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TRANSFER FUNCTION OF DOUBLE PENDULUM

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1 INTRODUCTION

In the process of considering the best method for connecting the suspension assembly to the optics platform, the idea of suspending the suspension assembly from the optics platform was brought up into focus. This will make the system so-called double pendulum. The first question regarding this system is whether or not the motion of the support structure should be damped with regard to the optic platform. In other words, the question is whether the suspension's damping system for the relative motion between the test mass and the support structure is effective enough to damp all the relevant mode. In this document, as the first step of investigating the question, the transfer function of the ideal double pendulum with the lower mass damped with respect to the upper mass will be analyzed.

2 TRANSFER FUNCTION

Let us think about the double pendulum system as shown in Fig. 1 The upper mass (m_1 in mass) is suspended by the wire (l_1 in length), and the lower mass (m_2 in mass) is suspended from the upper mass by the wire (l_2 in length). x_0, x_1, and x_2 denote the displacement of the suspension point, the upper mass, and the lower mass, respectively. The upper mass and the lower mass get the same amount of damping force with opposite sign, that is linear function of the relative position of the two masses. The damping force is mainly simple derivative of the relative position up to 12 Hz, then is steeply rolled off in gain for higher frequencies.

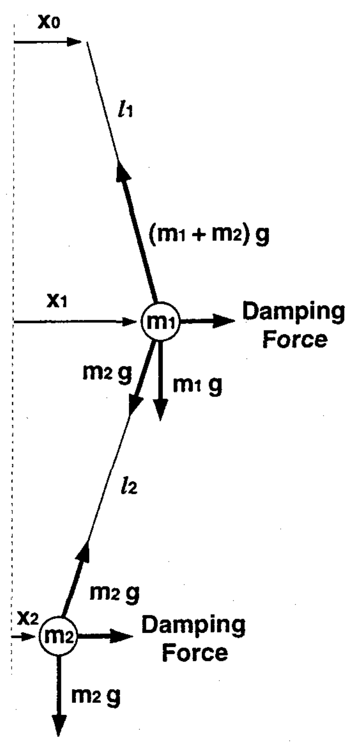

Figure 1: Ideal double pendulum system with damping force.

The equations of motion of the upper and the lower masses are, respectively:

$$-\frac{(x_1 - x_0)}{l_1}(m_1 + m_2)g - \frac{(x_1 - x_2)}{l_2}m_2 g - a\tilde{k}\left\{\frac{d}{dt}(x_1 - x_2)\right\} = m_1\frac{d^2x_1}{dt^2}$$

$$-\frac{(x_2 - x_1)}{l_2}m_2 g - a\tilde{k}\left\{\frac{d}{dt}(x_2 - x_1)\right\} = m_2\frac{d^2x_2}{dt^2},$$

where g is the acceleration of gravity, $\tilde{k}$ is the low-pass-filter operator for the damping force, and a is the coefficient of the damping force.

By taking the Laplace transform on the equations, we obtain the following equations:

$$-\frac{(X_1 - X_0)}{l_1}(m_1 + m_2)g - \frac{(X_1 - X_2)}{l_2}m_2 g - aKs(X_1 - X_2) = m_1 s^2 X_1$$

$$-\frac{(X_2 - X_1)}{l_2}m_2 g - aKs(X_2 - X_1) = m_2 s^2 X_2,$$

where $X_0(s)$, $X_1(s)$, $X_2(s)$, and $K(s)$ are the Laplace transforms of $x_0(t)$, $x_1(t)$, $x_2(t)$, and $\tilde{k}(t)$, respectively.

Instead of solving this complicated equations, we calculate the transfer function X_2/X_0 numerically using the "Block diagram method" in simulink of matlab. Fig. 2 shows the block diagram which reflects the two equations of motion described above. Inport, Outport, and Outport1 represent X_0, X_1, and X_2, respectively.

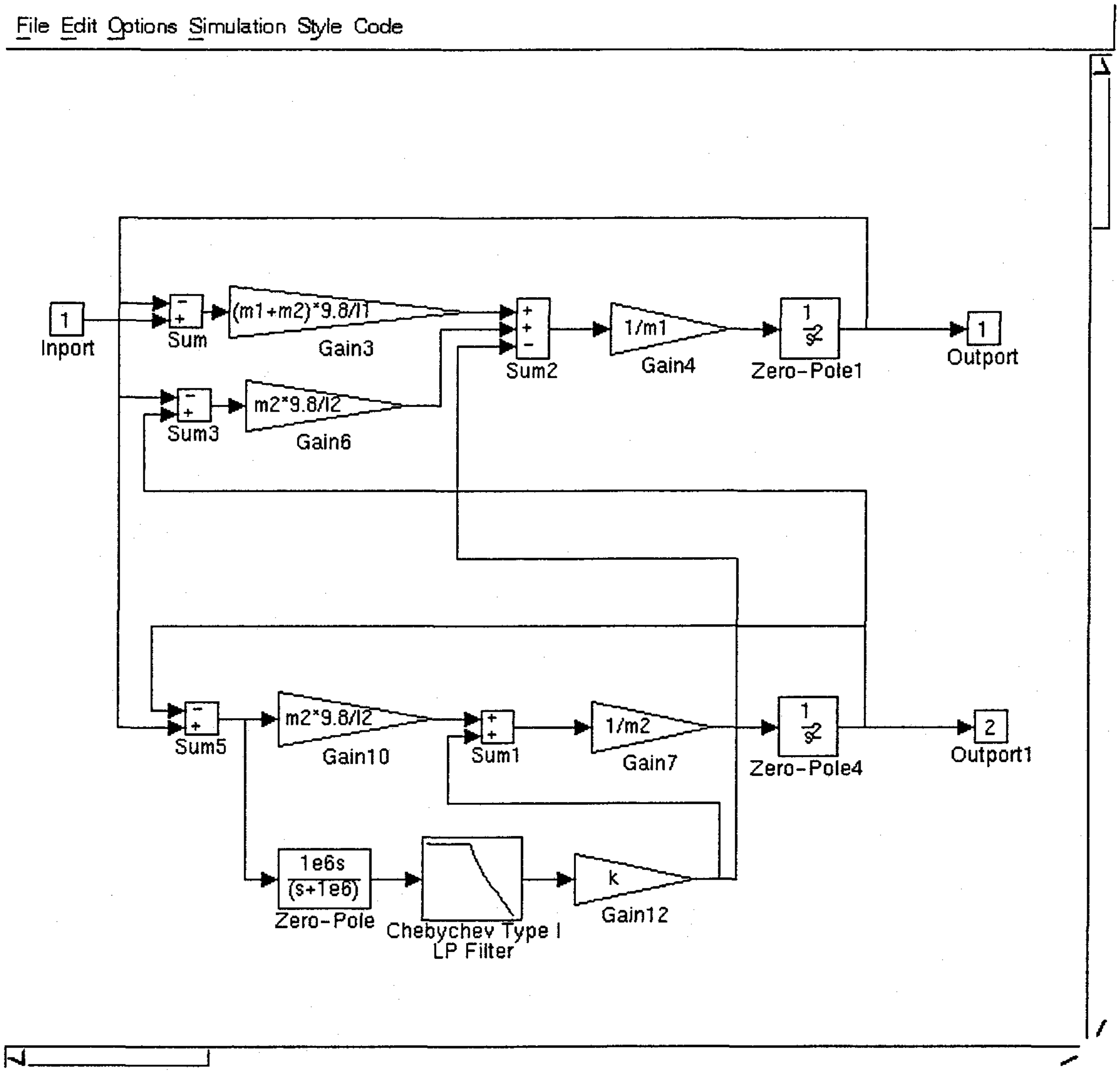

Figure 2: Block diagram reflecting the equations of motion of the double pendulum system.

The following parameters are realistically fixed:

- $l_2 = 0.45\text{m}$
- $M_2 = 10.7\text{kg}$
- K: Chebychev 10 pole, 1 dB, 12Hz

We obtained the transfer functions for the following free parameters:

- $l_1 = 0.5\text{m}$, $M_1 = 14\text{kg}$, a=30
- $l_1 = 0.2\text{m}$, $M_1 = 14\text{kg}$, a=50
- $l_1 = 0.5\text{m}$, $M_1 = 43\text{kg}$, a=30
- $l_1 = 0.2\text{m}$, $M_1 = 43\text{kg}$, a=40

The gain a is optimized for each case so that the bump in the transfer function is minimized. The transfer functions for each case are shown in Fig. 3.

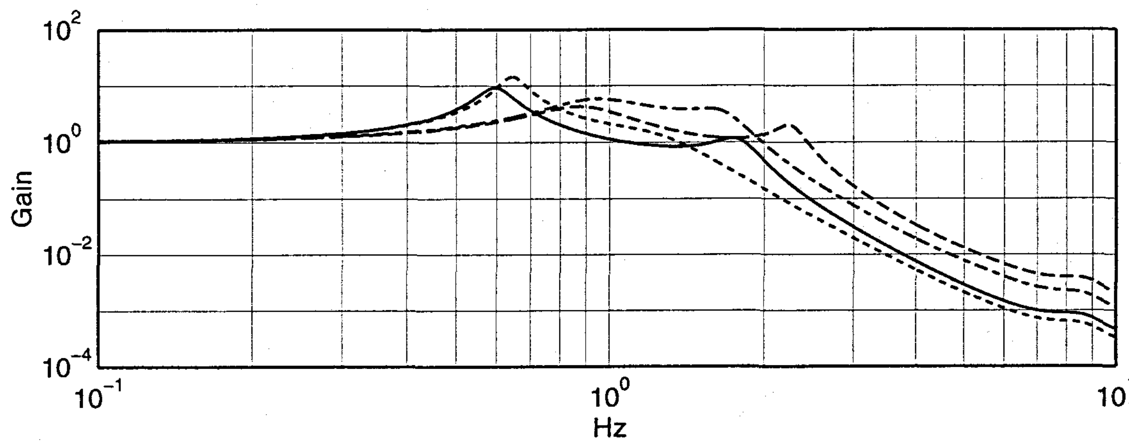

Figure 3: Transfer functions from x1 to x3 in the double pendulum system. Bold line: $l_1 = 0.5\text{m}$, $M_1 = 14\text{kg}$, a = 30, **Dashed line:** $l_1 = 0.2\text{m}$, $M_1 = 14\text{kg}$, **a = 50, Dotted Line:** $l_1 = 0.5\text{m}$, $M_1 = 43\text{kg}$, **a = 30, Long-short dotted line:** $l_1 = 0.2\text{m}$, $M_1 = 43\text{kg}$, **a = 40.**

3 CONCLUSION

In the ideal double pendulum with the damping force between the upper mass and the lower mass, the transfer function from the top suspension point to the lower mass shows only a mild bump (less than a factor of 10) around resonance frequencies with some realistic parameters of the pendulum system. This is promising enough to further analyze this system, incorporating the effect of pitch, yaw, etc.