed to be $\approx 1\%$.

- If possible, extend frequency range to DC and above 1 kHz

2 Design considerations

The classic Helmholtz 2-coil configuration can provide a quite uniform field in a region between the coils of approximate size limited by the coil radii. We have assumed that the region of interest is set by the test masses, hence is crudely a cube of side 20 cm. The first derivative $\partial B_z/\partial z$ vanishes in the central region in the Helmholtz configuration. In addition, the second derivative decreases rapidly with the ratio of separation to radius, and vanishes when the separation is equal to the radius.

Alternatively, a large gradient can be set up by reversing the current in one of the coils (the "anti-Helmholtz" case). The calculation of the fields for our chosen coil parameters are given in Section 2.3. Clearly, the two coils can assume any number of positions relative to the test chambers in order to achieve a broad range of field gradients. There are obvious experimental advantages in the capability for providing a uniform field, at least for initial investigations. We discuss another attribute of such a system.

In a perhaps overly simplified model, one can imagine there being two terms for the force on a test mass due to an external magnetic field $\vec{B}$. The first is the coupling of the dipole moment of the test mass permanent magnets to the field gradient. This force can be expressed as $\vec{F} = (\vec{m} \cdot \vec{\nabla})\vec{B}$, where $\vec{m}$ is the dipole moment of a permanent magnet. The other force is due to dipole-dipole interactions of the induced eddy current loops in nearby conductors with the magnet dipoles. Fred Raab, who made field measurements for the 40m prototype, discussed some of the issues with us. He showed data which apparently showed that the primary effect of external magnetic fields was eddy currents produced in the cage structure adjacent to the test mass. (In fact, he pointed out that LIGO went to stainless steel, as opposed to aluminum for the 40m, in large part to mitigate eddy current effects.)

A notable benefit of the two-coil configuration is to provide a technique, in addition to the frequency dependence (used for the 40m), for separating the two effects, *i.e.* the gradient-dipole effect and the eddy current effect. With a time varying, but spatially uniform, external field, the former effect should be negligible. The gradient can be increased in a controlled manner by changing the ratio of currents in the two coils from a Hemholtz configuration to an anti-Helmholtz configuration, in which the currents in the coils are oppositely directed. This is shown in Section 2.3.

This technique can be tried during the initial calibration with magnetometers placed within a test chamber. An open question is how an effect due to the applied fields might best be observed for the operating interferometer, for example in a controlled test.

2.1 Design equations

We use SI units. Hence, $\mu_0 = 4\pi \times 10^{-7}$ T-m-A^{-1}. The equations below are idealized in order to convey the basic dependencies.

A single coil of radius a, current I, and N turns is centered about the z axis. Let the origin be at the coil center. Consider a point at a distance r from the coil center. For $r >> a$, the field at this point is, in spherical coordinates, given approximately by

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

where m is the magnitude of the coil dipole moment:

$$m = NI\pi a^2$$

So, to set the scale, for a point on the z axis we have

$$B = B_z = (1.57 \times 10^{-5}\mathrm{T})\left[\frac{I}{1\mathrm{A}}\right]\left[\frac{N}{10^2}\right]\left[\frac{a}{0.5\mathrm{m}}\right]^2\left[\frac{1\mathrm{m}}{r}\right]^3 \quad (1)$$

If the coil has an axial length ℓ, then its self-inductance is (approx.)

$$L = BA/I = \mu_0 I(N/\ell)N\pi a^2/I = (50\mathrm{mH})\left[\frac{N}{10^2}\right]^2\left[\frac{a}{0.5\mathrm{m}}\right]^2\left[\frac{0.2\mathrm{m}}{\ell}\right] \quad (2)$$

The (magnitude of the) impedance of the coil is

$$Z = \left[R^2 + (\omega L)^2\right]^{1/2}$$

where R is the resistance and $\omega = 2\pi f$. For 12 gauge copper wire ($5.2 \times 10^{-3}\Omega$/m) we have

$$R = (1.6\Omega)\left[\frac{N}{10^2}\right]\left[\frac{a}{0.5\mathrm{m}}\right]$$

The diameter of 12 gauge copper wire is 2.052 mm. This can be compared with the skin depth of copper:

$$\delta = 66\mathrm{mm}/\sqrt{f}$$

which is 2.1 mm at 1 kHz. Hence, single-strand solid copper wire should be fine. The inductance clearly dominates the impedance for frequencies above DC. At 1 kHz, we have $Z \approx 320\ \Omega$. Hence, this will limit the current (and field) at high frequency.

2.2 Parameter choices

Our choice of parameters is driven primarily by the maximum field and the desire to choose a uniform field. The latter requires that the coil diameter be much larger than a LIGO test mass. Hence, conservatively we set the diameter to be 1 m:

$$a = 0.5\mathrm{m}$$

An additional benefit of relatively large radius is that, within the central region, all field configurations, uniform or not, are quite insensitive to small mis-alignments or displacements (compared to a) of the coils from ideal.

For 2 coils in the Helmholtz configuration, the field along the axis is twice that parameterized by Eq. 1. It is reasonable to design for a maximum field of 10^{-4} T when the coils are separated by 2 m ($r = 1$ m), thereby allowing the separation to double and still make the design specification. Also, it is conceivable that a field exceeding that due to the Earth ($\sim 5 \times 10^{-5}$ T) at DC could be useful. For 100 turns per coil this is achievable with a comfortable current of 3 A per coil. At DC the voltage drop (12 gauge copper wire) is reasonable.

However, $a = 0.5$ m and $N = 10^2$ imply a very large self-inductance. This constrains the maximum field at the highest frequencies. For example, a typical power amplifier can supply 30 V, thereby limiting the current to ~ 100 mA at 1 kHz, for a field of $\sim 3 \times 10^{-6}$ T. We can mitigate this by providing taps at smaller N. So, with the same amplifier, 10^{-5} T could be achieved with $N = 10$, so long as the turns cover the same axial extent, ℓ.

To obtain the smallest required field magnitude, it may not be practical to simply reduce the current, expecting a stable and easily measureable current at the level of 10 nA. A low-resistance ($\sim 10^{-1}$ Ω) voltage divider would help. But it would also be helpful for the coil to include a single-turn tap. Combined with a divider, about half the requisite ~ 9 orders of magnitude in field reduction could be achieved in this way, without changing r, so that the generateded currents do not have to be below ~ 10 μA.

Therefore, each coil should have taps at $N = 1$, $N = 10$, $N = 30$, and $N = 100$.

2.3 Field Calculations

Mathematica was used to perform the calculations and plot results. The fields were assumed to arise only from the coils, and vacuum is assumed elsewhere. Hence, there are neither magnetic materials nor eddy currents. It was easiest to calculate the fields in multipole approximation. We used $n = 6$ for the calculations here, which provides an excellent approximation so long as one is not interested in accuracy for the region $r < a$. The distance unit is meters and the fields, unless otherwise noted, are in units of 10^{-6} T.

The first two plots, Figs. 1 and 2, give the overall shape of the axial and transverse components of the field as a function of ρ and z for the Helmholtz configuration. The coils are placed at $z = \pm 2$ m. And $N = 10^2$, $I = 3$ A, and $a = 0.5$ for each coil. We see that the field is quite uniform for the central region.

The next two figures show how the dominant axial component of field varies spatially over the central region for various ratios of currents in the two coils. We see, as expected, that going from the Helmholtz to anti-Helmholtz configurations corresponds to a sizeable increase in field gradient.

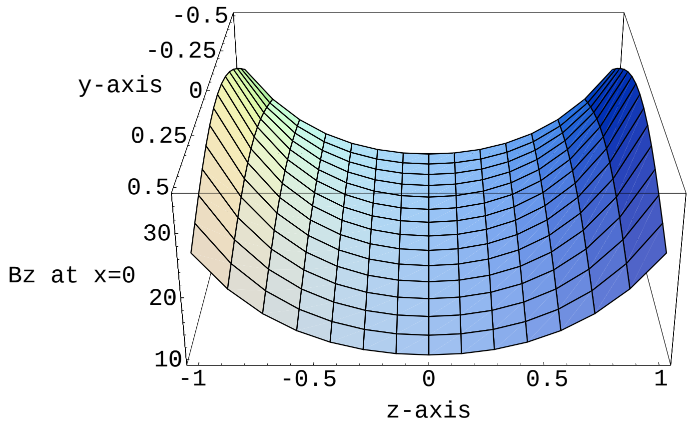

Figure 1: The axial field B_z as a function of (y, z) at $x = 0$ for the Helmholtz configuration with $N = 10^2$, $I = 3$ A. The coils are placed at $z = \pm 2$ m. The field is in units of 10^{-6} T.

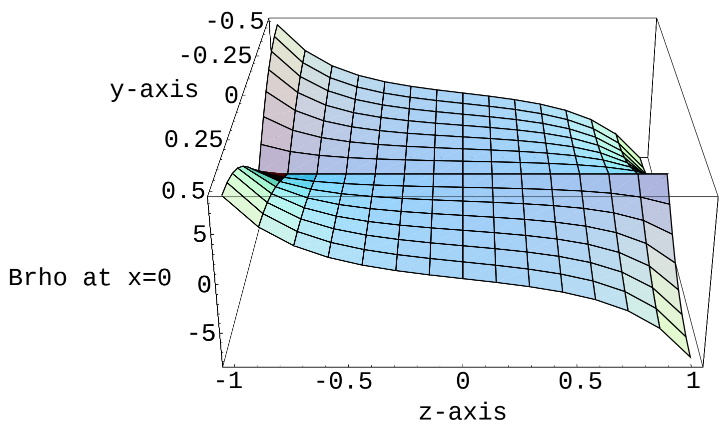

Figure 2: Same as previous figure but for the transverse field component, B_ρ.

3 Technical Specifications

Each coil is to consist of 12 gauge copper wire with a varnished coating wound on a spool of hardwood, which is easy to fabricate and is non-conductive. ("It's a Northwest thing.") The weight of the copper for each coil will be about 9.2 kg. The wooden frame will have a flange bolted to its center to allow the coil to be mounted to an aluminum stand of adjustable height. The maximum height of the coil axis can, in this way, be below or above the level of a test mass. The coils and frames are to be fabricated at Oregon. Oregon will provide the materials, the most significant of which is the wire. A quote for the wire is attached.

The coils are to be powered by a signal generator (already existing as part of the PEM equipment) which is input to a power amplifier, whose outputs are connected to

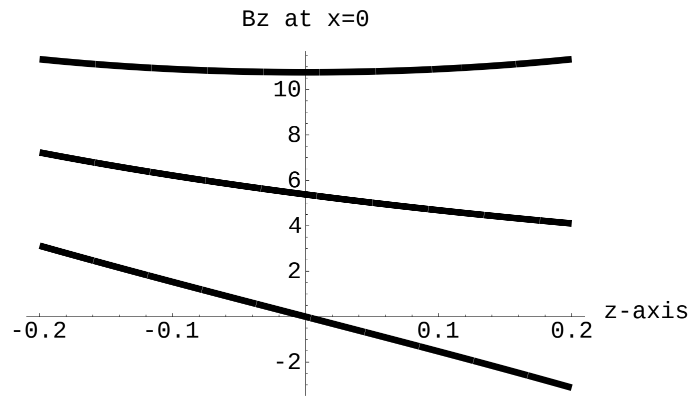

Figure 3: The axial field B_z in the central region as a function of z for 3 very different current ratios. Top curve: the Helmholtz (equal currents) configuration; middle curve: single coil (one coil off); lower curve: anti-Helmholtz (equal, but opposite, currents) configuration. The other parameters are as before, and again the field is in units of 10^{-6} T.

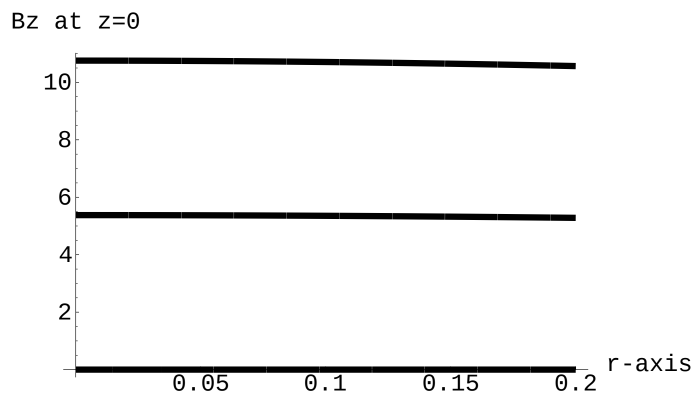

Figure 4: Same as previous figure, but B_z as a function of ρ.

the coils. Attached are some specifications and a quote for a viable power amplifier. It is a dual channel amplifier; the signal generator would be passively split, with each output channel powering one coil. It provides up to 8 A at 32 V (rms) per channel into loads which can be purely reactive with a frequency range from DC to 20 kHz. It can operate in current mode, which would be very desireable in this case, as it would allow one to sweep in frequency at fixed magnetic field. Unfortunately, the company has discontinued this product. Presumably it will not be difficult to find a similar product. (High power audio amplifiers are actually quite well suited for this purpose, with the exception that their frequency response typically does not go below about 20 Hz.)

It is important to be able to measure the currents delivered to the coils. For the range of currents envisioned, a typical DVM in AC current mode will be capable of these measurements. As discussed in Section 2.2, it should not be necessary to measure currents smaller than $\sim 10\ \mu$A, nor larger than ~ 3 A. One must check that the ammeters be able to handle circuits with load impedance as small as a few ohms. We will look into simple ammeters which would allow remote readout in case that becomes desireable in the future.