LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Document Type	**LIGO-T970074-00 -**	**D**	3/28/97

Secondary Light Noise Sources in LIGO

Jordan Camp

Distribution of this draft:

xyz

This is an internal working note
of the LIGO Project..

California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

Secondary Light Noise Sources in a Recycled Interferometer with Fabry-Perot Arms

JBC, HY, SEW

Caltech

Noise on the carrier and RF sideband frequencies of the laser light of the proposed LIGO produces strain noise at the gravitational wave signal output. We calculate the strain noise coupling to laser frequency and amplitude noise, RF oscillator noise, and scattered light phase noise. We find the noise on the RF sidebands generally dominates the strain output.

1 INTRODUCTION

The search for astrophysical sources of gravitational radiation will employ km-scale laser interferometers. A subset of these, LIGO and VIRGO, will have the configuration of a stabilized laser source injected into a mode cleaner filter cavity, followed by an asymmetric Michelson interferometer with Fabry-Perot arms. The frequency and amplitude stabilized light will probe the arm cavity lengths. Gravitational radiation will produce a differential length change in the arms, causing a signal at the output port.

The presence of noise at the interferometer output must be held below the desired strain sensitivity. The primary noise sources defining the interferometer sensitivity are seismic noise at.. frequencies, thermal noise at ... frequencies, and photon counting noise above.. Secondary noise sources are those which, if properly suppressed, will not limit the strain sensitivity. In particular, variations in the phase and amplitude of the laser electric field used to probe the cavity lengths must be carefully controlled. The Michelson configuration is insensitve to light variations; however, small imperfections such as differential losses or deviation from fringe center of the cavity lengths will couple to the light noise to produce an output at the GW port.

In this paper we calculate the induced GW noise from the following sources of noise on the laser light: frequency, amplitude, RF oscillator, and backscattering from the vacuum enclosure. The approach is to write the noise as frequency components of the light, phase modulate the light, and propagate the resultant frequency spectrum through the mode cleaner and to the GW signal output. In section 2 we show the effect of the noise on the frequency spectrum of the light, including the RF sidebands. In section 3 we derive the transfer function of the frequency spectrum from the mode cleaner input to the GW output. Section 4 demodulates the spectrum and section 5 compares the resultant signal with the interferometer sensitivity.

2 NOISE SPECTRUM OF LIGHT

The light noise can be written as audio frequency components about the carrier frequency ω_0, which produce additional audio sidebands when the light is phase modulated at the RF frequency Ω. We first look at frequency noise.

The electric field of the (noiseless) light is $E = E_0 e^{i\omega_0 t}$ where E_0 and ω_0 are the carrier amplitude and frequency.

Writing frequency noise as $\nu = \nu_0 + \delta\nu \cos \omega t$ (1)

where $\delta\nu$ and ω are the amplitude and Fourier frequency of the frequency variation,

the total laser electric field (E_L) phase variation is: $\phi(t) = 2\pi\upsilon_0 t + \frac{2\pi\delta\upsilon}{\omega}\sin\omega t$ (2)

$$E_L = E_0\left[e^{i\omega_0 t} + \frac{\pi\delta\upsilon}{\omega}(e^{i(\omega_0+\omega)t} - e^{i(\omega_0-\omega)t})\right] \quad (3)$$

Now the laser electric field of amplitude E_L is phase modulated to produce RF sidebands:

$$E = E_L e^{iA_{osc}} \text{ where } A_{osc} = \Gamma\cos\Omega t$$

$$= E_L\left[1 + \frac{i\Gamma}{2}(e^{i\Omega t} + e^{-i\Omega t})\right] \quad \text{for } \Gamma \ll 1 \quad (4)$$

$$= E_0\left[e^{i\omega_0 t} + \frac{\pi\delta\upsilon}{\omega}(e^{i(\omega_0+\omega)t} - e^{i(\omega_0-\omega)t})\right]\left[1 + \frac{i\Gamma}{2}(e^{i\Omega t} + e^{-i\Omega t})\right]$$

$$= E_0[(e^{i\omega_0 t} + \frac{\pi\delta\upsilon}{\omega}(e^{i(\omega_0+\omega)t} - e^{i(\omega_0-\omega)t}) + \frac{i\Gamma}{2}(e^{i(\omega_0+\Omega)t} + e^{i(\omega_0-\Omega)t})$$

$$+\frac{i\pi\delta\upsilon\Gamma}{2\omega}(e^{i(\omega_0+\omega+\Omega)t} - e^{i(\omega_0-\omega+\Omega)t} + e^{i(\omega_0+\omega-\Omega)t} - e^{i(\omega_0-\omega-\Omega)t})] \quad (5)$$

The above expression contains 9 frequency components: the carrier and 2 RF sidebands and their associated audio sideband pairs.

Figure 1 shows the frequency components of the light, including the noise audio sidebands. In this

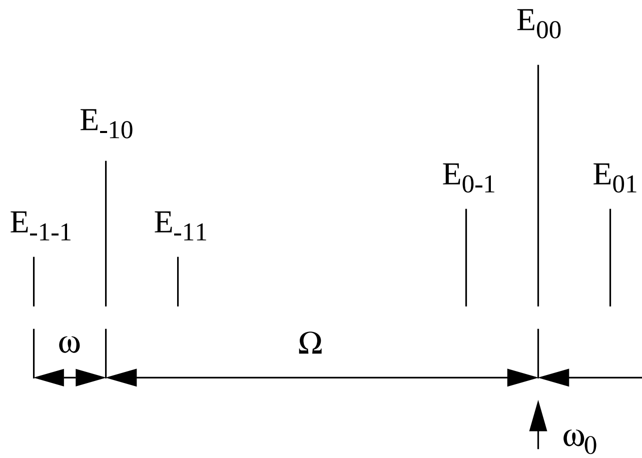


Figure 1: Frequency Spectrum of Light

diagram, E_{ab} refers to the electric field component of RF index *a* (where +/- 1 denotes the RF sidebands) and audio index *b* (where +/- 1 denotes the audio sidebands).

In a similar manner, we can construct the frequency spectrum for the following light noise sources:

Laser amplitude noise: $E = E_0 + \delta E \cos \omega t$

RF oscillator phase noise: $A_{osc} = \Gamma \cos (\Omega t + a_o \cos \omega t)$

RF oscillator amplitude noise: $A_{osc} = \Gamma (1 + (\delta A/A) \cos \omega t) \cos \Omega t$

We also consider the variation in the light from phase noise produced by carrier and sideband modulation in backscattering from the vacuum enclosure:

$E = E_0 + E_s k_0 x_{vac} \cos(\omega t)$ where E_s is the amplitude of the scattered light

Finally, we write the spectrum produced by carrier modulation from an arm cavity gravitational wave signal:

$E = E_0 + E_A k_0 x_A \cos(\omega t)$ where E_A is the amplitude of the arm cavity field

The audio sidebands of these field variations are listed in Table I. The last column of the table gives the overall source factor by which all amplitudes in the row corresponding to a noise source are multiplied.

Table 1: Audio sidebands on light

Noise Source	E_{-1-1}	E_{-10}	E_{-11}	E_{0-1}	E_{00}	E_{01}	E_{1-1}	E_{10}	E_{11}	Source factor
Laser ν noise	$\frac{-i\Gamma\pi\delta\upsilon}{2\omega}$	$\frac{i\Gamma}{2}$	$\frac{i\Gamma\pi\delta\upsilon}{2\omega}$	$\frac{-\pi\delta\upsilon}{\omega}$	1	$\frac{\pi\delta\upsilon}{\omega}$	$\frac{-i\Gamma\pi\delta\upsilon}{2\omega}$	$\frac{i\Gamma}{2}$	$\frac{i\Gamma\pi\delta\upsilon}{2\omega}$	E_0
Laser amp noise	$\frac{i\Gamma\delta E}{4E_0}$	$\frac{i\Gamma}{2}$	$\frac{i\Gamma\delta E}{4E_0}$	$\frac{\delta E}{2E_0}$	1	$\frac{\delta E}{2E_0}$	$\frac{i\Gamma\delta E}{4E_0}$	$\frac{i\Gamma}{2}$	$\frac{i\Gamma\delta E}{4E_0}$	E_0
Osc. ν noise	$\frac{\Gamma a_o}{4}$	$\frac{i\Gamma}{2}$	$\frac{\Gamma a_o}{4}$		1		$\frac{-\Gamma a_o}{4}$	$\frac{i\Gamma}{2}$	$\frac{-\Gamma a_o}{4}$	E_0
Osc. amp. noise	$\frac{i\Gamma\delta A}{4A}$	$\frac{i\Gamma}{2}$	$\frac{i\Gamma\delta A}{4A}$		1		$\frac{i\Gamma\delta A}{4A}$	$\frac{i\Gamma}{2}$	$\frac{i\Gamma\delta A}{4A}$	E_0
RC phase noise	$\frac{-\Gamma k_0 x_v}{4}$	$\frac{i\Gamma}{2}$	$\frac{-\Gamma k_0 x_v}{4}$	$\frac{ik_0 x_v}{2}$	1	$\frac{ik_0 x_v}{2}$	$\frac{-\Gamma k_0 x_v}{4}$	$\frac{i\Gamma}{2}$	$\frac{-\Gamma k_0 x_v}{4}$	E_{sr}
Arm phase noise		$\frac{i\Gamma}{2}$		$\frac{ik_0 x_v}{2}$	1	$\frac{ik_0 x_v}{2}$		$\frac{i\Gamma}{2}$		E_{sa}
Signal		$\frac{i\Gamma}{2}$		$\frac{ik_0 x_A}{2}$	1	$\frac{ik_0 x_A}{2}$		$\frac{i\Gamma}{2}$		E_A

3 TRANSFER FUNCTION OF LIGHT FREQUENCY SPECTRUM TO DARK PORT

3.1. Mode Cleaner

We propagate the light frequency spectrum through the mode cleaner and to the IFO dark port in the following way. Starting with the mode cleaner, we write the equilibrium cavity field equations (see fig. 2) as:

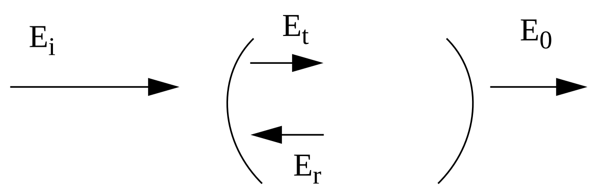


Figure 2: Mode Cleaner equilibrium fields

$$E_t = t_m E_i - r_m E_r \quad (6)$$

$$E_r = E_t e^{i\Phi} r_m \quad (7)$$

where Φ is the round trip phase change suffered by the light in traversing the cavity and t and r are the amplitude transmission and reflection of the mode cleaner mirror. The 2 equations may be combined to give:

$$E_0 = \frac{t_m{}^2 E_i e^{i\Phi/2}}{1 + r_m{}^2 e^{i\Phi}} \quad (8)$$

The round trip phase for the frequency components is calculated as follows. The resonance conditions for the carrier and RF sidebands are:

$$2k_0 L_m = \pi \quad (9)$$

$$\frac{2\pi c}{2L_m} = \Omega_{RF} \quad (10)$$

where L_M is the mode cleaner resonant length, k_0 is the carrier wavenumber, and Ω_{RF} is the RF frequency. Including the mode cleaner imperfections of a length offset from resonance $L'_m = L_m + dx_m$ and a sideband detuning from resonance dΩ we have:

$$\Phi_C = 2k_C L'_m = \frac{2}{c}(\omega_0 + \omega)(L_m + dx_m) = \frac{2}{c}[\pi + \omega L_m + \omega_0 dx_m] \quad (11)$$

$$\Phi_S = 2k_S L'_m = \frac{2}{c}[\pi + 2\pi + \omega L_m + \omega_0 dx_m + d\Omega L_m] \tag{12}$$

where ω is the audio frequency of the noise source under consideration.

Inserting these expressions in 8 and taking the near resonance approximation $e^{i\Phi} \sim 1 + i\Phi$ we obtain:

$$E_{co} = E_{ci}\left[1 + (iG_m)\frac{\left(2\frac{\omega L_m}{c} + 2k\,dx_m\right)}{\left(1 + i\frac{\omega}{\omega_m}\right)}\right] \tag{13}$$

$$E_{so} = E_{si}\left[1 + (iG_m)\frac{\left(2\frac{\omega\, L_m}{c} + 2kdx_m + d\Omega\, L_m\right)}{\left(1 + i\frac{\omega}{\omega_m}\right)}\right] \tag{14}$$

for the carrier and sideband field amplitudes after the mode cleaner. Here ω_m is the mode cleaner cavity pole, $G_m = \frac{1}{1 - r_m^2}$ and E_{ci} and E_{si} are the field amplitudes of the frequency components listed in table I.

3.2. Interferometer

We continue with the propagation of the fields of eqns 15-16 to the interferometer dark port. In fig. 3 we show the circulating fields in the interferometer.

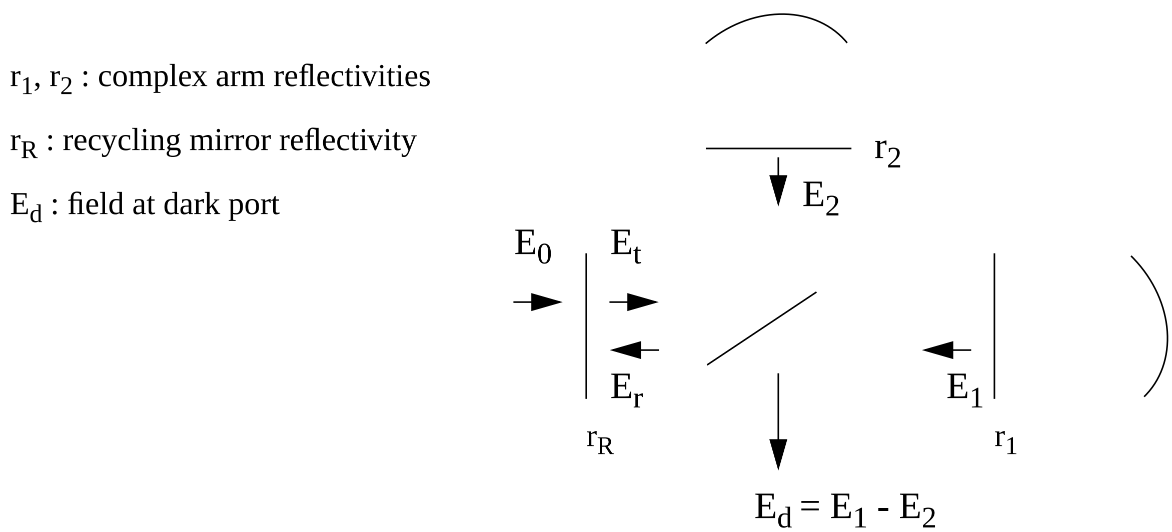


Figure 3: Interferometer circulating fields

we have $$E_r = \frac{E_t}{2}(r_2 e^{-i\phi_2} + r_1 e^{-i\phi_1}) \tag{15}$$

and $$E_t = E_{ic} t_R + \left(-\frac{E_t}{2}\right) r_R (r_2 e^{-i\phi_2} + r_1 e^{-i\phi_1}) \tag{16}$$

where ϕ is the round trip phase shift of the carrier in the recycling cavity. Also

$$E_d = \frac{E_t}{2}(r_2 e^{-i\phi_2} - r_1 e^{-i\phi_1}) \tag{17}$$

Combining eqns. 15 - 17 gives

$$E_d = \frac{\frac{E_{ic}}{2} t_R (r_2 e^{-i\phi_2} - r_1 e^{-i\phi_1})}{1 + \frac{r_R}{2}(r_1 e^{-i\phi_1} + r_2 e^{-i\phi_2})}$$

3.2.1. Carrier

We have $\phi_{1C} = \pi + k(dx_{r+} + dx_{r-})$, and $\phi_{2C} = \pi + k(dx_{r+} - dx_{r-})$, the resonance condition for the carrier in the recycling cavity, degraded by the common and differential mode offset. Then

$$E^c{}_d = \frac{\frac{E_{ic}}{2} t_R (\Delta r_c + i r_c k dx_{r-})}{1 + r_R r_c (1 + ikdx_{r+})} \tag{18}$$

where $\Delta r_c = r_2 - r_1$, and dx_R is the recycling cavity deviation from resonance. The circulating fields for the arm cavity give the following expression for the carrier arm reflectivity (see Appendix I):

$$r_c = \frac{r_0 + i r_F^{-1} \frac{\omega}{\omega_c} + i G^c{}_A k_0 dx_A}{1 + \frac{i\omega}{\omega_c}} \tag{19}$$

where $r_0 = r_F - \frac{t_F^2}{1 - r_F}$, $G^c{}_A = \frac{2}{1 - r_F}$, is the buildup of the carrier in the arm cavity, dx_A is the arm cavity deviation from resonance and $\omega_c \sim \frac{c}{2L} \cdot (1 - r_F)$ is the arm cavity pole frequency.

$$\text{Then } E^c{}_d \sim \frac{E_{co}}{2} t_R G^c{}_R \left(\frac{A_{cm}\left[1 + i\frac{\omega}{\omega_c}\right] + iG^c{}_A k dx_A}{\left(1 + \frac{i\omega}{\omega_{cc}}\right)} + ikdx_R \right) \tag{20}$$

where $A_{cm} = \Delta r_c$, the arm cavity reflectivity match which gives the interferometer common mode attenuation, $\omega_{cc} \sim \frac{\omega_c}{2} \cdot (1 + r_R r_0)$,which shows the filtering of noise on the carrier by the coupled recycling-arm cavity; $G^c{}_R = \frac{1}{1 + r_R r_0}$, the buildup of the carrier amplitude in the recycling cavity; and we have dropped the small ikx_{r+} term in the denominator.

3.2.1.1 RF sidebands

For the sidebands we take the arm cavity reflectivity = 1 and the following expressions for the recycling cavity phase shifts:

$$\phi_I = 2\ k\ l_I \tag{21}$$

$$= 2\left(\frac{\omega_0 + \Omega + \omega}{c}\right)\left(L_R + \frac{\delta}{2} + (dx_{r+} + dx_{r-})\right) \tag{22}$$

$$= \pi + \frac{\pi\omega}{\Omega} + \frac{\Omega\delta}{c} + \frac{\omega\delta}{c} + k(dx_{r+} + dx_{r-}) \tag{23}$$

where $\delta = l_I - l_P$ is the recycling cavity asymmetry and $2\ \Omega\ L_R = \pi$, the recycling cavity sideband resonant condition.

$$\text{Similarly, } \phi_P = \pi + \frac{\pi\omega}{\Omega} - \frac{\Omega\delta}{c} - \frac{\omega\delta}{c} + k(dx_{r+} - dx_{r-}) \tag{24}$$

Then eqn. 18 gives

$$E^s{}_d = -E_{so} t_R G^s{}_R i\left(\frac{\Omega\delta}{c} + \frac{\omega\delta}{c} - kdx_R\right) \tag{25}$$

where $G^s{}_R = \frac{1}{1 - r_R \cos\frac{\Omega\delta}{c}}$ is the sideband amplitude buildup in the recycling cavity and where we have neglected the recycling cavity pole $f_{rc} = (1-r_R)(\Omega/2\pi^2) \sim 60$ kHz.

3.2.2. Scattered light phase noise

The scattered light we consider enters the interferometer at the locations shown in figure 4, after backscattering from the vacuum enclosure. We must modify the above expressions to account for the change in noise source location, including the filtering and amplitude gain factors.

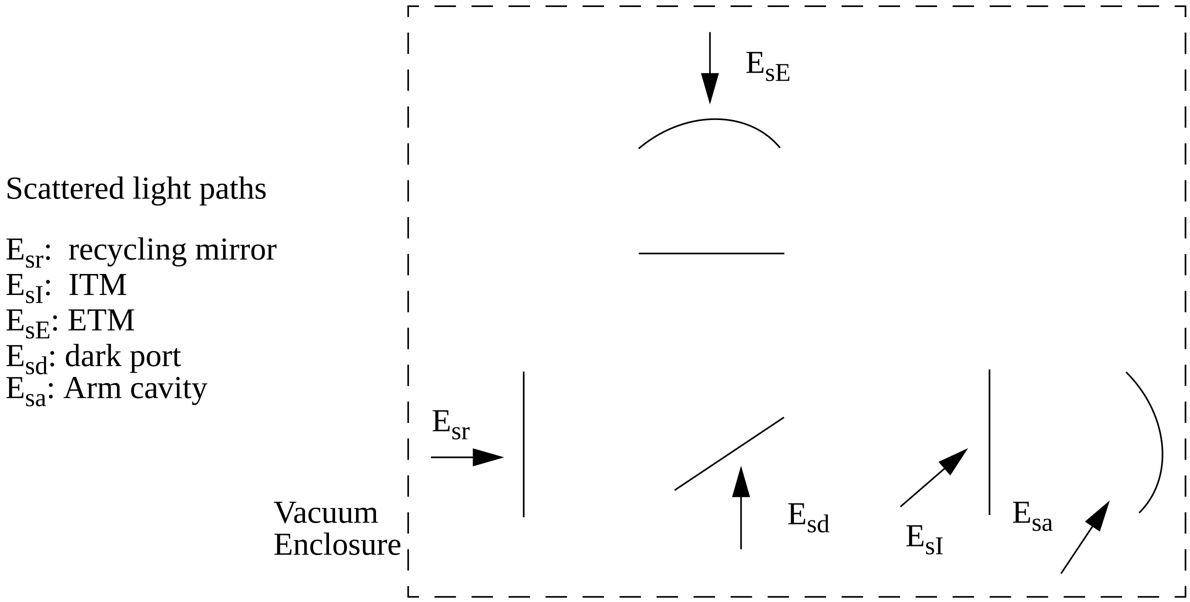


Figure 4: Scattered light source locations

3.2.2.1 Recycling mirror path

For a source of scattered light at this location, we replace in eqns 20 and 25 the source term $E_0\, t_r$ with $E_{sr}\, t_r$. The subsequent analysis is unchanged. Thus we have:

$$E^c{}_d = \frac{E_{sr}}{2} t_r G^c{}_R \left(\frac{A_{cm}\left[1 + i\frac{\omega}{\omega_c}\right] + iG^c{}_A k dx_A}{\left(1 + \frac{i\omega}{\omega_{cc}}\right)} + ik dx_R \right) \qquad (26)$$

$$E^s{}_d = -E_{sr} t_r G^s{}_R i\left(\frac{\Omega\delta}{c} + \frac{\omega\delta}{c} - k dx_R\right) \qquad (27)$$

3.2.2.2 Dark port path

Scattered light injected at this location couples to the recyling cavity in the following way. The recyling cavity is almost transparent to the sidebands, so that only 10% of the incident amplitude is reflected. However the carrier returns along the beam splitter symmetric path with almost no buildup or common mode attenuation. For the carrier we replace $E_0\ t_r / 2$ with E_{sdp} and also set $G^c{}_R = 1$, $A_{cm} = 1$, and apply the filtering of the *single* arm cavity pole. We obtain:

$$E^c{}_d = E_{sd}\left(\frac{\left[1 + i\frac{\omega}{\omega_c}\right] + iG^c{}_A k d x_A}{\left(1 + \frac{i\omega}{\omega_c}\right)} + ikdx_R\right) \tag{28}$$

$$E^s{}_d = -0.1 E_{sd} G^s{}_R\left(\frac{\Omega\delta}{c} + \frac{\omega\delta}{c} - kdx_R\right) \tag{29}$$

(30)

3.2.2.3 ITM path

For this path we find a large sideband buildup, and the carrier field dominated by a term which couples directly to the dark port and undergoes single cavity filtering with no common mode attenuation:

$$E^c{}_d = E_{sI}\left(\frac{\left[1 + i\frac{\omega}{\omega_c}\right] + iG^c{}_A k d x_A}{\left(1 + \frac{i\omega}{\omega_c}\right)} + ikdx_R\right) \tag{31}$$

$$E^s{}_d = 10 E_{sI} G^s{}_R i\left(\frac{\Omega\delta}{c} + \frac{\omega\delta}{c} - kdx_R\right) \tag{32}$$

3.2.2.4 Arm cavity scattering path

In this case the scattered carrier light enters directly into the arm cavity. We use the expression of Appendix II and replace the source term $E_0\left(\frac{t_R}{1 + r_R r_0}\right)\left(\frac{t_F}{1 - r_F}\right) ikX_a$ with $E_{sa}\ ikx_v$. We obtain:

$$E_d = \frac{E_{sa}}{t_F}\frac{G_A ik\,x_v}{\left(1 + \frac{i\omega}{\omega_c}\right)} \tag{33}$$

3.2.2.5 ETM path

This path is similar to the arm cavity path except we must multiply the source term by the ETM transmission. We have:

$$E_d = \frac{t_B E_{sE}}{t_F}\frac{G_A ik\,x_v}{\left(1 + \frac{i\omega}{\omega_c}\right)} \tag{34}$$

4 DEMODULATION OF PHOTOCURRENT

4.1. Photodetector current

If $E_1(\omega_1+\omega_0)$, E_2 $(\omega_2+\omega_0)$ are two electric fields incident on the photodetector where $\omega_1>\omega_2$ and ω_1, $\omega_2 << \omega_0$, the optical frequency, we can write:

$$E_p = Re[(E_1 e^{i\omega_1 t} + E_2 e^{i\omega_2 t})e^{i\omega_0 t}] \qquad (35)$$

$$= Re(Ae^{-i\omega_0 t}) \qquad (36)$$

and $i_p = \overline{EE^*} = AA^* = [E_1 E_2{}^* e^{i(\omega_1-\omega_2)t} + cc] = 2Re[E_1 E_2{}^* e^{i(\omega_1-\omega_2)t}]$ (37)

where i_p is the photodetector current and we have omitted terms which do not mix the two frequencies.

Refering to figure 1 and writing the contributions to i_p for all the fields separated by $\Omega \pm \omega$ which will give a demodulated in-band output we have:

$$i_p = 2Re[E_{00}E^*_{-1-1}e^{i(\Omega+\omega)t} + E_{00}E^*_{-11}e^{i(\Omega-\omega)t} + E_{11}E^*_{00}e^{i(\Omega+\omega)t} + E_{1-1}E^*_{00}e^{i(\Omega-\omega)t}]$$

$$+2Re[E_{0-1}E^*_{-10}e^{i(\Omega-\omega)t} + E_{10}E^*_{0-1}e^{i(\Omega+\omega)t} + E_{01}E^*_{-10}e^{i(\Omega+\omega)t} + E_{10}E^*_{01}e^{i(\Omega-\omega)t}] \qquad (38)$$

The 1st 4 terms represent mixing of the carrier with the RF audio sidebands

2nd 4 terms represent mixing of the carrier audio sidebands with the RF sidebands

4.2. Demodulation

The gravity wave port mixer output is the product of the photodetector signal with a quadrature phase reference:

$$V_{out} = \int i_p \sin\Omega t dt \qquad (39)$$

With *T* the RF modulation period,

$$\frac{2}{T}\int_t^{(t+T)} Re[ae^{i(\Omega+\omega)t}]\sin\Omega t dt = -Re(a)\sin\omega t - Im(a)\cos\omega t \qquad (40)$$

$$= \int Re[-a^* e^{i(\Omega-\omega)t}] \sin\Omega t dt \quad = \quad -Im[ae^{i\omega t}] \tag{41}$$

Thus we can write:

$$V = -Im\Big(E_{00}(E^*_{-1-1} - E^*_{1-1}) + E^*_{00}(E_{11} - E_{-11})$$

$$+ E_{01}(E^*_{-10} - E^*_{10}) + E^*_{0-1}(E_{10} - E_{-10})\Big)e^{i\omega t} \tag{42}$$

and therefore $|V| =$

$$\left|E_{00}(E^*_{-1-1} - E^*_{1-1}) + E^*_{00}(E_{11} - E_{-11}) + E_{01}(E^*_{-10} - E^*_{10}) + E^*_{0-1}(E_{10} - E_{-10})\right|$$

Carrier x RF audio sidebands   Carrier audio sidebands x RF

4.2.1. Demodulation Phase Noise

Phase noise on the local oscillator demodulation produces audio sidebands about the demodulator frequency.

The quadrature demodulation spectrum is:

$$A = \Gamma\,[\sin\Omega t + (a_o/2)(\cos(\Omega+\omega)t + \cos(\Omega-\omega)t)] \tag{43}$$

Then the mixer output is:

$$V = \int Re[(E_{00}E_{-10}^* + E_{10}E_{00}^*)e^{i\Omega t}]\frac{a_o}{2}(\cos(\Omega+\omega)t + \cos(\Omega-\omega)t)dt$$

$$= Re(E_{00}E_{-10}^* + E_{10}E_{00}^*)\frac{a_o}{2}\cos\omega t \tag{44}$$

5 NOISE OUTPUT AND COMPARISON WITH SIGNAL

In this section we add the terms of eqn 39 for each noise source in Table I to form the mixer output, and compare with the output from an arm cavity signal at the limit of the interferometer sensitivity. To summarize, we use the the audio sideband amplitudes and source factors in Table I, apply the phase shifts, filtering and gain factors from passage through the mode cleaner and interferometer (eqns. 13-14, 20, 25, 26-30), and demodulate the resultant frequency spectrum according to eqn. 39. As many terms are involved, the sums were evaluated symbolically with the use of a *Mathematica* code.

For each noise source we list the largest noise couplings for both the carrier and sideband noise. We then compare the dominant noise term to the arm cavity signal at the interferometer sensitivity limit.

5.1. Arm Cavity Signal

With the carrier field of appendix II we can immediately write the mixer output as:

$$V = \frac{\Gamma E_0^2}{2} t_R^2 G^c{}_R G^s{}_R \cdot \frac{k_0 G_A X}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}} \tag{45}$$

5.2. Laser Frequency Noise

5.2.1. Carrier noise

$$V = \frac{\Gamma E_0^2}{2} t_R^2 G^c{}_R G^s{}_R \cdot \frac{\pi\delta\upsilon}{\omega} A_{cm} \frac{\omega}{\omega_c} \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_{cc}}\right)^2}} \tag{46}$$

5.2.2. Sideband noise

$$V = \frac{\Gamma E_0^2}{2} t_R^2 G^c{}_R G^s{}_R \cdot A_{cm} \frac{\pi\delta\upsilon}{\omega} \tag{47}$$

5.2.3. Comparison to signal

$$\frac{V_{\delta\nu}}{V_{signal}} = A_{cm} \frac{\delta\nu}{\nu_0} \frac{L}{X} \tag{48}$$

5.3. Laser Amplitude Noise

5.3.1. Carrier Noise

$$V = \frac{\Gamma E_0^2}{2} t_R^2 G^c{}_R G^s{}_R \cdot \frac{\delta E}{2E_0}\left(\frac{k_0 G_A dx_A + k_0 dx_R + i\frac{G_m \omega L_m}{c}}{\sqrt{1+\left(\frac{\omega}{\omega_{cc}}\right)^2}}\right) \tag{49}$$

5.3.2. Sideband noise

$$V = \frac{\Gamma E_0^2}{2} t_R^2 G^c{}_R G^s{}_R \cdot \frac{\delta E}{2E_0}\left(k_0 G_A dx_A + k_0 dx_R + i\frac{G_m \omega L_m}{c}\right) \tag{50}$$

5.3.3. Comparison to signal

$$\frac{V_{\delta E}}{V_{signal}} = \frac{\delta E}{2E_0} \frac{dx_A}{X} \frac{\omega}{\omega_c} \tag{51}$$

5.4. Oscillator Phase Noise

We consider noise on the RF modulator and also the local oscillator demodulation (eqn 40.)

5.4.1. Sideband noise

$$V = \frac{\Gamma E_0^2}{2} t_R^2 G^c{}_R G^s{}_R \cdot \frac{a_o}{4}\left[\frac{\omega\delta}{c}(k_0 G_A dx_A) + \frac{A_{cm} G_m \omega L_m k dx_r}{c}\right] \tag{52}$$

5.4.2. Comparison to signal

$$\frac{V_{\delta a}}{V_{signal}} = \frac{a_o}{4} \frac{\omega\delta}{c} \frac{dx_A}{X} \tag{53}$$

5.5. Oscillator Amplitude Noise

5.5.1. Sideband noise

$$V = \frac{\Gamma E_0^2}{2} t_R^2 G^c{}_R G^s{}_R \cdot [k_0 G_A dx_A + k_0 dx_R] \tag{54}$$

5.5.2. Comparison to signal

$$\frac{V_{\delta A}}{V_{signal}} = \frac{\delta A}{2A}\frac{dx_A}{X}\frac{\omega}{\omega_c} \tag{55}$$

5.6. Scattered light Phase Noise - RM path

5.6.1. Carrier noise

$$V = \frac{\Gamma E_0}{2}t_R^2 E_{sr} G^c{}_R G^s{}_R \cdot k_0 x_v \left(\frac{A_{cm}\frac{\omega}{\omega_c}}{\sqrt{1+\left(\frac{\omega}{\omega_{cc}}\right)^2}}\right) \tag{56}$$

5.6.2. Sideband Noise

$$V = \frac{\Gamma E_0}{2}t_R^2 E_{sr} G^c{}_R G^s{}_R \cdot k_0 x_v (A_{cm}) \tag{57}$$

5.6.3. Comparison to Signal

$$\frac{V_{sr}}{V_{signal}} = \frac{E_{sr}}{E_0}\frac{\sqrt{2}A_{cm}x_v}{G_A X} \tag{58}$$

5.7. Scattered light Phase Noise - Dark Port path

5.7.1. Carrier Noise

$$V = \Gamma E_0 t_R E_{sd} G^s{}_R \cdot k_0 x_v \left(\frac{\frac{\omega}{\omega_c}}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}\right) \tag{59}$$

5.7.2. Sideband Noise

$$V = \frac{\Gamma E_0}{2}t_R E_{sd} G^c{}_R (0.1) G^s{}_R \cdot k_0 x_v (A_{cm}) \tag{60}$$

5.7.3. Comparison to Signal

$$\frac{V_{sd}}{V_{signal}} = \frac{2\sqrt{2}E_{sd}}{t_R {G^c}_R E_0} \frac{x_v}{G_A X} \tag{61}$$

5.8. Scattered light phase noise - ITM path

5.8.1. Carrier noise

$$V = \Gamma E_0 t_R E_{sI} {G^s}_R \cdot k_0 x_v \left(\frac{\frac{\omega}{\omega_c}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \right) \tag{62}$$

5.8.2. Sideband Noise

$$V = \frac{\Gamma E_0}{2} t_R 10 E_{sI} {G^c}_R {G^s}_R \cdot k_0 x_v (A_{cm}) \tag{63}$$

5.8.3. Comparison to Signal

$$\frac{V_{sI}}{V_{signal}} = \frac{2\sqrt{2}E_{sI}}{t_R {G^c}_R E_0} \frac{x_v}{G_A X} \tag{64}$$

5.9. Scattered light phase noise - ETM path

5.9.1. Carrier noise

$$V = t_B t_F E_{sE} \Gamma E_0 t_R {G^s}_R \cdot \frac{G_A k_0 x_v}{\sqrt{1 + \frac{\omega}{\omega_c}^2}} \tag{65}$$

5.9.2. Comparison to Signal

$$\frac{V_{sE}}{V_{signal}} = \frac{2 t_F t_B E_{sE}}{t_R {G^c}_R E_0} \frac{x_v}{X} \tag{66}$$

5.10. Arm Cavity Phase Noise

5.10.1. Carrier Noise

$$V = t_F E_{sa} \frac{\Gamma E_0 t_R G^s{}_R G_A ikx_v}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \tag{67}$$

5.10.2. Comparison to Signal

$$\frac{V_{sa}}{V_{signal}} = \frac{t_F E_{sa}}{t_R G^c{}_R E_0} \frac{x_v}{X} \tag{68}$$

6 DERIVED LIMITS ON NOISE SOURCES

We assume all of the above noise sources must be held to <10% of the arm cavity signal at the level of the interferometer sensitivity at f = 100 Hz. In the following table we assume the following interferometer imperfection and list the derived limits on the noise sources.

Imperfections

$A_{cm} = 10^{-2}$

$dx_A = 10^{-12}$ m

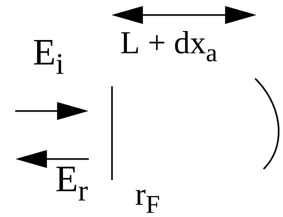
$dx_r = 10^{-10}$ m

Table 2: Derived limits on Light Noise Sources

Noise Source	Parameter	Value
Laser frequency	$\delta\nu$	10^{-7} Hz / Hz$^{1/2}$
Laser amplitude	$\delta E / E_0$	10^{-8} / Hz$^{1/2}$
Oscillator Phase	a_0	10^{-4} / Hz$^{1/2}$
Oscillator Amplitude	$\delta A / A$	10^{-8} / Hz$^{1/2}$
RM scattered light	E_{sr} / E_0	10^{-5}
Dark Port scattered light	E_{sd} / E_0	3.5×10^{-7}
ITM scattered light	E_{sI} / E_0	3.5×10^{-7}
ETM scattered light	E_{sE} / E_0	5×10^{-6}
Arm scattered light	E_{sa} / E_0	7×10^{-8}

APPENDIX 1 ARM CAVITY REFLECTIVITY

$$r_c = \frac{E_r}{E_i}$$



With the usual circulating field analysis we can write:

$$r_c = r_F + \frac{t_F^2 e^{-i\Phi}}{1 + r_F e^{-i\Phi}}$$

where $$\Phi = 2k(L + dx_a) = 2\left(\frac{2\pi\nu_0 + \omega}{c}\right)(L + dx_a)$$

$$= \pi + \frac{2L\omega}{c} + 2k_0 dx_a$$

Thus

$$r_c = \frac{r_0 + r_F^{-1} i\left(\frac{\omega}{\omega_c} + k_0 G_a dx_a\right)}{1 + i\left(\frac{\omega}{\omega_c} + k_0 G_a dx_a\right)}$$

where $\omega_c = \frac{c}{2L} \cdot \frac{1 - r_F}{r_F}$, $G^c{}_A = \frac{2}{1 - r_F}$, and $r_0 = r_c(\Phi = \pi)$

Also with $\Delta x_{rms} < 10^{-11}$ m and $r_F \sim 1$ we have:

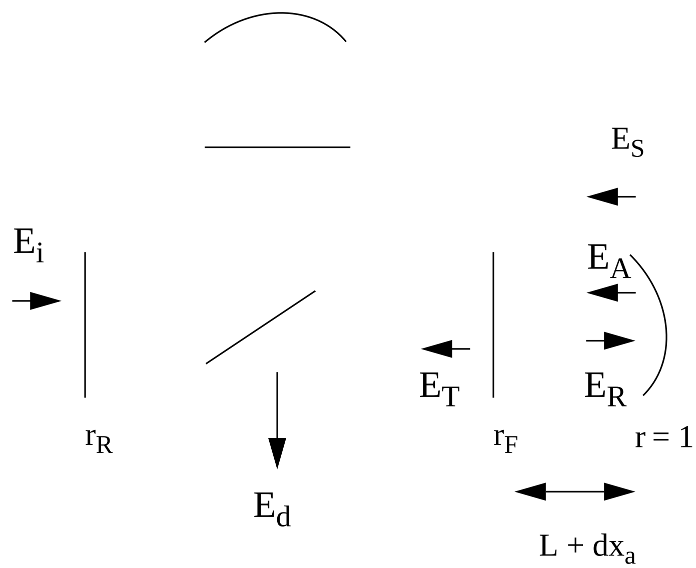
$$r_C \sim \frac{r_0 + i\left(\frac{\omega}{\omega_c} + k_0 G_a \Delta x_a\right)}{1 + i\frac{\omega}{\omega_c}} \qquad (69)$$

(70)

For $\omega \gg \omega_c$ this expression reduces to 1, as is the case for the arm cavity reflectivity for the RF sidebands.

APPENDIX 2 ARM CAVITY SIGNAL

We solve for the circulating field in the recycled IFO, where the audio sidebands are seen as a source field to be added to the circulating field:



Then $$E_A = E_S + E_R = E_s + E_A e^{-i2k(L+dx_a)}(-r_F) \qquad \text{where } k = \frac{2\pi\nu_0 + \omega}{c}$$

so that $$E_A = \frac{E_s}{(1 - r_F)\left(1 + \frac{i\omega}{\omega_c}\right)}$$

and $$E_d = \frac{E_s t_F (1 + ikdx_a)}{(1 - r_F)\left(1 + \frac{i\omega}{\omega_c}\right)}$$

Now E_s = source field of audio sidebands

$$= \frac{E_0}{2}\left(\frac{t_R}{1 + r_R r_0}\right)\left(\frac{t_F}{1 - r_F}\right) ikX_a$$

Thus $$E_d = \frac{E_0}{2} t_R G_R^C G_A \frac{ikX_a}{\left(1 + \frac{i\omega}{\omega_c}\right)} \qquad (71)$$

This expression contains the arm cavity pole and the recycling cavity field amplification.