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MODELLING THE PERFORMANCE OF AN INITIAL-LIGO DETECTOR WITH REALISTICALLY IMPERFECT OPTICS

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The satisfactory performance of interferometric detectors in the Laser Interferometer Gravitational-Wave Observatory (LIGO) project will depend upon exceedingly high-quality optical components. In order to accurately predict the response of real detectors, we have written a grid-based simulation program which models the steady-state laser fields in a complete LIGO interferometer with multiply-coupled-cavities, for a wide variety of possible optical imperfections. Using measurements of exceptionally smooth mirrors obtained from industry, we show how feasibly obtainable levels of mirror deformations may degrade the sensitivity of the LIGO detector to astrophysically-generated gravitational waves.

1 Introduction and Background

The LIGO project [1] will use long-baseline interferometers (IFO's) to detect astrophysically-generated gravitational waves (GW's) via their perturbing forces on the interferometer mirrors. In order to detect these extraordinarily small GW-induced mirror motions ($\Delta L/L \sim 10^{-21}$), several limiting noise sources must be controlled, especially seismic, thermal, and photon shot noise.

Imperfections in the IFO optics will inhibit the detection of GW's by causing a reduction in the amount of resonating power available for the sensing of mirror positions, and by increasing the amount of unmodulated (i.e. non-signal-bearing) light which emerges from the IFO signal port and contributes to the shot noise. The net result is a degradation of the *shot-noise-limited* part ($\nu_{GW} \geq 100$ Hz) of the expected LIGO sensitivity envelope, $h(f)_{SN}$.

To quantify these effects, we have developed a computer code to perform detailed numerical simulations of an initial-LIGO IFO, with the capability of simulating a wide variety of IFO imperfections. Here we introduce the code and present a selection of results; a detailed description may be found elsewhere [2].

2 The LIGO Simulation Program

The program is a Fortran code, adapted for execution on the massively-parallel Paragon supercomputers at Caltech. As a grid-based program, the fundamental objects it manipulates are complex, 2-D maps representing mirror profiles and transverse slices of the laser beam electric field, sampled at various points in the IFO. Using the *paraxial approximation*, beam propagations become simplified procedures primarily involving FFT's[3]. Reflections and transmissions at mirrors are performed using a small-distance approximation[4], reducing them to pixel-by-pixel multiplications of an electric field map with a mirror map.

The program can incorporate many different optical imperfections, including, for example: (i) Deformations in surface figure and substrate homogeneity profiles, (ii) Finite mirror apertures and realistic beam clipping, (iii) Mirror displacements, tilts, curvature errors and beam mismatch, (iv) Pure losses into which we lump our estimates of high-angle scattering and power absorption.

We simulate a static IFO, neglecting the dynamics of control systems and power buildup. An iterative procedure relaxes the electric fields to their steady-state distributions. The code simultaneously implements a number of parameter optimizations to ensure that all of the proper resonance conditions are achieved, and that the interferometer is optimally configured for GW detection. Finally, sideband frequency beams for the LIGO heterodyne detection scheme are modelled, so that we can explicitly compute $h(f)_{SN}$ for the IFO.

We have obtained 2 maps of real mirror deformations from industry: a fused-silica substrate homogeneity map from Corning, and a surface figure map (of a polished but uncoated substrate) from Hughes-Danbury. To create enough substrate and surface maps for all of the IFO mirrors, Fourier transform techniques were used to convert each of the source maps into a family of maps with identical power spectra but different, uncorrelated structure. Finally, the initial family of surface maps (w/RMS deformations of $\sim .6$ nm $\sim \lambda_{YAG}/1800$ in the central portion of the source map) were scaled up by constant factors to create surface profile families of $\lambda/1200$, $\lambda/800$, and $\lambda/400$, for conservative estimation given poorly known mirror coating homogeneity limitations.

3 Results and Discussion

Five baseline runs are presented here to characterize the effects of realistically deformed optics upon LIGO's GW sensitivity: one (control) run with perfectly smooth surfaces and substrates, and 4 runs with (respectively) $\lambda/1800$, $\lambda/1200$, $\lambda/800$, or $\lambda/400$ surface maps on all mirrors, plus the deformed substrates.

Fig. 1 plots strain noise spectral density, $h(f)$, vs. GW frequency f. The five shot-noise curves are computed[2] for each of the simulation runs, and are shown against the overall *GW-strain-equivalent noise requirement envelope*[5] for the initial-LIGO interferometers. All of the runs except for the very worst case ($\lambda/400$ surfaces) meet the LIGO requirement.

Fig. 2 shows the effects of deformed mirrors upon LIGO's sensitivity to the periodic GW's emitted by a non-axisymmetric pulsar. The noise curves

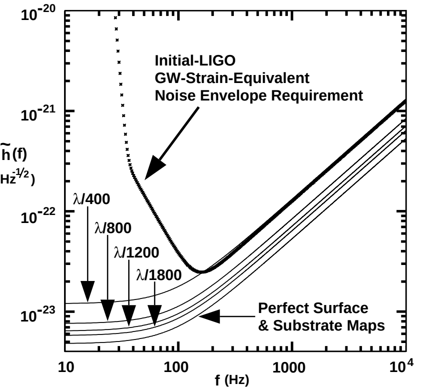

Figure 1: Initial-LIGO requirement vs. simulation-derived shot noise curves.

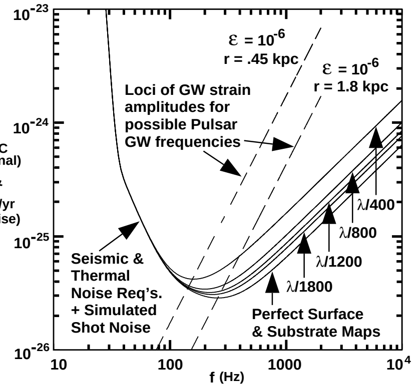

Figure 2: Effects of deformed mirrors on LIGO's sensitivity to GW's from pulsars.

are the quadratic sums of LIGO's seismic and thermal noise *requirements* plus the simulation-derived shot noise curves of Fig. 1. They have been converted to $h_{3/yr}(f)$ [6], representing high-confidence coincident detection in all 3 LIGO IFO's. The dashed lines represent the characteristic signal strengths, $h_c(f)$ [6], for pulsars with ellipticity $\epsilon = 10^{-6}$ at different distances (r) from the earth. Estimating roughly by setting $h_c = h_{3/yr}$ at the frequency of peak sensitivity, going from worst case to best gains a factor of ~4 in 'lookout distance' r, yielding a potential event rate increase ($\propto r^2$ in galactic disk) of ~16.

To summarize: (1) Our simulation program can be used to drive specifications for LIGO optics [7], (2) The sensitivity goals of the initial IFO's can be met with feasibly obtainable mirrors (pending acceptable coatings), (3) Significant benefits to LIGO science are gained with extremely high quality optics.

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