LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Document Type	LIGO-T970189-00 -	D	today

Pitch Adjustment Magnets for LOS and SOS

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1 ABSTRACT

The suspension will use small permanent magnets to provide damping and attitude control of the optics. This document summarizes a number of calculations which were done to ensure that the use of magnets was thoroughly understood. These include

- comparison of the measured magnet strength with the spec-sheet value,
- a first-principles calculation of eddy current damping,
- a check on the method of calculation by comparison with the experiment results of Miyoki (LIGO-T970073-00-D),
- eddy current damping calculations for the optic magnets to the gold-plated Macor, for the optic magnets to the pitch-adjustment magnets, and for the pitch-adjustment magnets to the wire loop supporting the optic,
- pitch as a function of screw position for the pitch-adjustment magnets.

2 KEYWORDS

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3 OVERVIEW

Both the LOS and SOS designs will use magnets bonded to the optics together with magnetic coils in the sensor actuator heads to provide damping and attitude control for the optics. In addition, the LOS design will use pitch-adjustment magnets (PAMs) opposing those on the optics to provide extra static torque to bridge the gap between the precision attainable by manual adjustment and the dynamic range of the coil actuators. (The SOS design does not include PAMs at the time of writing but it would be possible to retrofit them without too much difficulty.)

There are a number of potential problems with this scheme which need to be considered. The first is one of design: choosing a suitable position adjustment system and strength for the PAMs so that precision and dynamic range requirements are satisfied. The second is the eddy-current damping between the magnets and various metal objects both on the optic and in the surrounding space. Miyoki measured the eddy current damping between a magnet and an aluminium duplicate of the SOS sensor/actuator head holder. (The damping in Al is 30 times greater than for stainless steel because of the lower resistivity and thus easier to measure.) He did not attempt to estimate the damping analytically. However there are many other combinations of magnet and conductor to be checked, and so the remainder have been calculated, using Miyoki's result as a check on the method of calculation.

The third potential problem is cross-coupling between the pitch, yaw and axial degrees of freedom due to forces from the pitch adjustment magnets, especially in the presence of strength and position mismatches in the PAMs. These are calculated and shown to be negligible.

4 ESTIMATION OF THE MAGNET STRENGTH

4.1. Method 1: Using data in spec sheet

The magnets chosen to be bonded to the optic in both LOS and SOS are l=3.175 mm long and a=0.9525 mm in radius, made of NEO-35. The dipole moment of a volume V of uniform magnetization M is

$$p_m = MV = \pi M l a^2$$

The magnetization can be estimated in terms of the residual induction, which is about 1.25 T for NEO-35:

$$M \approx \frac{B_{resid}}{\mu_0}$$

In fact, B_{resid} is the value of B at a point on the hysteresis curve where H has just been reduced to zero, whereas the H field inside a bar magnet is substantial. A calculation using formulae in M.A. Plonus, "Applied Electromagnetics”, shows that the average axial $B/(\mu_0 H)$ inside the magnet is -3.6, which, by inspection of the hysteresis curve for NEO35, is still (just) on the flat, top part of the curve, so the equilibrium B field should still be quite close to B_{resid}. Thus, p_m=0.0090 J/T. This is an upper bound - small regions of high negative H near the pole faces may result in partial local demagnetization.

4.2. Method 2: Pick-up test

In this method, one magnet was used to pick up another, and the distance between them at the point the gravitational force was overcome was used together with the mass to calculate the dipole moment using the formula

$$mg = F_z = \frac{3\mu_0 p_m^2}{2\pi z^4}$$

The value from this method was $p_m = 0.0079$ J/T.

4.3. Method 3: Tube test

In this test two magnets were confined in a narrow vertical glass tube so as to align them and the distance at which the upper one floated above the lower one was measured. The formula of the previous section was used to derive $p_m = 0.0073$ J/T.

4.4. Preferred value

As expected the measured values were somewhat lower than the value derived (naively) from the spec sheet. The tube and pick-up tests give similar results. The tube result is used in subsequent analysis.

5 ANALYSIS OF MIYOKI'S EXPERIMENT

5.1. Estimation of the magnetic field and EMF

The sensor/actuator head holder used by Miyoki consists of a bar 395 mm long, 65 mm tall and 16 mm thick, with a 27 mm diameter hole slightly below the midline about 1/3 of the way along. The magnet was placed on and parallel to the axis of the hole. Although the geometry of distant parts of the holder is complicated, the strength of the interaction falls very strongly with distance and so only the axially symmetric part around the hole contributes significantly. Thus it is convenient to use cylindrical coordinates. Consider then a loop of radius r parallel to the x-y plane, centered on the z axis, and moving in the z direction with velocity v_z. The emf in the loop can be calculated either as the flux of B_r cut per unit time or as the negative rate of change of the B_z integrated over the area of the loop:

$$e = \oint E \cdot dl = \int B_r \times v_z \cdot dl = -\frac{\partial}{\partial t}\int B_z \cdot dS$$

Let the dipole be at the origin with its axis along the z direction. Then the axial and radial components of the magnetic field are

$$B_z = \frac{\mu_0 m_z}{4\pi} \frac{r^2 + 2z^2}{(r^2 + z^2)^{5/2}}$$

$$B_r = \frac{3\mu_0 m_z}{4\pi} \frac{rz}{(r^2 + z^2)^{5/2}}$$

Since the loop is circular, B_r is constant in magnitude over the perimeter and it is convenient to use the flux-cutting formula, so that the EMF is

$$e = 2\pi r B_r v_z$$

(Integrating the time derivative of B_z numerically over the area of the loop using Mathematica gives the same EMF.)

5.2. Calculation of the current and energy dissipation

Now replace the loop by an annulus of infinitesimal cross-sectional area dS and consider the current induced in it. If the resistivity of the metal is ρ, then the conductance of the loop is

$$dG = \frac{dS}{2\pi r\rho} = \frac{dr dz}{2\pi r\rho}$$

The power dissipated in the annulus is then

$$dP = e^2 dG$$

and the contribution to damping force per unit velocity is

$$db = \frac{dP}{v_z^2} = \frac{9\mu_0^2 m_z^2}{8\pi\rho}\frac{r^3 z^2 dr dz}{(r^2+z^2)^5}$$

Integrating from z=10.59 mm (the separation of 9 mm plus half the magnet length) to z=26.59 mm (the same plus the holder thickness) and from r=13.5 mm (the hole radius) to r=28.5 mm (the largest circle centred on the hole and fully contained in the holder metal) and using ρ_{Al}=2.65×10^{-8} Ω m gives $b = 4.5\times10^{-6}$ N/(m/s).

5.3. Comparison with Miyoki's value

Eddy current damping is velocity dependent, so that the lag angle ϕ at any particular frequency, (on- or off- resonance) is given by

$$\phi(\omega) = \frac{\omega\gamma}{\omega_0^2} = \frac{\omega b}{\omega_0^2 m}$$

Miyoki specifies a value for the damping at resonance $\phi_0 = \phi(\omega_0)$ of the test pendulum of $\phi_{meas} = 5.3\times10^{-5}$ (or, restoring a significant figure, 5.27×10^{-5}). Given m=5.7 g and $\omega_0/(2\pi) = 0.9$ Hz, $b_{meas} = 1.7\times10^{-6}$ N/(m/s). This is 2.6 times smaller than estimated above. The reason for the discrepancy has not been identified yet.

5.4. Derivation of damping factor for SOS

Digression: from the above, Miyoki estimates the damping and thermal noise in the real system with four magnets instead of one, a frequency of 1.0 Hz instead of 0.9, and a stainless steel holder of 30 times lower conductivity. He does not give the details of this calculation but it can be reconstructed as follows. The damping force under these conditions is

$$b_{SOS} = 4b_{meas}\frac{\rho_{Al}}{\rho_{SS}} = 2.26\times10^{-7} \qquad N/(m/s)$$

which implies a damping factor of

$$\phi_{0(SOS)} = \frac{2\pi b_{SOS}}{\omega_0 m} = 1.44\times10^{-7}$$

However Miyoki's quoted value is 1.8×10^{-7}, which is different by a factor of 0.9^2, apparently due to a mistake. Miyoki's values for thermal noise are consistent with his quoted value, so if the mistake is his, not mine, the real situation is approximately 20% less stringent than he calculates.

6 EDDY CURRENT CALCULATIONS FOR OTHER SITUATIONS

7 DESIGN OF PITCH ADJUSTMENT MAGNET SYS-

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7.1. Design requirements

The requirements for this subsystem have evolved with time. The original motivation for adding PAMs (as reflected in the name, “pitch adjustment magnet”) was that setting the pitch of the optic on a single loop suspension is a very fiddly manual adjustment, requiring much trial and error even to approach the required tolerance. Therefore a system for adding static corrective pitch torques to the optic was conceived. However yaw was to be managed via bodily rotations of the LOS structure relative to the optical table provided by the LOS Alignment Fixture.

It was later appreciated that the mechanism for rotating the LOS structure in yaw would be prohibitively expensive if it had to be precise enough to satisfy the initial yaw tolerance by itself. Therefore the requirement on the LOS Alignment Fixture was relaxed (LIGO-C980070-00-D) and responsibility for fine initial yaw alignment was transferred to the PAM subsystem.

Of the final requirements, the ones that drive the design are those for 3 mrad(pp) of range in pitch (S. Kawamura, private communication) and 0.02 mrad (4 arc second) resolution in yaw (D. Coyne, private communication). If these are achieved, the requirements for 0.1 mrad(pp) range in yaw (LIGO-C980070-00-D) and 0.5 mrad initial pitch tolerance (LIGO-T950011-19) are trivially satisfied.

7.2. Design concept

We need to be able to add independently a small constant pitch torque and/or a small constant yaw torque, while minimising any other forces, torques and crosscouplings. In order to reduce the coupling between the magnets on the optic and any stray fields, the orientation of the optic magnets has previously been chosen so that those on one diagonal have one polarity and those on the other diagonal have the opposite polarity. Thus the net dipole and quadrupole moments are zero. To take advantage of this, the PAMs are aligned in the same direction so they simulate a uniform field, which the optic is already resistant to. If the PAMs are mounted on screws, static torques can then be produced by moving the PAMs closer to and further from the optic magnets, keeping the same average separation.

Adjusting a single PAM screw by itself gives a complex combination of pitch torque, yaw torque and axial force which is not very useful. Useful adjustments involve all four screws at one. Since there are four magnets, the space of adjustments needs a basis of four linearly independent perturbations. The following combinations are convenient:

A = “all”: all magnets are closer than the nominal distance to their partners on the optic by some given amount ε

H = “horizontal”: left magnets are closer and right magnets further than nominal by ε

V = “vertical”: top magnets are closer and bottom magnets further than nominal by ε

D = “diagonal” top-left and bottom-right magnets are above and bottom-left and top-right magnets are further than nominal by ε

Somewhat counterintuitively, the adjustment which gives a pure pitch adjustment is “H” (whereas one might naively have expected it to be “V”). “V” adjustment gives yaw torque and “D” adjustment gives axial force. “A” adjustment does not produce forces or torques itself but increases the sensitivity of the other adjustments. (The torque produced by a given ε goes as the inverse fifth power of the initial separation between magnet centres.) Because it will not be cost-effective to add mechanism to coordinate these adjustments, the operator will have to follow a strict protocol to avoid applying “D” or “A” adjustments inadvertently. Therefore there needs to be an easy-to count unit of adjustment for each.

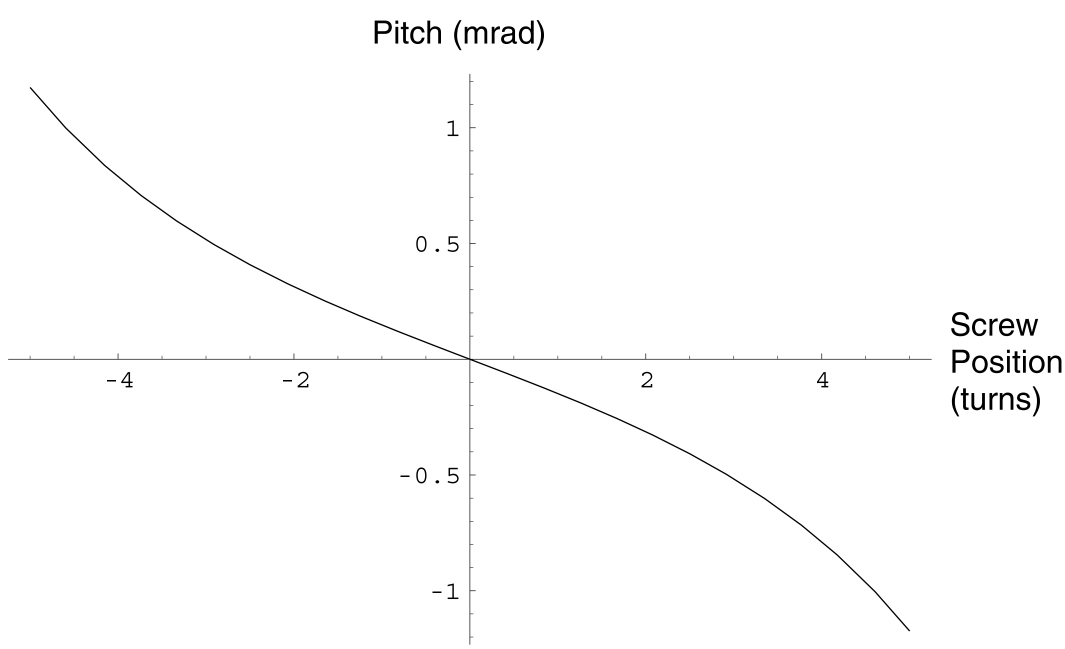
7.3. Detailed design

In the first version of the requirements the desired range in pitch was 3 mrad(pp) and the desired precision was 0.1 mrad. For LOS, the pitch frequency is 0.6 Hz and the moment of inertia is approximately 0.051 km m^2 so the angular elasticity is 0.72 Nm/rad. This led to the following choice of parameters which was used in the LOS Final Design:

- Magnet strength: 0.0073 J/T (as for optic magnets)
- Initial separation: 10 mm (between pole faces)
- Thread pitch: 32 tpi
- Range: +/-4 mm or +/- 5 turns
- Adjustment unit: 1/4 turn on each screw

With these parameters, the expected pitch as a function of screw position is as in Figure 1. If required the range of the adjustment can be increased substantially by moving all the PAMs slightly closer to the optic. However once the operating point is chosen, the response to matched adjustments of all the screws is relatively linear over a wide range. The variations from linearity are not to an extent that would make adjustment tricky.

Figure 1: Pitch of optic as a function of PAM adjustment screw position



7.4. Effect of strength and position imbalances in the magnets

The PAM system relies on cancellations between the forces on the magnets to keep spurious couplings low. This may be upset if the position and strength of some of the magnets are different from the nominal values, so it is necessary to calculate the coupling factors. Displacements of the PAMs transverse to the magnet axis are likely to be small (due to tight manufacturing tolerances) and have only a second order effect on the axial force. Therefore the important variables are (normalised) magnet strength, s, and axial position, z. This gives a total of eight perturbations, s_A, s_H, ..., z_V and z_D.

The effect of the perturbations was calculated using a lever arm for each magnet of 80.8 mm in both the x and y directions, as for the LOS1 support structure.

7.4.1. Static torques and forces

Table 1 gives the static torques in pitch and yaw and the static axial force for strength perturbations of 0.1 (i.e., 10%) and position perturbations of one screw turn (i.e., 1/32'' = 0.79 mm). 75% of the entries are identically zero. The non-zero values can be compared to those required to produce 0.1 mrad of pitch (7.21E-5 N.m), 0.1 mrad of yaw (5.01E-5 N.m) and 10 μm of axial displacement (2.33E-3 N). The non-zero entry for pitch torque next to z_V represents the purpose of the system. The non-zero entries opposite z_H and z_D allow adjustments to yaw and position. This uncalled-for functionality may be a nuisance if the operator miscounts the turns of the screws and perturbs the optic in yaw while trying to correct the pitch. However it is not a major problem and might be useful in some circumstances. Any of the non-zero terms will tend to cause long-term drifts via the temperature coefficients of the magnets (0.12%/K) but for fluctuations of a few degrees these are well within the range of the actuators and thus unimportant.

Table 1: Static torques and forces from perturbations to the magnets (10% in strength or 1/32" in position)

	Pitch torque (N.m)	*Yaw torque (N.m)*	*Axial force (N)*
s_A	0	0	0
s_H	4.29E-5	0	0
s_V	0	4.29E-5	0
s_D	0	0	5.51E-4
z_A	0	0	0

Table 1: Static torques and forces from perturbations to the magnets (10% in strength or 1/32” in position)

	Pitch torque (N.m)	*Yaw torque (N.m)*	*Axial force (N)*
z_H	1.05E-4	0	0
z_V	0	1.05E-4	0
z_D	0	0	1.29E-3

7.4.2. Cross-couplings from linear and angular displacement to pitch and torque in the presence of perturbations

As well as static forces, perturbations to the magnets can produce cross-couplings from motion of the optic (or the support structure holding the PAMs). Tables 2-4 give the cross-coupling coefficients from motions of the optic to torque and force in the usual three degrees of freedom. The table entries are not in common units and cannot be compared directly. The general pattern to the units is <force or torque unit> per <linear or angular displacement unit>. Values in N.m/rad can be compared to the angular restoring force of 0.72 N.m/rad for pitch and 0.5 N.m/rad for yaw. Values in N/m can be compared to the pendulum restoring force of 230 N/m. Values in N.m/m or N/rad can be compared to the z to pitch cross-coupling due to the support being a distance d_{pitch} above the centre of mass: $8.7\times10^{-2}\omega^{2}$ where ω is a typical angular frequency. For $\omega = 1$, this is 3.4 N.m/m. The table entries are less than the comparison values by two orders of magnitude or more. This means that they are also substantially

Table 2: Pitch torque from motions of the optic in the presence of perturbations to the magnets (10% in strength or 1/32” in position)

	N.m/rad(pitch)	*N.m/rad(yaw)*	*N.m/m(axial)*
s_A	0	-0.0010657	0
s_H	0	0	-0.0131893
s_V	0	0	0
s_D	-0.0010657	0	0
z_A	0	-0.00409884	0

Table 2: Pitch torque from motions of the optic in the presence of perturbations to the magnets (10% in strength or 1/32" in position)

	N.m/rad(pitch)	*N.m/rad(yaw)*	*N.m/m(axial)*
z_H	0	0	-0.0507282
z_V	0	0	0
z_D	-0.00409884	0	0

Table 3: Yaw torque from motions of the optic in the presence of perturbations to the magnets (10% in strength or 1/32" in position)

	N.m/rad(pitch)	*N.m/rad(yaw)*	*N.m/m(axial)*
s_A	-0.0010657	0	0
s_H	0	0	0
s_V	0	0	-0.0131893
s_D	0	-0.0010657	0
z_A	-0.00409884	0	0
z_H	0	0	0
z_V	0	0	-0.0507282
z_D	0	-0.00409884	0

Table 4: Axial force from motions of the optic in the presence of perturbations to the magnets (10% in strength or 1/32" in position)

	N/rad(pitch)	*N/rad(yaw)*	*N/m(axial)*
s_A	0	0	0
s_H	-0.0131893	0	0
s_V	0	-0.0131893	0
s_D	0	0	-0.163234
z_A	0	0	0

Table 4: Axial force from motions of the optic in the presence of perturbations to the magnets (10% in strength or 1/32” in position)

	N/rad(pitch)	*N/rad(yaw)*	*N/m(axial)*
z_H	-0.0507282	0	0
z_V	0	-0.0507282	0
z_D	0	0	-0.627824