# LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY <br> - LIGO - <br> CALIFORNIA INSTITUTE OF TECHNOLOGY <br> MASSACHUSETTS INSTITUTE OF TECHNOLOGY 

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| PO Beam Waist Size and Location on the |
| ISC Table |
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## 1 OVERVIEW

The PO beam waist location on the ISC table can be varied by defocusing the ISC telescope. An optical schematic drawing of the PO beam optical train, which includes the 8 X reflective telescope inside the vacuum housing and the 3.3X ISC telescope on the ISC optical table is shown in 1.The input beam waist is inside the IFO, approximately 900 m from the PO telescope. The ISC telescope is separated from the PO telescope by approximately 3 m . The input beam waist is transformed by the lens train to an output beam waist of size $\mathrm{w}_{042}$, located in the vicinity of the ISC table at a distance $\mathrm{z}_{42}$ from the end of the ISC telescope.


ISC Table

Figure 1: Optical schematic of PO beam optical train

The size and location of the output beam waist can be determined by using Gaussian beam transformation theory.

## 2 GAUSSIAN BEAM TRANSFORMATION THEORY

An analysis of the dependence of the output beam waist and location on the defocusing of the ISC telescope was made using the following parameters.

### 2.1. Telescope Parameters

## PO Telescope

primary focal length, $\mathrm{f}_{1} \quad 1524 \mathrm{~mm}$
secondary focal length, $\mathrm{f}_{2} \quad-190.5 \mathrm{~mm}$
input beam waist position, $\mathrm{z}_{11}$
input beam waist parameter, $\mathrm{w}_{011}$ distance to ISC telescope, d
$9 \times 10^{5} \mathrm{~mm}$
36.4 mm

3000 mm

## ISC Telescope

primary focal length, $\mathrm{f}_{3}$
secondary focal length, $\mathrm{f}_{4}$
251.8 mm
$-74.172 \mathrm{~mm}$

### 2.2. Gaussian Beam Transformation Equations

$$
\begin{aligned}
& \mathrm{z}_{12}:=\mathrm{f}_{1}+\mathrm{f}_{1}{ }^{2} \cdot \frac{\left(\mathrm{z}_{11}-\mathrm{f}_{1}\right)}{\left[\left(\mathrm{z}_{11}-\mathrm{f}_{1}\right)^{2}+\left(\pi \cdot\left(\frac{\mathrm{w}_{011}}{\lambda}\right)^{2}\right]^{2}\right]} \\
& \mathrm{w}_{012}:=\left[\frac{1 \cdot\left(1-\frac{\mathrm{z}_{11}}{\mathrm{f}_{1}}\right)^{2}}{\mathrm{w}_{011}{ }^{2}}+\frac{1 \cdot\left(\pi \cdot \frac{\mathrm{w}_{011}}{\lambda}\right)^{2}}{\mathrm{f}_{1}{ }^{2}}\right]^{-0.5} \\
& { }^{\mathrm{w}} 021 \text { := }{ }^{\mathrm{w}} 012 \\
& \mathrm{z}_{21}:=\mathrm{f}_{1}+\mathrm{f}_{2}-\mathrm{z}_{12} \\
& \mathrm{z}_{22}:=\mathrm{f}_{2}+\mathrm{f}_{2}{ }^{2} \cdot \frac{\left(\mathrm{z}_{21}-\mathrm{f}_{2}\right)}{\left.\left[\left(\mathrm{z}_{21}-\mathrm{f}_{2}\right)^{2}+\left(\pi \cdot \frac{\mathrm{w}_{021}}{\lambda}\right)^{2}\right]^{2}\right]} \\
& \mathrm{w}_{022}:=\left[\frac{1 \cdot\left(1-\frac{\mathrm{z}_{21}}{\mathrm{f}_{2}}\right)^{2}}{\mathrm{w}_{021}{ }^{2}}+\frac{1 \cdot\left(\pi \cdot \frac{\mathrm{w}_{021}}{\lambda}\right)^{2}}{\mathrm{f}_{2}{ }^{2}}\right]^{-0.5}
\end{aligned}
$$

$$
\begin{aligned}
& { }^{\mathrm{w}} 031 \text { : }{ }^{\mathrm{w}} 022 \\
& \mathrm{z}_{31}:=\mathrm{d}-\mathrm{z}_{22} \\
& \mathrm{z}_{32}:=\mathrm{f}_{3}+\mathrm{f}_{3}{ }^{2} \cdot \frac{\left(\mathrm{z}_{31}-\mathrm{f}_{3}\right)}{\left[\left(\mathrm{z}_{31}-\mathrm{f}_{3}\right)^{2}+\left(\pi \cdot \frac{\mathrm{w}_{031}}{\lambda}\right)^{2}\right]} \\
& \mathrm{w}_{032}:=\left[\frac{1 \cdot\left(1-\frac{\mathrm{z}_{31}}{\mathrm{f}_{3}}\right)^{2}}{\mathrm{w}_{031}{ }^{2}}+\frac{1 \cdot\left(\pi \cdot \frac{\mathrm{w}_{031}}{\lambda}\right)^{2}}{\mathrm{f}_{3}{ }^{2}}\right]^{-0.5} \\
& { }^{\mathrm{w}} 041 \text { := }{ }^{\mathrm{w}} 032 \\
& \mathrm{z}_{41}\left(\Delta_{\text {ISC }}\right):=\mathrm{f}_{3}+\mathrm{f}_{4}+\Delta \text { ISC }^{-\mathrm{z}_{32}} \\
& \mathrm{z}_{42}\left(\Delta_{\text {ISC }}\right):=\mathrm{f}_{4}+\mathrm{f}_{4}{ }^{2} \frac{\left(\mathrm{z}_{41}\left(\Delta_{\text {ISC }}\right)-\mathrm{f}_{4}\right)}{\left[\left(\mathrm{z}_{41}\left(\Delta_{\text {ISC }}\right)-\mathrm{f}_{4}\right)^{2}+\left(\pi \cdot \frac{\mathrm{w}_{041}}{\lambda}\right)^{2}\right]^{2}} \\
& \mathrm{w}_{042}(\Delta \text { ISC }):=\left[\frac{1 \cdot\left(1-\frac{\mathrm{z}_{41}(\Delta \text { ISC })}{\mathrm{f}_{4}}\right)^{2}}{\mathrm{w}_{041}{ }^{2}}+\frac{1 \cdot\left(\pi \cdot \frac{\mathrm{w}_{041}}{\lambda}\right)^{2}}{\mathrm{f}_{4}{ }^{2}}\right]^{-0.5}
\end{aligned}
$$

Raleigh range of ISC output, mm

$$
\mathrm{z}_{\mathrm{R}}\left(\Delta_{\text {ISC }}\right):=\pi \cdot \frac{{ }^{\mathrm{w}} 042\left(\Delta^{(\Delta S C}\right)^{2}}{\lambda}
$$

### 2.3. Results: Output Beam Waist Size and Location

The location of the ISC telescope output beam waist, as measured from the output lens, can be varied by defocussing the ISC telescope as shown in figure 2. The Rayleigh range associated with the output beam waist is also shown in figure 2 . The beam waist size also varies as the telescope is defocussed, as shown in figure 3.


Figure 2: Beam waist position and Raleigh range versus ISC defocus, $d=\mathbf{3 0 0 0} \mathbf{m m}$


Figure 3: Beam waist size versus ISC defocus, $d=3000 \mathrm{~mm}$

### 2.3.1. Locating the Far Field of the ISC Telescope

It is desireable to minimize the effect of the position of the ISC telescope on the Guoy phase at the WFS detector, by placing the first Guoy lens of the WFS system in the far field of the ISC telescope output. This can be accomplished by defocussing the ISC telescope approximately -0.6 mm , which will place the apparent output beam waist at the location -3000 mm (refer to figure 2). And since the Rayleigh range is +3000 mm , the output beam will be in the far field as it emerges from the ISC telescope.

