LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Document Type	**LIGO-T960058-03 -**	**D**	17 July 97

Length Sensing & Control (LSC) Design Requirements

P. Fritschel

Distribution of this draft:

ISC Group

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

1 INTRODUCTION

1.1. Purpose

The purpose of this document is to define the content of the Length Sensing and Control (LSC) subsystem, and to give the design requirements for the LSC.

1.2. Scope

The LSC (including the CDS component) comprises the sensors and control systems for establishing and maintaining the interferometeric lengths at their operating points, and for providing a calibrated readout of the interferometer strain (gravity wave output). The subsystem includes:

- sensors and controls for maintaining the interferometer lengths during operations, including any special optics required for the sensors, mounting provisions, and any control electronics and software for this function
- sensors and controls for stabilizing the frequency of the Input Optics output light to the required level
- any hardware and/or algorithms for achieving the length operating point of the interferometer (lock acquisition)
- the signal source for any modulation frequency used to detect degrees-of-freedom (length or alignment) in the main interferometer (Core Optics degrees-of-freedom), including any phase shifters between the signal source and demodulators
- any hardware and/or algorithms for calibrating the gravity wave readout

The LSC specifically does not include:

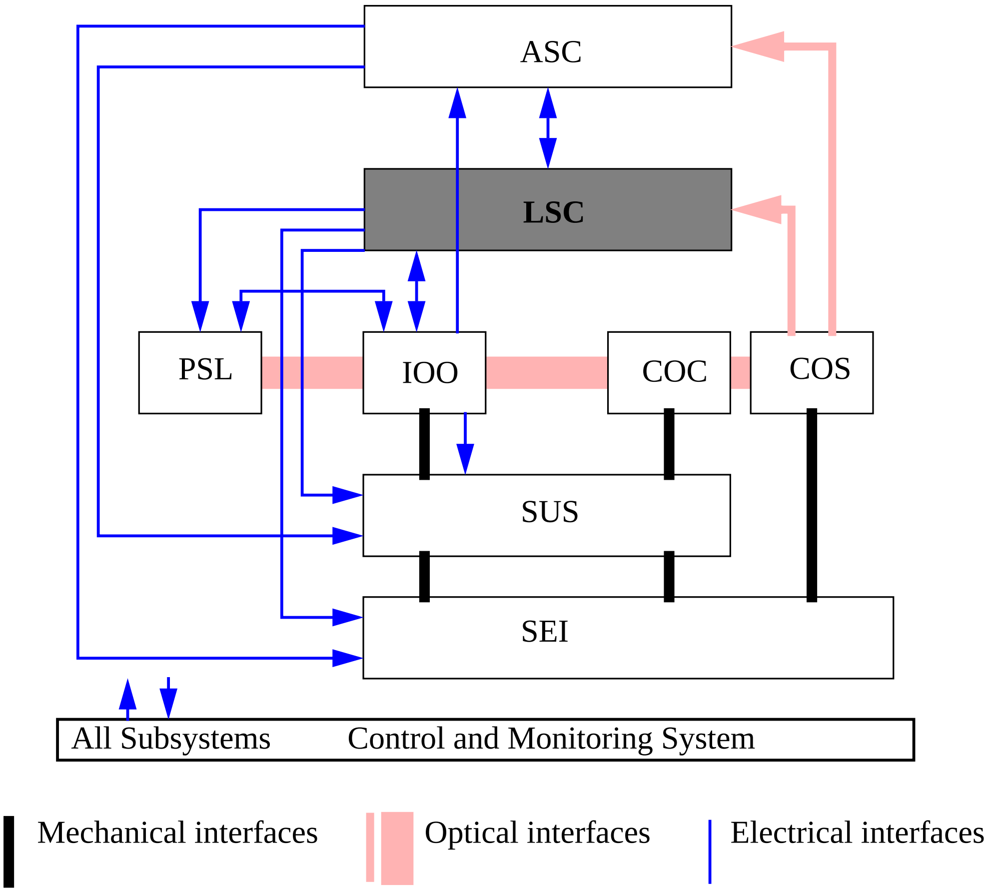
- any length or frequency actuators
- phase modulators
- signal sources for modulation frequencies used elsewhere than in the main interferometer

2 GENERAL DESCRIPTION

2.1. Specification Tree

This document is part of an overall LIGO detector requirement specification tree; refer to the SYS DRD for a diagram of the tree.

2.2. Product Perspective



2.3. Product Functions

A main function of the LSC is to bring the interferometer into resonance with the input light and maintain a degree of length and laser frequency stability that allows the interferometer to operate with a performance consistent with LIGO’s primary science requirements. The other main function is to provide a readout of the gravity wave signal, as well as the other interferometer length degrees-of-freedom. The various operating conditions of the interferometer make it useful to define two modes of operation for the LSC. These modes and the functions that the LSC provides in each are described below.

2.3.1. Acquisition Mode

Acquisition Mode refers to the state in which the interferometer lengths are brought into resonance from their initial uncontrolled values. The PSL and IOO subsystems are assumed to be fully operational in this mode. The primary function of the LSC in this mode is to lock the interferometer. After lock, a settling period is required. Wire and mirror resonances are permitted to settle down (or are actively damped), filters allowed to equilibrate, control ranges are adjusted, and self-tests are completed to verify that residual excitations do not exceed Detection mode limits.

2.3.2. Detection Mode

In this mode the interferometer lengths are maintained at a level of stability which allows detection of strain signals within the LIGO sensitivity specifications. The functions in this mode are:

- sense and control the four interferometer lengths and the input light frequency
- provide a measure of the residual deviations of the four lengths and the light frequency
- provide a calibrated readout of the interferometer strain

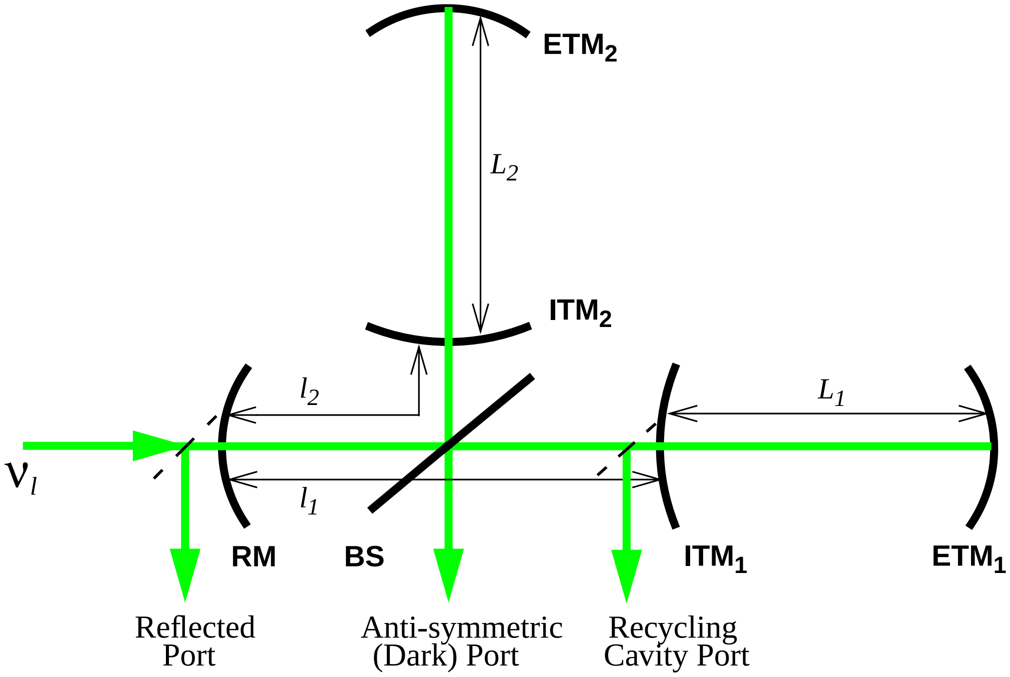
2.3.3. Diagnostic/Calibration Mode

This is a mode (in fact a set of modes) that may be accessed from the preceding modes. The functions of this mode are:

- provide diagnostic capability to determine the performance of the LSC
- enable implementation of calibration procedures within the LSC (e.g., determination of the sensor sensitivities)
- support diagnosis of other subsystems

2.4. Definition of Length Degrees-of-freedom

The four independent lengths in the interferometer are defined in Figure 1; for reference, the Michelson asymmetry and the interferometer output ports are also defined. The ‘δ’ symbol is prepended to a length symbol to indicate a deviation of that length from the nominal operating point.



Name	*Symbol*	*Definition*	*Deviation Symbol*
Differential Arm Length	L_-	$L_1 - L_2$	δL_-
Common Arm Length	L_+	$L_1 + L_2$	δL_+
Michelson Length	l_-	$l_1 - l_2$	δl_-
Recycling Cavity Length	l_+	$l_1 + l_2$	δl_+
Michelson Asymmetry	Δl	$(l_1 - l_2)/2$	N.A.

Figure 1 Definitions of length degrees-of-freedom and output ports. ‘ITM’ stands for input test mass, ‘ETM’ for end test mass, ‘BS’ for beamsplitter, and ‘RM’ for recycling mirror. The frequency, wavelength and propagation constant of the laser light is denoted by ν_l , λ_l , and k_l respectively.

2.5. Assumptions and Dependencies

2.5.1. LIGO SRD requirements

The following requirements on the LIGO detector sensitivity and availability, as given in the Science Requirements Document, directly influence some of the requirements for the LSC described in section **3**.

1. Sensitivity: The initial LIGO displacement sensitivity requirement is given in the SRD. The displacement requirement, **x**, is defined such that the strain sensitivity is $h(f) = \mathbf{x}(f)/ L$, where L = 4 km; i.e., $\mathbf{x}(f)$ is the differential arm length sensitivity.
2. Availability goals: 90% for single interferometer operations; 85% for double coincidence; 75% for triple coincidence.

2.5.2. Detector Noise Budget Allocation

Each noise mechanism originating within the LSC or through an interaction between the LSC and another Detector subsystem(s) is to be controlled so that the equivalent displacement sensitivity as given by the SRD displacement curve is degraded by no more than 0.5% in the gravity wave band of 40 Hz - 10 kHz. The specific noise mechanisms which fall in this category, preceded by the subsystem(s) which contribute, are:

1. LSC. Residual deviations in the anti-symmetric port degree-of-freedom, coupling with the (LSC) oscillator amplitude noise.
2. LSC. Residual deviations in the arm cavity common mode length, producing a power drop in the arm cavities.
3. LSC. Residual deviations in the recycling cavity length, producing a power drop in the interferometer.
4. LSC. Electronics noise in the GW readout channel.
5. LSC. Sensing and control noise in the auxiliary length degrees-of-freedom.
6. LSC & COC. Residual frequency noise (LSC), coupling with imperfections in the COC optics (specifically, amplitude and storage time unbalance between the arm cavities).
7. LSC & COC/Beam Tubes. Scattered light from the beam tubes, interacting with spatial nonuniformity in the LSC detector to produce noise.
8. LSC & COS. Back-scattered light from the LSC photodetector(s), propagating back through the interferometer to the anti-symmetric port (COS) to produce noise.
9. LSC & PSL. GW-band intensity noise in the interferometer input beam (PSL), coupling with the residual deviation in the anti-symmetric port degree-of-freedom (LSC).
10. LSC & IOO. Oscillator phase noise (LSC) coupling with an offset of the sideband frequency from the mode cleaner resonance (IOO) to produce sideband amplitude noise at the mode cleaner output.

2.5.3. Detector Subsystem Parameters

In determining the design requirements in section **3**, the following assumptions concerning the performance of the other Detector subsystems have been made. A change in any of these assumptions may necessitate a change in the LSC requirements.

2.5.3.1 PSL performance

- Fractional power fluctuations in the GW-band at the interferometer input are to be controlled to level less than or equal to *twice* the curve given in Figure 3 of this document.

2.5.3.2 COC parameters

- The maximum difference in the round trip loss between the two arm cavities is assumed to be 75 ppm..
- The maximum total power at the anti-symmetric port during normal operation (locked and aligned interferometer) is assumed to be 600 mW, for 6 W total input power.
- Storage time unbalance between the arms. The storage time difference can be as large as 5%, and still not affect the frequency noise requirement, which will remain dominated by the round trip loss difference quoted above.

2.5.3.3 Mode Cleaner – match of free-spectral-range to modulation frequency

We assume that the RF modulation frequency is within 100 Hz of the center of the mode cleaner resonance through which it passes. This involves initial setup of the mode cleaner length and the oscillator frequency, and also the long term stability of the length of the mode cleaner. Specifically, if there is no post-setup control of the oscillator frequency, the length of the mode cleaner must be stable to ~50 µm.

3 PERFORMANCE REQUIREMENTS

The LSC subsystem derives its requirements from the top-level LIGO requirements for sensitivity and availability, and from secondary requirements imposed by interactions with other IFO subsystems and the LIGO facilities. These secondary requirements are defined and allocated in conjunction with Detector Systems Engineering. The performance requirements for the LSC are conveniently grouped into requirements for each mode of the LSC, as described in section **2.3.**

3.1. Acquisition Mode

The time span of the Acquisition Mode must be compatible with the LIGO availability requirement. The acquisition process is best considered as a combined process of the LSC and the ASC to bring the interferometer to the proper operating point. We require that the time duration of the acquisition process not significantly impact the Detector availability.

3.2. Detection Mode

3.2.1. Residual Length Deviations

In Detection Mode, the four lengths of the interferometer must be controlled so that their residual (rms) deviations from the operating point are no greater than the levels given in Table 1. The foundations of these requirements are discussed in Appendix 4.

Degree-of-freedom	*Residual Deviation Requirement, rms*	*Mechanism*
Differential	$\left(\delta L_- + \frac{\pi}{2F}\delta l_-\right) \le 1\times10^{-13}$ m	Amplitude Noise Coupling
Common Arm Phase	$\delta(k_l \cdot L_+) \le 9\times10^{-6}$ radian	Arm Cavity Power Drop
Michelson Length	$\left(\delta l_- + \frac{\pi}{2F}\delta L_-\right) \le 1\times10^{-9}$ m	Amplitude Noise Coupling
Recycling Cavity Length	$\delta l_+ \le 1.25\times10^{-10}$ m	Recycling & Arm Cavity Power Drop

Table 1 Requirements on the residual deviations of the four LSC degrees-of-freedom.

3.2.2. Frequency Stability

The requirement on the gravity-wave-band frequency fluctuations of the light incident on the interferometer is shown in Figure 2. The derivation of this requirement is given in Appendix 5. This level of frequency noise would produce a noise level in the gravity wave readout that is 10% in amplitude of the noise corresponding to the initial LIGO SRD sensitivity, in the presence of a difference in the arm cavity resonant-reflectivities of 0.5 % (corresponding to a round-trip loss difference of 75 ppm).

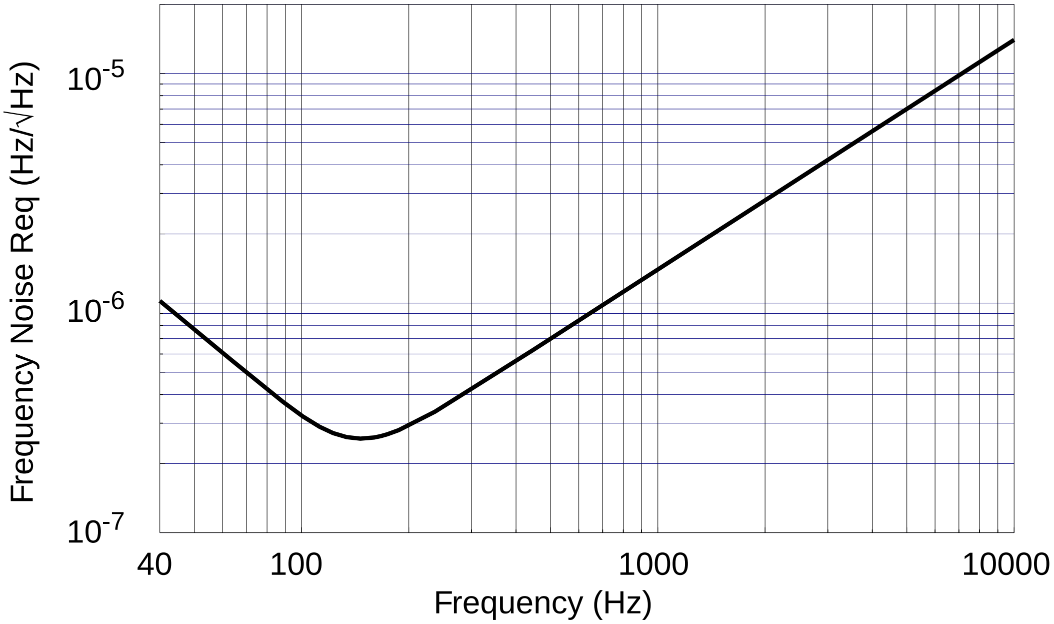


Figure 2 Frequency noise requirement on the light incident on the recycling mirror, derived assuming a 0.5% difference in the resonant-reflectivities of the two arm cavities. See also Appendix 5**.**

3.2.3. Modulation Source

The requirements for the amplitude and phase noise of the modulation source used for the gravity wave readout phase modulation are given in Figure 3. The foundations of these requirements are discussed in Appendix 5. The oscillator amplitude noise requirement arises from a coupling with the differential length deviation, and the phase noise requirement arises from a coupling with an offset in the modulation frequency (with respect to the resonant frequency in the mode cleaner). The amplitude and phase noise levels given in Figure 3 each produce a noise level in the gravity wave readout that is 10% in amplitude of the noise corresponding to the initial LIGO SRD sensitivity.

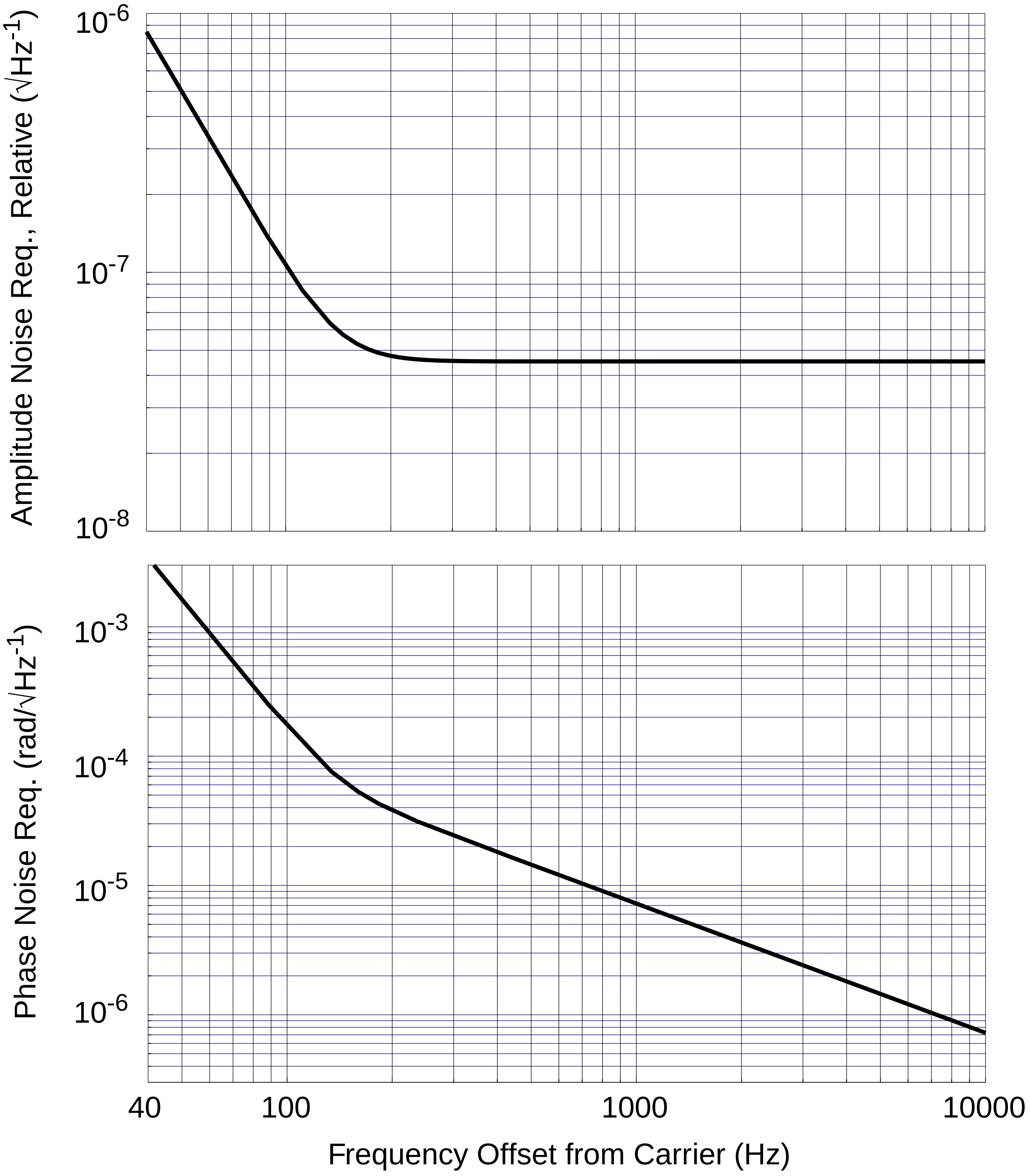


Figure 3 Amplitude and phase noise requirements on the RF phase modulation source.

3.2.4. Sensing & Control System Noise – Gravity Wave Readout

The LSC electronics noise in the gravity wave readout channel must be no greater than 10% of the signal corresponding to the LIGO SRD sensitivity, **x**(*f*), in the gravity wave band of 40 Hz - 10 kHz. As one component of this loop, the suspension driver, is not part of LSC, the noise contribution from the driver (part of SUS) is considered separately. For purposes of assessing the LSC performance against this requirement, the suspension driver is considered to be noise-less. Note that this requirement encompases electronics noise in the gravity wave photodetector, in addition to all LSC electronics from the mixer onwards.

3.2.5. Sensing & Control System Noise – Auxiliary Degrees-of-freedom

The total noise produced in the grravity wave readout by the LSC sensing and control system for the auxiliary length degrees-of-freedom must be no greater than 10% of the signal corresponding to the LIGO SRD sensitivity, $\mathbf{x}(f)$. No further restriction is made on the sources making up this noise contribution (whether they arise from shot noise, electronics noise, etc.).

3.2.6. Globally Uncontrolled Degrees-of-freedom

Since there are six optics in the interferometer, and only four interferometrically sensed lengths, there are two position degrees-of-freedom that must be only locally controlled (four for the 2 km interferometer). To prevent upconversion of signals produced by spurious light paths into the gravity wave band, we require that the maximum velocity of locally controlled degrees-of-freedom, with respect to the input laser table, must be held to < 5 λ/second.

3.2.7. Photodetector Performance

Several requirements apply only to the ‘gravity wave’ photodetector – i.e., the detector at the anti-symmetric port.

3.2.7.1 Quantum Efficiency

The quantum efficiency η_{pd} of the gravity wave photodetector must be greater than 80%. The quantum efficiency is defined as $\eta_{\mathrm{pd}} = (hc/e\lambda_l)R$, where R is the ratio of the total average photocurrent in the detector to the average laser power exiting the COS delivery optics at the anti-symmetric port, where the averaging is over a time > 1 sec, and ‘detector’ refers to the detector assembly (not an individual photodiode).

3.2.7.2 Electronics Noise

The requirement on the electronics noise of the gravity wave photodetector is covered by the more global gravity wave readout electronics noise requirement given in 3.2.4.

3.2.7.3 Spatial Uniformity

The spatial uniformity of the photodetector reponse at the RF modulation frequency must be less than 2% rms over spatial scales of order the beam radius (TBR).

3.2.7.4 Optical Power Handling

The above three requirements must all be met in the presence of an average optical power incident on the gravity wave photodetector assembly of at most 600 mW.

3.2.7.5 Transient Protection

Adequate protection must be provided against damage from expected loss-of-lock power transients. Recovery time from such a transient must not significantly impact the availability requirement. The first three performance requirements above do not apply during such transients.

3.2.7.6 Scattered Light

The back-scattered light from the LSC photodetectors into the interferometer must not overly degrade the sensitivity. Specifically, the phase noise produced by this scattered light can be no more than 10% in amplitude of the LIGO SRD phase sensitivity. Several factors involved in the generation of scattered light phase noise are under control (to some extent) of the LSC: the scattering properties of the detector surfaces; the beam size at the detector scattering surfaces; the motion of the detector scattering surfaces; the number of detectors. The following factor, which is linear in the equivalent phase noise, involves only LSC parameters:

$$x_i^{sc}(f) \cdot m_i \cdot \sqrt{\mathrm{BRDF}_i(\theta)}$$

where for the *i*th detector, $x^{sc}{}_i$ is the motion of the scattering surface(s), m_i is the ratio of the beam waist in the interferometer to the beam size at the scattering surface(s), and $\mathrm{BRDF}_i(\theta)$ is the bidirectional reflectance distribution function of the scattering surface(s) at the beam angle of incidence θ. Exactly how this factor figures in the generation of phase noise is discussed in Appendix 7. For multiple LSC detectors, the root-square-sum of the above factor over all detectors is the relevant figure-of-merit; it must meet the requirement shown in Figure 4.

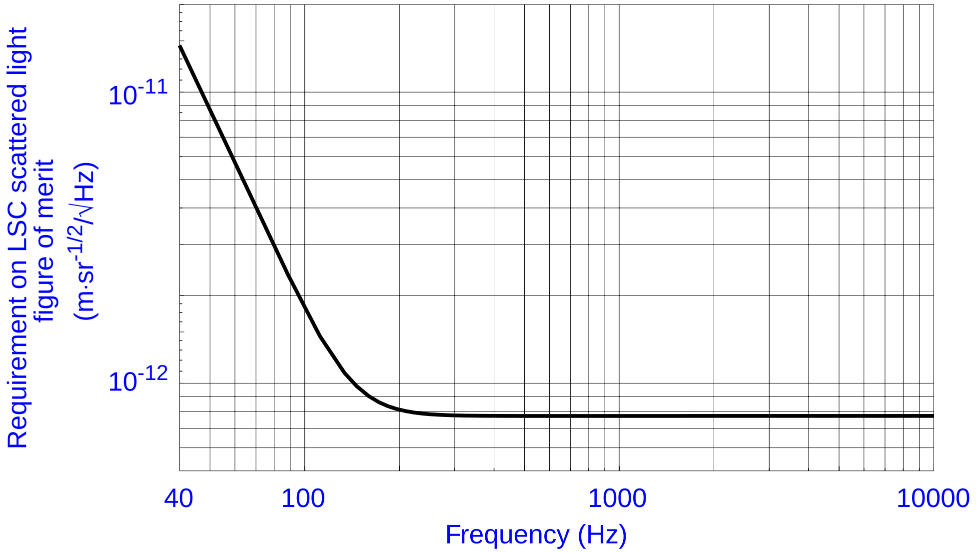


Figure 4 Requirement on the scattered light factor $\left[\sum_i \left(x_i^{sc}\cdot m_i\cdot\sqrt{\mathrm{BRDF}_i(\theta)}\right)^2\right]^{1/2}$.

The requirement is shown assuming that $A = 1$ (see Appendix 7 for the definition of A). An attenuation factor (A less-than-unity) for a particular path can be included by multiplying the appropriate scattered light factor for that path by A before comparing to the requirement curve in the plot.

3.2.8. Calibration

The following requirements apply to the accuracy of the calibration of the gravity wave readout channel, in the band of 40 Hz – 10 kHz:

- Amplitude: better than ± 5%
- Timing: better than ± 50 microseconds
- Stability: the long term stability of the calibration must be sufficient that invasive re-calibration procedures do not significantly impact the interferometer availability requirement

The readout of the three auxiliary lengths must be accurate to within ± 10% in amplitude and ± 100 microseconds in timing (in the 40Hz – 10 kHz band).

3.2.9. Commissioning & Diagnostic Requirements

The LSC must be able to perform diagnostics to determine the proper functioning of the LSC in the Detection Mode, and to support the operation of the interferometer in a subset of alternate optical configurations (single cavity, etc.). The following functions must be provided for:

1. Determination of closed loop transfer functions of the control loops.
2. Determination of offsets in the lock-points.
3. Determination of gw-detection band noise produced by the LSC.

4. Monitoring of feedback forces applied to the controlled optics.

The LSC must also be able to support more global diagnostics involving other Detector subsystems.

3.2.10. Induced Environment

The LSC subsystem shall comply with the guidelines and procedures laid out in LIGO-E960036-02-E, **LIGO EMI CONTROL PLAN AND PROCEDURES**.

4 DESIGN AND CONSTRUCTION

4.1. Materials and Processes

Mounting surfaces will be designed to make a well-defined plane of contact. Kinematic mounts will be used where possible.

Any in-vacuum components must be prepared with only approved vacuum-compatible materials and manufactured, cleaned, and handled according to procedures approved for in-vacuum equipment.

4.2. Component Naming

All components shall be identified using the LIGO Detector Naming Convention (LIGO-T950111-01-E). This shall include identification physically on components, in all drawings and in all related documentation.

4.3. Transportability

All items shall be transportable by commercial carrier without degradation in performance. As necessary, provisions shall be made for measuring and controlling environmental conditions (temperature and accelerations) during transport and handling. Special shipping containers, shipping and handling mechanical restraints, and shock isolation shall be utilized to prevent damage. All containers shall be movable for forklift. All items over 100 lbs. which must be moved into place within LIGO buildings shall have appropriate lifting eyes and mechanical strength to be lifted by cranes.

4.4. Safety

This item shall meet all applicable NSF and other Federal safety regulations, plus those applicable State, Local and LIGO safety requirements. A hazard/risk analysis shall be conducted in accordance with guidelines set forth in the LIGO Project System Safety Management Plan LIGO-M950046-F, section 3.3.2.

4.5. Preparation for Delivery

Equipment shall be appropriately prepared. For example, vacuum components shall be prepared to prevent contamination.

Procedures for packaging shall ensure cleaning, drying, and preservation methods adequate to prevent deterioration, appropriate protective wrapping, adequate package cushioning, and proper containers. Proper protection shall be provided for shipping loads and environmental stress during transportation, hauling and storage.

Appropriate identification of the product, both on packages and shipping containers; all markings necessary for delivery and for storage, if applicable; all markings required by regulations, statutes, and common carriers; and all markings necessary for safety and safe delivery shall be provided.

4.6. Documentation

4.6.1. Design Documents

Design documents will be provided throughout the design phases. The Final Design Document, issued at the Final Design Review, will include the complete design of the LSC and will supercede all previous design documents.

4.6.2. Engineering Drawings and Associated Lists.

4.6.2.1 Procedures

Procedures shall be provided for, at a minimum:

- Initial installation and setup of equipment
- Normal operation of equipment
- Normal and/or preventative maintenance
- Troubleshooting guide for any anticipated potential malfunctions

4.6.2.2 Manuals (TBD)

4.6.3. Documentation Numbering

All documents shall be numbered and identified in accordance with the LIGO documentation control numbering system LIGO document TBD.

4.6.4. Test Plans and Procedures

All test plans and procedures shall be developed in accordance with the LIGO Test Plan Guidelines, LIGO document TBD.

4.7. Logistics

The design shall include a list of all recommended spare parts and special test equipment required.

5 QUALITY ASSURANCE PROVISIONS

5.1. Special Tests

5.1.1. Engineering Tests

A prototype unit shall be tested to meet specifications, and to determine the extent to which it exceeds specifications, before the Final Design Review.

5.1.2. Reliability Testing

Reliability evaluation/development tests shall be conducted on items with limited reliability history that will have a significant impact upon the operational availability of the system.

5.2. Configuration Management

Configuration control of specifications and designs shall be in accordance with the LIGO Detector Implementation Plan.

Appendix 1 Definitions

$x(f)$ Curve of initial LIGO differential displacement sensitivity, as given in the Science Requirements Document

F finesse of the arm cavities ($F = 210$)

Appendix 2 Acronyms and Abbreviations

ASC	Alignment Sensing and Control
CDS	Control and Data System
COC	Core Optics Components
COS	Core Optics Support
ETM	End Test Mass
IOO	Input/Output Optics
ITM	Input Test Mass
LSC	Length Sensing and Control
PSL	Prestabilized Laser
RM	Recycling mirror

SRD Science Requirements Document

SUS Suspension System

SYS Detector Systems Engineering

TBR To Be Reviewed

Appendix 3 Applicable Documents

LIGO Documents

LIGO-E950018-02-E LIGO Science Requirements Document

LIGO-E960112-05-D Detector Subsystems Requirements

LIGO-T970098-00-D Proposed interferometers parameters

LIGO-T970084-00-D Frequency Response of the LIGO Interferometer

LIGO-T970071-00-D Core Optics Support Design Requirements Document

LIGO-G970067-00-D Viewgraphs - Core Optics Support Design Requirements Review

LIGO-T970101-A-D Strain Calibration in LIGO

LIGO-T960067-00-D Length Control RMS Deviations from Resonance

LIGO-E960036-02-E LIGO EMI Control Plan and Procedures

LIGO-T950111-01-E LIGO Naming Conventions

Non-LIGO Documents

None.

APPENDIX 4 RESIDUAL DEVIATION REQUIREMENTS

Differential degree-of-freedom, anti-symmetric port.

The requirement on this degree-of-freedom is determined by coupling with laser and RF-oscillator amplitude noise, producing a GW-band noise term. The signals produced at the anti-symmetric port by input light and oscillator amplitude fluctuations and by differential arm length fluctuations are given in **LIGO-T970084-00-D**. The ratio of the laser power signal, $S_{\delta P}$, to the length signal, $S_{\delta L-}$, and of the oscillator amplitude signal, $S_{\delta\Gamma}$, to the length signal, as a function of the audio frequency f_a is:

$$\frac{S_{\delta P}}{S_{\delta L-}} = \left[1+\frac{1}{(1+s_{cc})(1+s_c)}\right]\left(\frac{\delta P}{2P}\right)(\delta L_- + \pi\delta l_-/2F)_{\mathrm{rms}} \cdot \left(\frac{1+s_c}{\delta L_-(f)}\right)$$

$$\frac{S_{\delta\Gamma}}{S_{\delta L-}} = \left(\frac{\delta\Gamma}{\Gamma}\right)(\delta L_- + \pi\delta l_-/2F)_{\mathrm{rms}} \cdot \left(\frac{1+s_c}{\delta L_-(f)}\right)$$

where $s_x = i\, f_a/f_x$ (f_c is the arm cavity pole frequency and f_{cc} is the coupled cavity pole frequency); $\delta\Gamma/\Gamma$[1] and $\delta P/P$ are the relative fluctuations in the RF-oscillator amplitude and input laser power; $(\delta L_- + \pi\delta l_-/2F)_{\mathrm{rms}}$ is the rms deviation in this degree-of-freedom; $\delta L_-(f)$ is the displacement noise (neglecting the l_- contribution).

For each case the requirement is that the ratio be less than 0.1 when $\delta L_- = \mathbf{x}(f)$. The oscillator amplitude noise coupling involves a trade-off that is completely within the LSC. We choose to enforce $(\delta L_- + \pi\delta l_-/2F)_{\mathrm{rms}} \le 10^{-13}\ \mathrm{m}$, which implies the requirement $\delta\Gamma/\Gamma \le 4.5\times10^{-8}$ above 200 Hz.

The requirement on the input power fluctuations is 2× looser than the oscillator amplitude noise requirement (i.e., the fractional power fluctuations can be up to twice as large as the fractional oscillator amplitude fluctuations).

At this level of residual deviation, other effects are negligible. If all the deviation is due to cavity arm length deviation, the power drop in the arm cavities due to being off-resonance is much less than 1%. The phase difference between the arms due to this level of length difference is 1.5×10^{-4} rad; the fractional carrier power at the anti-symmetric port due to this offset is thus $(\phi_d/2)^2 = 2.5\times10^{-8}$, which is negigible compared to the static contrast defect.

Differential degree-of-freedom, recycling cavity or reflected port.

This signal is proportional to $\delta l_- + \pi\delta L_-/2F$. Any δl_- will be corrected in the anti-symmetric signal loop by a corresponding $\delta L_- = -\delta l_-(\pi/2F)$, and this can be done up to $\delta L_- = 1.5\times10^{-10}\ \mathrm{m}$, at which point the power in the cavities drops below 99.5% of the maximum. Thus we could allow $\delta l_- \le 2\times10^{-8}\ \mathrm{m}$. But then amplitude noise would couple to this sensor at the 10^{-15} m/√Hz level. This is about 5× higher than the shot noise sensitivity of this sensor, when taken from the recycling cavity pick-off (the ITM ghost beam). In order not to spoil the shot noise limited sensitivity of this signal, we tighten the requirement to: $(\delta l_- + \pi\delta L_-/2F)_{\mathrm{rms}} \le 10^{-9}\ \mathrm{m}$.

---

[1] Strictly speaking, this should be the fractional change in the gain factor $J_0(\Gamma)J_1(\Gamma)$, which for small modulation index reduces to the fractional change in Γ. The nominal modulation index is estimated to be $\Gamma = 0.45$, at which point the fractional change in $J_0(\Gamma)J_1(\Gamma)$ is about 15% smaller than the fractional change in Γ. We neglect this difference.

Common degrees-of-freedom.

The deviations in the two common mode degrees-of-freedom are both determined by the allowed reduction in power build-up as cavities are detuned from resonance. Considering first the arm cavities, the carrier power in the arms drops to 99% of maximum for a common length offset of $\delta L_{+} = 1.5\times10^{-12}\ \mathrm{m}$. Since both the laser frequency and the common arm length are sensed and controlled, we express the requirement as applying to the common mode phase, $\Phi_{+} = kL_{+}$. In this case, we require: $\delta\Phi_{+} \le 9\times10^{-6}\ \mathrm{rad}$.

Deviations in the recycling cavity length affect the build-up of the carrier and the sidebands. Using the criterion that the S/N ratio is degraded by no more than 0.5% leads to the requirement: $\delta l_{+} \le 1.25\times10^{-10}\ \mathrm{m}$.

For the common degrees-of-freedom, other effects such as amplitude noise coupling are negligible at these levels. The detailed expressions used to derive the numbers for the common degrees-of-freedom can be found in **LIGO-T960067-00-D.**

APPENDIX 5 FREQUENCY STABILITY REQUIREMENT

The signals produced at the anti-symmetric port by input light frequency fluctuations and by differential arm length fluctuations are given in **LIGO-T970084-00-D**. The ratio of the frequency signal, $S_{\delta\nu}$, to the length signal, $S_{\delta L-}$, as a function of the audio frequency f_a is:

$$\frac{S_{\delta\nu}}{S_{\delta L-}} = \left(\frac{1+s_c}{1+s_{cc}}\right)\left(\frac{\delta\nu}{2r_c' k\,\delta L_{-}}\right)\left[\frac{8\pi\Delta l}{c}(r_c+s_c) + \frac{\Delta\tau}{\tau}\left(\frac{1-r_c}{1+s_c}\right)\frac{1}{f_c} + \frac{\delta r_c}{f_{cc}}\right]$$

where $s_x = i\, f_a/f_x$ (f_c is the arm cavity pole frequency and f_{cc} is the coupled cavity pole frequency); $\delta\nu$ is the magnitude of the frequency fluctuation; r_c and r_c' are the arm cavity field reflectivity and its derivative w.r.t. the round trip phase; k is the light wavenumber; $\Delta\tau/\tau$ is the fractional storage time unbalance of the arm cavities; Δl is the Michelson asymmetry; δr_c is the field reflectivity difference between the arm cavities (on resonance).

We require that $S_{\delta\nu}/S_{\delta L-} < 0.1$ when $\delta L_{-} = \mathbf{x}(f)$. Using the interferometer parameters given in **LIGO-T970098-00-D**, we then get a requirement on δf. For the storage time unbalance, we use $\Delta\tau/\tau = 1\%$; for the reflectivity difference, we use $\delta r_c = 0.5\%$ (this corresponds to 75 ppm round trip loss difference between the arms). The signs of these two terms are set to add coherently, but in fact the factor involving the reflectivity difference dominates completely (the Michelson asymmetry term is also negligible). We then arrive at the δf requirement shown in Figure 2.

APPENDIX 6 OSCILLATOR NOISE REQUIREMENTS

The amplitude noise requirement on the RF oscillator used for the gravity-wave-detecting modulation and demodulation is a trade-off with the residual deviation in the anti-symmetric port degree-of-freedom; this is explained in Appendix 4 above.

The phase noise requirement derives from noise coupling when the RF sidebands pass through the mode cleaner. The most critical effect occurs when the sideband frequency is not exactly equal to an integral number of the mode cleaner free-spectral-range. Changes in the modulation frequency then result in changes in the sidband field amplitude at the mode cleaner output. Thus phase noise of the oscillator couples with the frequency offset to produce sideband amplitude noise, which couples to the GW-signal in the same way as the oscillator amplitude noise.

We begin with the derivative of the magnitude of the mode cleaner field transmission coefficient with respect to the round trip phase shift ϕ, expanded to first order in the offset ϕ_0:

$$\frac{d}{d\phi}\left|t_{mc}(\phi)\right| = \frac{R}{T^2}\cdot\phi_0$$

where $\phi = 2\omega L_{mc}/c$ is the round trip phase shift (ω is the light frequency), R and T are the power reflection and transmission coefficients, respectively, of the input and output mirrors (assumed identical). The phase offset ϕ_0 is related to the frequency offset between the sidebands and the resonance, $\phi_0 = 4\pi\Delta f_{\mathrm{sb}}L_{mc}/c = 5\times10^{-7}\Delta f_{\mathrm{sb}}$. Similarly the small fluctuating round trip phase is related to the phase noise in the oscillator: $d\phi = 5\times10^{-7}(f_a\cdot\delta\phi_{\mathrm{osc}})$, where $\delta\phi_{\mathrm{osc}}$ is the phase fluctuation of the oscillator (in radians) at the audio frequency f_a.

For the mode cleaner design parameters T = 0.2%, R = 0.998, the change in transmitted sideband amplitude is then given by:

$$d\left|t_{mc}\right| = \frac{R}{T^2}\cdot\left(\frac{4\pi L_{mc}}{c}\right)^2\cdot\Delta f_{\mathrm{sb}}\cdot f_a\cdot\delta\phi_{\mathrm{osc}} = 6.3\times10^{-8}\cdot\Delta f_{\mathrm{sb}}\cdot f_a\cdot\delta\phi_{\mathrm{osc}}$$

The requirement on the oscillator phase stability, $\delta\phi_{\mathrm{osc}}$, is determined by requiring that the fluctuation in sideband amplitude be less than that due to the oscillator amplitude fluctuations, when the frequency offset is $\Delta f_{\mathrm{sb}} = 100$ Hz. This requirement is shown in

APPENDIX 7 PHOTODETECTOR BACKSCATTERING

The phase noise produced by light scattered back into the interferometer from elements in the path of the interferometer output beams is discussed in **LIGO-G970067-00-D** & **LIGO-T97xxxxx**. For a scattering path which ultimately results in scattered light showing up at the anti-symmetric port, the signal produced by this scattering process (S_{sc}), relative to the signal corresponding to the LIGO SRD sensitivity expressed as phase noise ($S_{\phi\mathrm{SRD}}$), is:

$$\frac{S_{sc}}{S_{\phi_{SRD}}} = \frac{4\pi x_{sc}(f)}{\lambda} \cdot \sqrt{\frac{P_{sc}}{P_{bs}}} \cdot \frac{1}{\delta\phi_{SRD}}$$

where x_{sc} is the motion of the scattering surface along the direction of the beam propagation vector, P_{sc} is the scattered light power at the anti-symmetric port (in the mode of the main beam), P_{bs} is the power incident on the beamsplitter, and $\delta\phi_{SRD}$ is the phase noise corresponding to the LIGO SRD sensitivity. This last quantity is related to the SRD displacement sensitivity by:

$$\delta\phi_{SRD} = \frac{8F}{\lambda} \frac{\mathbf{x}(f)}{\sqrt{1 + (f / f_c)^2}}$$

where F is the arm cavity finesse and f_c the arm cavity pole frequency.

There are nominally two output beams for which scattered light from the LSC could show up at the anti-symmetric port: the anti-symmetric port beam and the ITM ghost beam (serving as the recycling cavity pick-off). The scattered light power P_{sc} at the anti-symmetric port from one of these beams is

$$P_{sc} = P_{port} \cdot \mathrm{BRDF}(\theta) \cdot m^2 \Omega_{ifo} \cdot A$$

where P_{port} is the power in the main beam at the output port, BRDF(θ) is the value of the bidrectional-reflectance-distribution function for the scattering surface at the angle of incidence θ, m is the ratio of the beam size at the beamsplitter to the beam size at the scattering surface, Ω_{ifo} is the solid angle of the beam in the interferometer, and A is any power attenuation factor between the scattering surface and the beamsplitter.

We require that the signal ratio (scattered light-to-phase noise) above be less than 0.1. The parameters in the domain of the LSC are x_{sc}, m, and BRDF(θ). The requirement on these factors is then expressed as

$$x_{sc} \cdot m \cdot \sqrt{\mathrm{BRDF}(\theta)} \le 0.1\,\frac{\lambda}{4\pi}\sqrt{\frac{P_{bs}}{P_{AS}\Omega_{ifo}A}}\ \delta\phi_{SRD} = \frac{1.1\times 10^{-2}}{\sqrt{A}}\ \delta\phi_{SRD}$$

where the number is given for the anti-symmetric port beam, assuming P_{AS} = 0.6 W, P_{bs} = 250 W, and $\Omega_{ifo} = 2.5 \times 10^{-10}$. If more than one port (path) is signficant, then the incoherent sum of the quantity on the l.h.s. for all ports must meet the requirement on the r.h.s.