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- LIGO -
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Misalignment-Beam Jitter Coupling in LIGO

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ABSTRACT

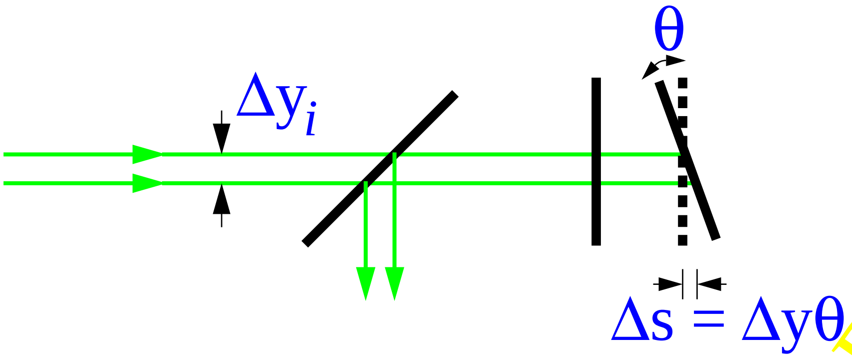
When an angular degree-of-freedom in the interferometer is misaligned, the length error signals may become first-order sensitive to the direction of the incident laser beam (in the absence of misalignment they are second-order sensitive). A model for the sensitivity of the interferometer to fluctuations in the input laser beam direction has been made. This model is described, and the results of applying the model to the initial LIGO interferometer are given. The model uses a modal expansion of the input beam and the interferometer fields, treating the input beam, fluctuating in direction at frequency *f*, as containing sidebands at frequencies $\pm f$ from the optical frequencies (carrier and sidebands), but which are spatially in an orthogonal, TEM_{10} mode.

1 INTRODUCTION

The rms misalignment tolerance of the LIGO interferometer mirrors has been calculated based on the degradation of the shot-noise limited sensitivity in the presence of misalignment [1]. Another important effect is the coupling of interferometer mirror misalignment (static or at low frequency) with fluctuations in the input beam direction (at gw frequencies). This can produce differential phase shifts in the interferometer arms, leading to a signal at the gravity wave readout. Such a mechanism leads to a combined requirement on the misalignment tolerance and the input beam direction fluctuations in the gw band.

2 OVERVIEW OF THE PHYSICAL PROCESS

The effects of laser beam direction fluctuations were first examined by the Garching group [2], which looked at a simple Michelson interferometer. In this case a simple geometrical picture can be used to see how the misalignment-‘jitter’ mechanism arises, and to calculate its effect. In the figure the two arms of the Michelson are folded along the same direction; the two arms are shown as being of unequal length, but they could be of equal length and the same picture would apply. This shows how a fluctuating beam lateral shift (Δy_i) leads to extra phase shift in the Michelson arm which is misaligned. The misalignment is static or slowly varying, while the beam shift is



fluctuating at a frequency in the gw-band, leading to a fluctuating phase shift at a gw-band frequency.

Modal expansion picture. When the optical configuration is more complicated than a simple Michelson, it is natural to perform the calculation using the modal expansion technique. Before describing the full model, it is useful to gain some insight by examining how this noise mechanism is described by the modal expansion model.

Since we are interested in how this noise mechanism produces small phase shifts (comparable to the shot noise phase sensitivity), the problem is restricted to the regime where the mirror misalignments and the beam tilts and shifts are small compared to the beam divergence angle and beam waist size; therefore expanding the fields in the fundamental and first-order Hermite-Gaussian modes is sufficient. Taking the same example of a Michelson interferometer (this time explicitly of equal arm length), the shifted input beam is described by a superposition of TEM_{00} and TEM_{10} modes; in one dimension, the input field is:

$$E_i = E_0 \begin{bmatrix} \sqrt{1 - x^2 - \alpha^2} \\ x - i\alpha \end{bmatrix}$$

This uses the same notation as Hefetz and Mavalvala [3]: E_0 is the input field; x and α are the beam shift and tilt in units of the beam radius and divergence angle, respectively. A mirror which is tilted by an angle θ with respect to the TEM_{00} mode is represented by the matrix:

$$M = \begin{bmatrix} \sqrt{1 - 4\theta^2} & -2i\theta \\ -2i\theta & \sqrt{1 - 4\theta^2} \end{bmatrix}$$

I’ll consider the above input field to be defined at the beamsplitter, and the two arm mirrors to be located also at the position of the beamsplitter; this is to simplify the example, so that the propagation from the point where shifts and tilts are defined to the arm mirrors and beamsplitter does not have to be accounted for. Consider then that one of the arm mirrors, M_1, is tilted as above, and that the other arm mirror is aligned so that $M_1 = I$. At the interferometer output there are two fields which are subtracted: $E_1 = M_1 E_i/2 = E_i/2$ and $E_2 = M_2 E_i/2$. The output field is then:

$$E_1 - E_2 = \frac{E_0}{2}\begin{bmatrix} \sqrt{1-x^2-\alpha^2}\cdot\left(1-\sqrt{1-4\theta^2}\right) + 2i\theta x + 2\theta\alpha \\ x - i\alpha + 2i\theta\sqrt{1-x^2-\alpha^2} - \sqrt{1-4\theta^2}\cdot(x-i\alpha) \end{bmatrix}$$

$$\approx E_0 \begin{bmatrix} -\left(1-\frac{x^2}{2}-\frac{\alpha^2}{2}\right)\theta^2 + i\theta x + \theta\alpha \\ i\theta\left(1-\frac{x^2}{2}-\frac{\alpha^2}{2}\right) + \theta^2(x-i\alpha) \end{bmatrix}$$

There are two factors which are linear in the mirror misalignment and beam (mis-) direction: the terms $i\theta x E_0$ and $\theta\alpha E_0$ in the TEM_{00} component. Recall that a differential phase shift in the interferometer (due to a differential mirror motion, e.g.) produces an imaginary field at the output. It is thus the imaginary component of the above field - the term proportional to $i\theta x$ - that would be interpreted as a phase shift.

So in the modal expansion picture the misalignment-jitter mechanism for this example is described by a conversion of the fluctuating input TEM_{10} mode (due to beam jitter) into a fluctuating TEM_{00} mode, via the mirror misalignment. The imaginary component of the output field is then detected as a phase shift.

3 POWER RECYCLED, FABRY-PEROT ARM MICHELSON

The misalignment-jitter calculation for the LIGO interferometer uses a one-dimensional, single higher order mode modal model of the power recycled, Fabry-Perot arm Michelson interferometer [4]. The input beam is rf-modulated with a single rf-frequency, and only the first set of rf sidebands are kept; an asymmetry in the Michelson enables detection of the differential phase shift by rf-demodulation of the dark port light power. The parameters of the interferometer used in the model are given in Table 1.

Parameter	Unit	arm (ITM)	arm (ETM)	recycl. (RM)	BS
length (common / differential)	m	4002.5		7.5 / 0.14	
power transmission	%	3	0	4	0.49995
losses	ppm	100	100	100	100
radius of curvature	m	–14540	7400	–9851	∞
modulation frequency	MHz	30.0			
modulation depth	Γ	0.5			
wave length	µm	1.064			

Table 1 Interferometer Parameters

The coordinate system is defined in the ASC DRD [5]. The common-differential basis is used here for the mirror angles. In this basis the mirror angles are related to the angles of the individual mirrors by a rotation:

$$\begin{bmatrix} \Delta\theta_{ETM} \\ \Delta\theta_{ITM} \\ \overline{\theta_{ETM}} \\ \overline{\theta_{ITM}} \\ RM \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} \theta_{ITM1} \\ \theta_{ETM1} \\ \theta_{ITM2} \\ \theta_{ETM2} \\ \theta_{RM} \end{bmatrix}$$

When the field at the input has a finite radius curvature (i.e., not at a waist) the description of a lateral shift of the beam must be modified; this is described in Appendix A.2 of reference [4]. The field description of a shifted beam now depends on the distance to the waist; the following substitution is made:

$$x \rightarrow x(1 + i(z/z_0))$$

where z_0 is the beam Rayleigh length. For the LIGO geometry, $(z/z_0) \approx -0.58$.

The modulated input beam shift and tilt are given by:

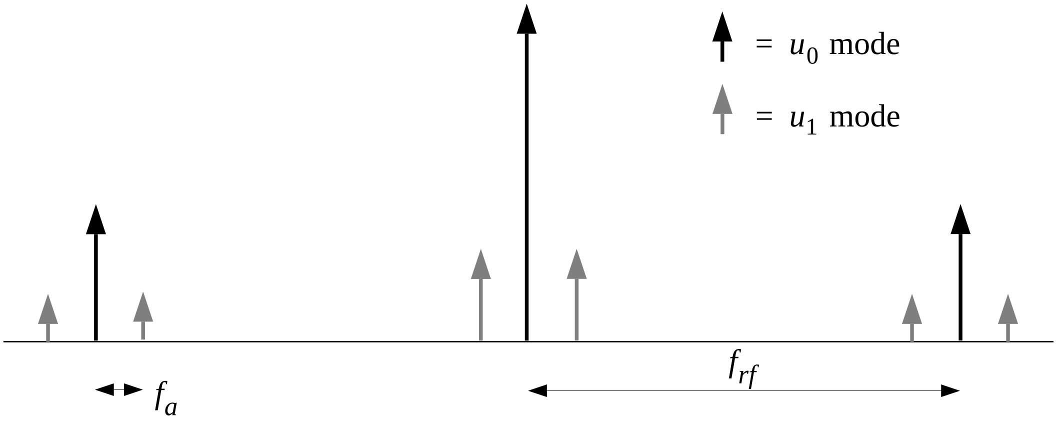
$$x(t) = x_0 \cos(\omega_a t) \quad \text{and} \quad \alpha(t) = \alpha_0 \cos(\omega_a t)$$

The displacements and angles are again in units of the beam radius and divergence angle, and ω_a is the audio frequency of the direction modulation.

The input beam (prior to rf-modulation) is then expanded into the fundamental and first-order modes, denoted by u_0 and u_1, respectively:

$$E_0(t) = \left\{\left(1 - \frac{x_0^2}{2} - \frac{\alpha_0^2}{2}\right) \cdot u_0 + (x_0(1 + i(z/z_0)) - i\alpha_0) \cdot \cos\omega_a t \cdot u_1\right\} e^{i\omega t}$$

In the frequency domain, the u_1 component is expressed as two audio sidebands at frequencies of $\pm\,\omega_a$ with respect to the u_0 component, each of amplitude $(x_0(1 + i(z/z_0)) - i\alpha_0)/2$. There are then three discrete frequencies, and the phase-modulation adds a pair of rf-sidebands to each of these, resulting in nine frequencies incident on the interferometer; this is shown graphically as:



Each of these frequency components is propagated through the interferometer, to the output and reflected ports. The total intensity at a port is then demodulated with the phase which gives maximal sensitivity to the degree-of-freedom in question. The misalignment-jitter mechanism is studied by applying either an input beam shift or tilt of a specified amplitude, along with a misalignment of one of the five angular degrees-of-freedom of the interferemeter. The transfer function is then calculated between the demodulation component at the frequency f_a and the input beam shift or tilt, over a frequency range of f_a = 1 - 1000 Hz. The demodulation output is calibrated by comparing with the output produced by a change in the degree-of-freedom in question.

3.1. GW Signal

In this case the fields at the anti-symmetric port are detected and demodulated in Q-phase. It turns out that the each of the transfer functions were essentially independent of frequency. This is understandable because the u_1 modes are not resonant in the arm cavities; the sideband u_1 modes are resonant in the recycling cavity, but the width of this cavity is much larger than 1 kHz.

The result can be written compactly as the equivalent differential displacement signal ($\delta L_- = hL$, where h is the strain sensitivity) produced when there are misalignments and input beam shifts and tilts:

$$\delta L_- = 1.95\times10^{-20}\left[\left(\frac{\Delta\theta_{\mathrm{ETM}} + 0.45\Delta\theta_{\mathrm{ITM}}}{10^{-8}\ \mathrm{rad}}\right)\left(\frac{\alpha}{10^{-8}/\sqrt{\mathrm{Hz}}}\right) + 0.16\left(\frac{\Delta\theta_{\mathrm{ETM}} + 0.5\Delta\theta_{\mathrm{ITM}}}{10^{-8}\ \mathrm{rad}}\right)\left(\frac{x}{10^{-8}/\sqrt{\mathrm{Hz}}}\right)\right]\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$$

The interferometer is mostly sensitive to tilt fluctuations of the input beam. The terms which are proportional to the other degrees-of-freedom (the recycling mirror and common mode angles) are negligible compared to the test mass differential angles.

The transfer function is shown in Figure 1 for the case of end test mass differential misalignment and input beam tilt; it is nearly independent of frequency.

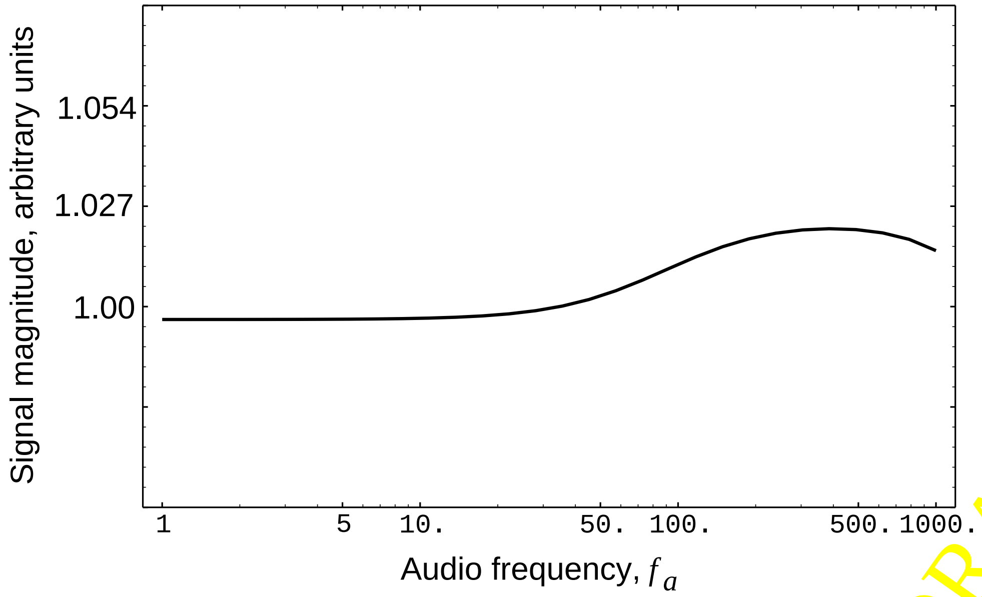


Figure 1 Frequency response of output signal to input beam tilt in the presence of a differential ETM misalignment

Beam direction fluctuation requirements for LIGO. The stability required of the beam direction is determined by requiring that the noise produced by this mechanism is equivalent to no more than 10% of the displacement sensitivity given in the Science Requirements Document initial LIGO noise curve [5] ($\delta L_{-}\ (150\ \mathrm{Hz})\ =\ 10^{-19}\mathrm{m}/\sqrt{\mathrm{Hz}}$). The misalignment will be held to a level of $\Delta\theta_{\mathrm{ETM}} + 0.45\Delta\theta_{\mathrm{ITM}} \approx 10^{-8}\ \mathrm{rad}$, according to reference [6]. Including the existence of two dimensions, and assuming that the beam fluctuations in tilt and shift are of roughly the same level, the requirements on beam direction fluctuation are:

$$x(f > 150\,\mathrm{Hz}) < 3.5\times10^{-9}\sqrt{\mathrm{Hz}}^{-1}$$
$$\alpha(f > 150\,\mathrm{Hz}) < 3.5\times10^{-9}\sqrt{\mathrm{Hz}}^{-1}$$

At 40 Hz the direction fluctuations can be 10× higher.

3.2. Laser Frequency (or common arm) signal

The error signal arising from detection of the light reflected from the interferometer, demodulated in I-phase, is used to stabilize the laser frequency. The misalignment-beam jitter coupling represents a noise source in this detection. The result from the modal model is written in terms of the equivalent frequency noise produced by the misalignment-beam jitter signal:

$$\delta\nu\ =\ 3.6\times10^{-8}\left(\frac{\theta_{\mathrm{RM}} - 0.73\,\overline{\theta_{\mathrm{ITM}}}}{10^{-8}\ \mathrm{rad}}\right)\left(\frac{\alpha + 0.39x}{10^{-7}/\sqrt{\mathrm{Hz}}}\right)\frac{\mathrm{Hz}}{\sqrt{\mathrm{Hz}}}$$

Since the frequency noise requirement for the interferometer is $\delta\nu \approx 10^{-7}\ \mathrm{Hz}/\sqrt{\mathrm{Hz}}$, if the beam jitter magnitude is held to the level given above, the effect on the frequency noise measurement (and stabilization) is negligible.

4 SINGLE FABRY-PEROT CAVITY

In this section I compare the above results with the jitter-misalignment sensitivity of a single arm cavity and I apply the analysis to the mode cleaner, where jitter-misalignment coupling presents a noise term to the frequency stabilization. I also compare single cavity results with an expression previously developed by A Abramovici.

This section to be completed.

REFERENCES

[1] D. Sigg, "Modal Model Update 2, GW-Sensitivity to Angular Misalignments," LIGO-T960114-00-D (1996).

[2] A. Rudiger, et al., "A mode selector to suppress fluctuations in laser beam geometry," *Optica Acta*, **28**, 641-658 (1981).

[3] Y. Hefetz and N. Mavalvala, "Demonstration of Automatic Alignment of a Suspended Fabry-

Perot Cavity.”
[4] Y. Hefetz, N. Mavalvala and D. Sigg, “Principles of Calculating Alignment Signals in Complex Optical Interferometers,” LIGO-T960005-00R (1996).
[5] LIGO-E950018-02-E, LIGO Science Requirements Document.
[6] LIGO-T952007-02-I, ASC Design Requirements Document.