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Draft **LIGO-T960158-00 - D** Sep. 26, 96

RESPONSE OF SUSPENSION SYSTEM TO PITCH MOTION OF SUSPENSION POINT

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Distribution of this draft:

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Let us think about the suspension system shaken in pitch around the suspension point. As shown in Fig. 1, the mass (of mass M) is suspended by the wire (of length l) at the release point which is distance d above the center of mass. The suspension sensor/actuators are mounted on the suspension support structure at a distance of a above and below the center of mass. These sensor/actuators damp the mass pseudo-critically using a signal proportional to the relative position of the mass to the sensor itself. The actuators also apply a DC force on the mass, which depends on this relative position. Therefore, the motion of the sensor/actuators causes the test mass to shake. θ_1, θ_2, and θ_3 denote the pitch angle displacement of the suspension support structure, the test mass, and the wire, respectively. x_2, and x_3 denote the position displacement of the test mass, and an average position displacement of the suspension's sensor/actuators.

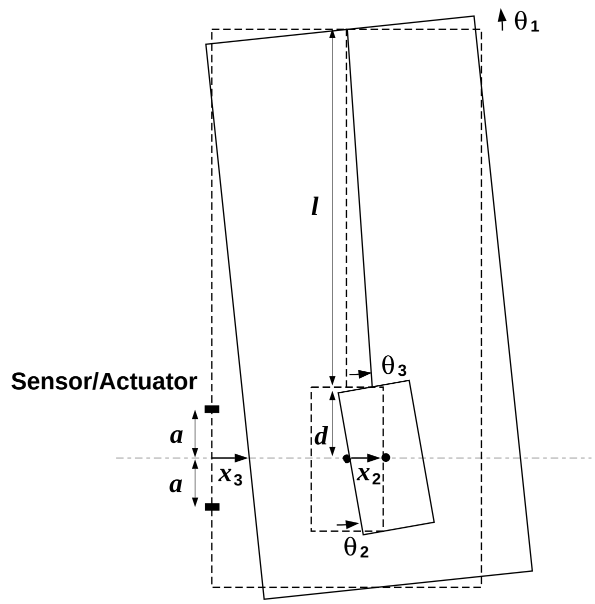

Figure 1: Side view of the suspension system showing the pitch motion of the suspension support structure around the suspension point.

The equations of motion of the mass for x_2 and θ_2 are, respectively:

$$-g\theta_3 - \tilde{k}_x \cdot \frac{d}{dt}(x_2 - x_3) + \alpha(x_2 - x_3) = \frac{d^2 x_2}{dt^2}$$

$$g(\theta_3 - \theta_3)d - \tilde{k}_\theta \cdot \frac{d}{dt}(\theta_2 - \theta_1) + \alpha(\theta_2 - \theta_1)a^2 = I \cdot \frac{d^2\theta}{dt^2},$$

where I is a normalized[1] moment of inertia, g is the acceleration of gravity, $\tilde{k}_x$ and $\tilde{k}_\theta$ are normalized low-pass-filter operators for damping force along x and θ, respectively, α is a normalized DC force change per unit displacement, i.e., $\alpha = \frac{\left(\frac{dF_{DC}}{dx}\right)}{M}$. There are simple geometrical relations:

$\theta_3 = \frac{x_2 - d\theta_2}{l}$ and $x_3 = \theta_1(l + d)$.

The DC force and torque are neglected. The wire resonances are not considered here.

The parameters in our case are:

- $I = \frac{D^2}{16} + \frac{L^2}{12} = 4.740 \times 10^{-3}\mathrm{m}^2$ (D=0.25 m: diameter, L=0.1 m: Thickness)
- $l = 0.45\mathrm{m}$ (The pendulum frequency is 0.74 Hz.)
- $d = 0.0068\mathrm{m}$ (The pitch frequency is 0.60 Hz.)
- $K_x = \frac{a}{(s - p_1)(s - p_2)\ldots(s - p_n)}$ (Laplace transform of $\tilde{k}_x$);

  $a = 8.980 \times 10^{16}$, $p_n = -1.688 \pm 75.19j$, $-4.898 \pm 67.83j$, $-7.634 \pm 53.83j$, $-9.623 \pm 34.56j$, $10.66 \pm 11.91j$, (Chebychev 10 pole, 1 dB, 12Hz; Gain at DC is 4 for pseudo-critical damping.)
- $K_\theta = \frac{a}{(s - p_1)(s - p_2)\ldots(s - p_n)}$ (Laplace transform of $\tilde{k}_\theta$);

  $a = 1.122 \times 10^{14}$, $p_n = -1.688 \pm 75.19j$, $-4.898 \pm 67.83j$, $-7.634 \pm 53.83j$, $-9.623 \pm 34.56j$, $-10.66 \pm 11.91j$, (Chebychev 10 pole, 1 dB, 12 Hz; Gain at DC is 0.005 for pseudo-critical damping.)
- $a = 0.085\mathrm{m}$
- $\alpha = 0.093\mathrm{N/mkg}$ (F_{DC} =0.01 N (corresponding to 40 μm), 10%change per 1mm; $\frac{\left(\frac{dF_{DC}}{dx}\right)}{F_{DC}} = 10^2/\mathrm{m}$, M=10.7 kg)

1. For this document, “normalized” is defined as “divided by the mass”.

The transfer functions from θ_1 to x_2 and θ_2 were calculated using the "block diagram method" in simulink of matlab (Fig. 2)[1].

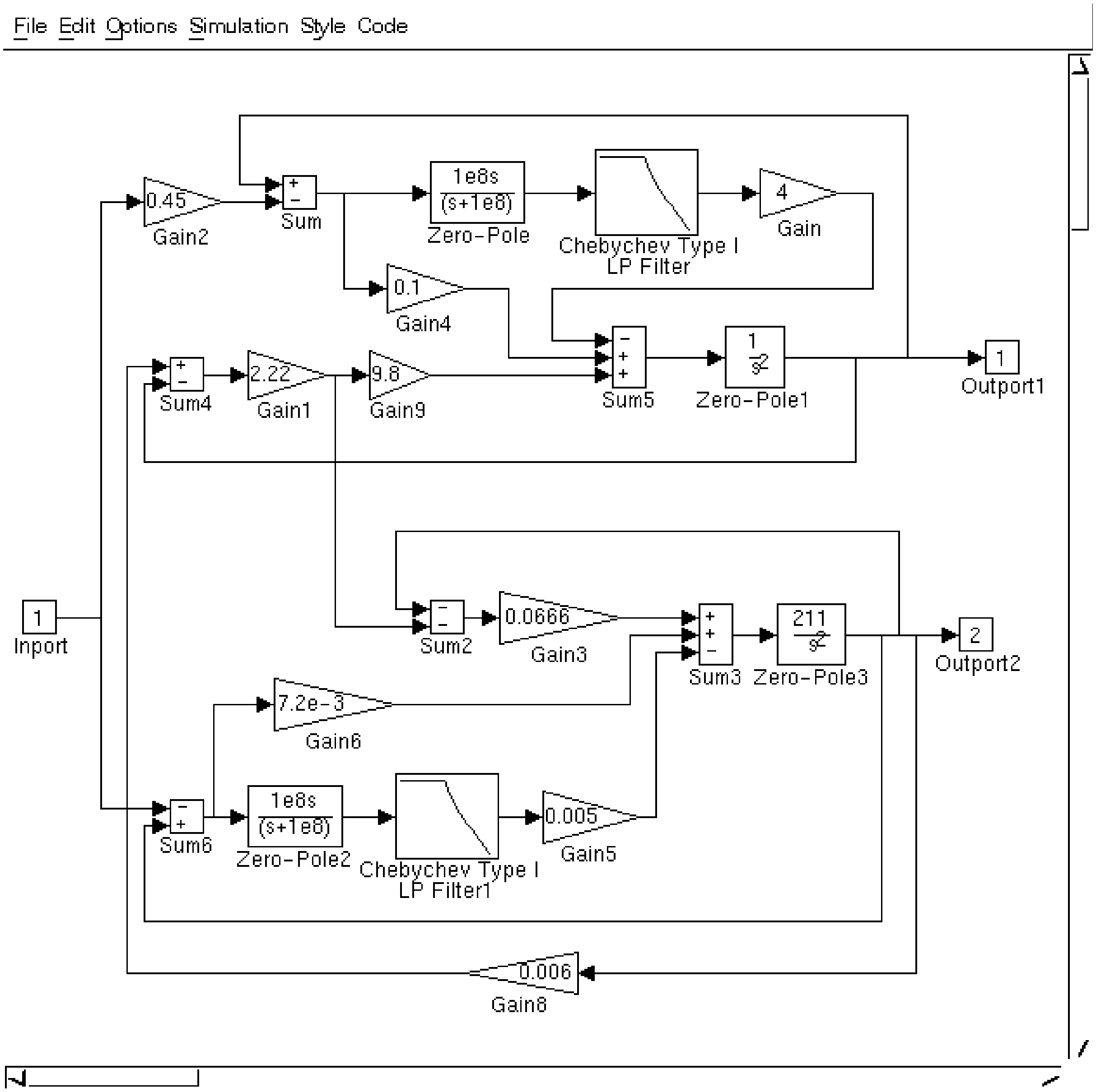

Figure 2: Block diagram representing the two equations of motion;
Inport: θ_1, Outport1: x_2, Outport2: θ_2.

1. ~seiji/DETsuspen/pendresponse/pitchto.m is the simulink file for it.

The two transfer functions obtained are shown in Fig. 3 together with the transfer functions without the DC force changing effect. Some important features of the transfer functions are:

- Around the resonance frequencies, the test mass motion is comparable with that of the sensor/actuators both in position and pitch.
- At low frequencies, both transfer functions approach asymptotes due to the DC force changing effect.
- At high frequencies, both transfer functions have slopes of f^{-2} due to the DC force changing effect.
- The transfer function from θ_1 to θ_2 at 40 Hz is $\sim 3 \times 10^{-5}$.
- The transfer function from θ_1 to x_2 at 40 Hz is $\sim 10^{-6}$.

It can be shown by the last feature[1] that in order to suppress the test mass displacement noise below $5 \times 10^{-20} \mathrm{m}/\sqrt{\mathrm{Hz}}$ at 40 Hz, the pitch motion of the stack top plate must be less than $5 \times 10^{-14} \mathrm{rad}/\sqrt{\mathrm{Hz}}$ at 40 Hz.

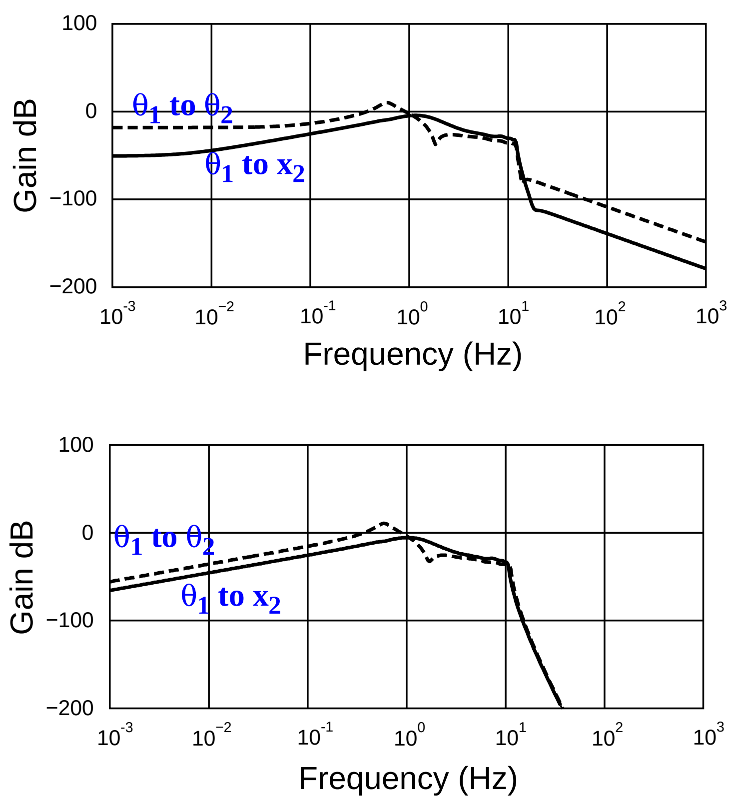

Figure 3: Transfer function from θ_1 to θ_2 and x_2 with the DC force changing effect (top) and without it (bottom).

1. The transfer function from θ_1 to θ_2 at 40 Hz of $\sim 3 \times 10^{-5}$ does not set a stringent requirement, assuming that the beam spot offset from the center of mass is 1 mm.