LIGO-T960034-00-D

HYTEC-TN-LIGO-03
03/01/96

BSC Stack Design Trend Study

Eric Ponslet
March 1, 1996

Abstract

A number of stack designs for the beam splitter chambers (BSC) have been generated based on various spring concepts. Designs using the original VITON springs are defined as baseline cases. A number of spring variations are then presented together with their effect on stack design and performance. The propagation of weight savings from the top structure to the entire system is also examined. This study provides valuable insight into the stack design problem and leads to specifications for the design of novel spring concepts.

HYTEC Inc.
127 Eastgate Dr., Ste. 103
Los Alamos, NM 87544
(505) 662-0080

LIGO PROJECT

1. Description

The Beam Splitter Chamber (BSC) is shown schematically in Fig. 1. Terminology and tentative dimensions are included in the drawing. For the stack design, we will refer to the *downtube structure* which includes the top plate, downtube, and optical table, and the *top structure* (in blue in the sketch) which includes the downtube structure and the payload. We use a payload mass estimate of 500 lbs (includes test mass and all instrumentation attached to optical table). The words *total mass* refer to the combined mass of all stack elements and the top structure (all blue and red components in Fig. 1).

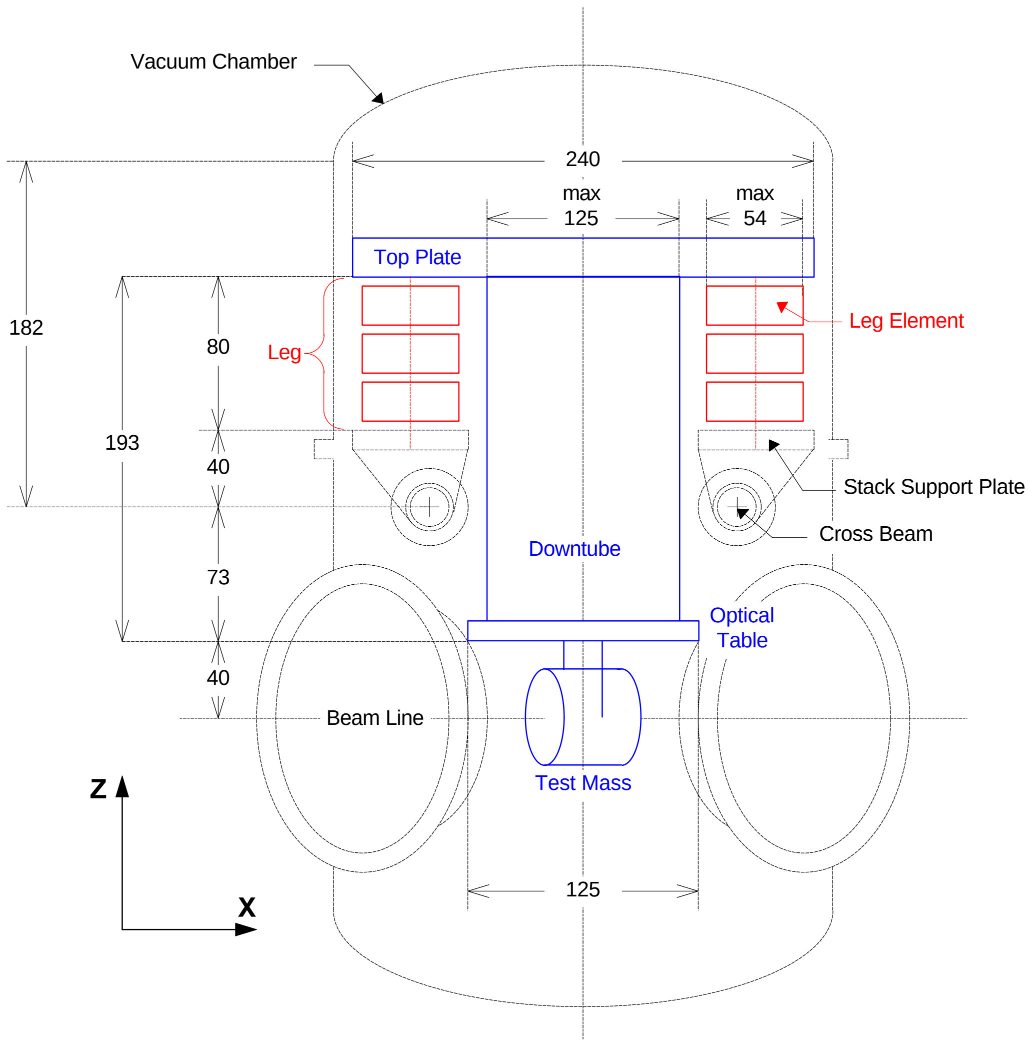


Figure 1: BSC chamber and isolation stack; terminology and baseline dimensions (cm); most dimensions from [1] and [2].

The isolation stack will consist of 3 or 4 identical legs resting on a support plate, itself supported by 2 cross-beams. The stack has 4 stages (4 layers of elastic elements) and each leg has 3 elements. The whole system is enclosed in a vacuum chamber. All elements of the system (cross beams, support table, leg elements, and top structure) are assumed infinitely stiff in this initial design study.

2. Designs with 3700 lbs Downtube Structure

First, a 3700 lb estimate[2] of the mass of the downtube structure was adopted. This leads to a 4200 lb mass for the top structure (downtube structure + payload). Inertia properties were roughly estimated assuming the following breakdown of the downtube structural mass: 62% for the upper plate, 22% for the downtube, and 16% for the optical table. The stack is assumed to rest on an infinitely stiff support structure (flexibility of cross-beams and support plate temporarily neglected). Following the optimal stack design approach described in an previous technical note[3], we generate a number of designs using various spring concepts. The total mass of the stack and top structure will be limited to a maximum of 13512 lbs for all designs (4 legs × 3 elements × 776 lbs + 3700 lbs + 500 lbs, see [2]). Note that the actual total mass of some designs will be less than that limit due to the discreteness of the design problem (integer number of springs at each stage, see [3]). The absolute vertical transmissibility at 100Hz from the support plate to the centerpoint of the optical table is used to compare the various designs.

All designs are listed in Table 1. The table gives: k_{ax} and P_{max} of the springs at 100 Hz; the number of legs n_{leg}; for each stage, the number of springs per leg, the leg element masses M_i, and the static spring loading in % of capacity P_i; the total mass of the system and the total number of springs used; the one-dimensional prediction of vertical transmissibility ($log_{10}(T_{zz})$) and the corresponding performance index PI_{zz} defined as the ratio of the transmissibility of the baseline design (1) and that of the current design; Horizontal and Vertical transmissibilities obtained from 3-dimensional MATLAB simulations are also listed. The 3D performance indices PI_{xx} and PI_{zz} are based on 3D transmissibilities of design 1.

2.1 VITON Springs

The original VITON spring design is used to generate baseline stack designs. These springs have a dynamic axial stiffness at 100 Hz equal to 4757.1 lb/in[3] and are limited to a maximum static axial compression load of 125 lbs each (d_{max}=0.0263" at 100 Hz). Designs with 3 and 4 legs are generated (Table 1, designs 1 and 2). Note how both designs have almost exactly the same performance ($log_{10}(T_{zz(100)})=-4.18$) and use about the same number of springs. The 4 legged design has the advantage that the legs' lower stage rest closer to the cross beams on the support plate. This should result in a stiffer support structure. Also, for a given stack element diameter, the 4 legged stack is shorter than the 3-legged one. For these reasons, the 4-leg stack will be taken as the baseline design.

A 3D model of the 4 legged stack was created in MATLAB (Fig. 2). Predicted vertical and horizontal absolute transmissibilities are plotted in Fig. 3. The isolation achieved at 100Hz are about 10^{-4} for T_{ZZ} and 10^{-6} for T_{XX}. These levels are very similar to the measured performance of the 'MIT prototype' stack.

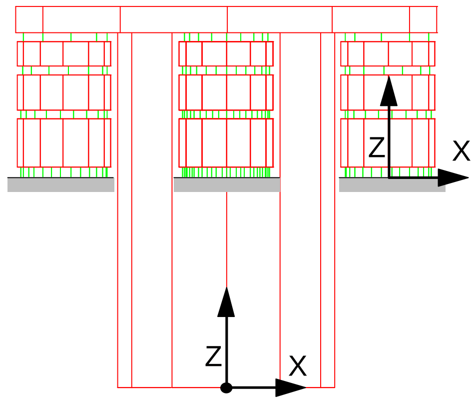


Figure 2: MATLAB model of 4 leg design with VITON springs and 3700 lbs top structure (Table 1, design 1); transmissibilities are computed from pure translations of the stack s floor to centerpoint of the optical table.

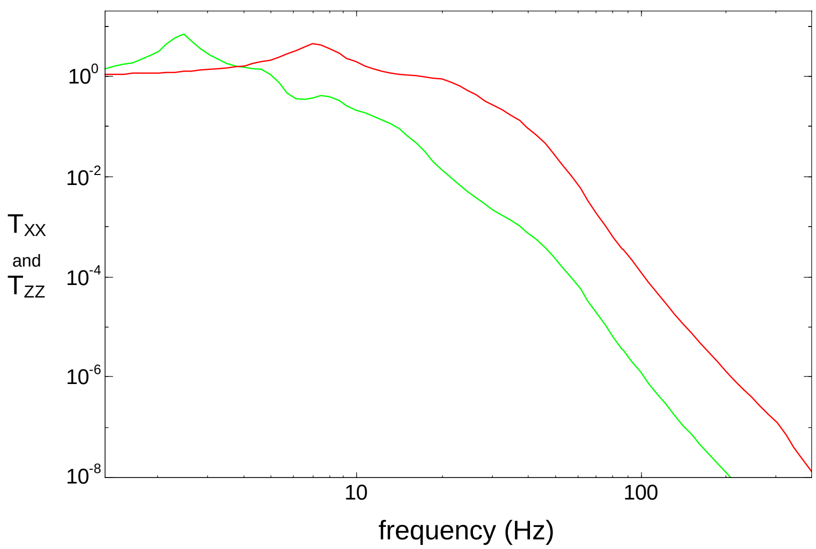
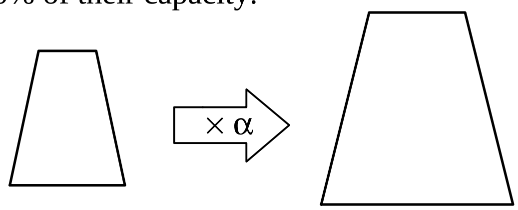


Figure 3: predicted transmissibilities (T_{xx}, T_{zz}) from base to optical table centerpoint, 3 dimensional MATLAB model, 4 leg design with VITON springs (design 1 of Table 1).

2.2 Scaled-Up VITON Springs

The performance of the stack using VITON springs is limited by their low static load capacity (P_{max}) and relatively high stiffness (k_{ax}). Scaling up the springs by a factor α (>1, Fig. 4) leads to performance improvement by increasing the ratio of load capacity to stiffness: for equal admissible strain, the load capacity is proportional to the cross sectional area of the spring ($P_{max} \propto A \propto \alpha^2$) while the stiffness is proportional to $A/l \propto \alpha$ (l is the length of the spring) so that $P_{max} / k_{ax} = \delta_{max} \propto \alpha$. We have shown[3] that $T_{zz} \sim 1 / \delta_{max}{}^n$, where n is the number of stages. This implies that scaling the springs up by α should produce a reduction in T_{zz} by a factor α^n. To test this idea, we generate 3 and 4 leg designs using VITON springs scaled by 200% from the original dimensions (all proportions remain the same). This enlarged spring has an axial stiffness at 100 Hz equal to k_{ax} = 2 × 4757.1 lb/in = 9514.2 lb/in. and a load capacity P_{max} = 4 × 125 lb = 500 lb. The resulting designs (Table 1, designs 3 and 4) show 1 dimensional performance indices (ratio of T_{zz} to baseline T_{zz}) equal to 10 and 14. The theoretical improvement 2^4=16 is not achieved because of discreteness (integer number of springs) which prevents the springs from being loaded to 100% of their capacity.



Stiffness k_{ax}	$\times \alpha$
load capacity P_{max}	$\times \alpha^2$
$d_{max} = P_{max} / k_{ax}$	$\times \alpha$
number of springs	$\times 1/\alpha^2$
Total exposed area	$\times 1$
Total volume	$\times \alpha$
Vertical isolation T_{zz}	$\times \alpha^n$

Figure 4: scale effects on rubber springs and performance of *n*-stage isolation stack.

For the 4 leg design, predictions from a 3 dimensional MATLAB model are plotted in Fig. 5 and compared to baseline. These results confirm the vertical performance improvement expected from the 1 dimensional approximation. The horizontal transmissibility is also improved by about the same factor (see Table 1).

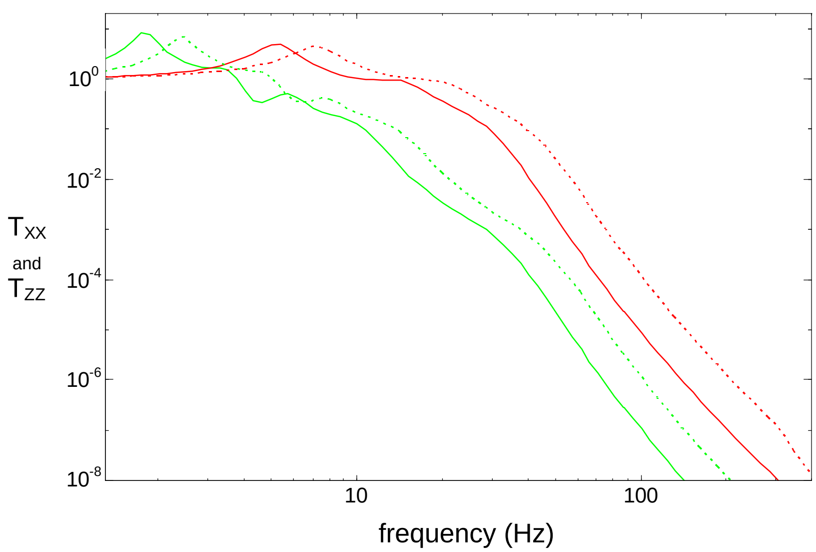


Figure 5: 3 dimensional MATLAB predictions of transmissibilities of 4 leg design with scaled up VITON springs (design 3, T_{xx}, T_{zz}) compared to baseline (design 1, ---T_{xx}, ---T_{zz}).

Also note that the total exposed rubber area of design 3 is essentially the same as in design 1 (72×4=284, compared to 272). The total volume of rubber however is about doubled.

2.3 Hybrid VITON/Steel Coil Springs

Improving vertical performance requires increasing $d_{max} = P_{max} / k_{ax}$. The axial load capability P_{max} of the VITON springs is limited by their strain capacity. Helical coil springs on the other hand have large static load and deflection capacities but no damping. Combining a VITON rubber spring with a longer steel coil spring may lead to large increases in d_{max}. The key is using a coil spring with a free length longer than that of the VITON spring. The coil spring can then support most of the static load without proportionally increasing stiffness. Figure 6 represents a hypothetical hybrid spring build on this principle, assuming a coil spring with k_{ax} = 1709 lb/in (equal to DC stiffness of VITON spring) and a static load capacity of 1250 lbs (10 times larger than the VITON spring). Note that only rectangular cross section springs can achieve those values.

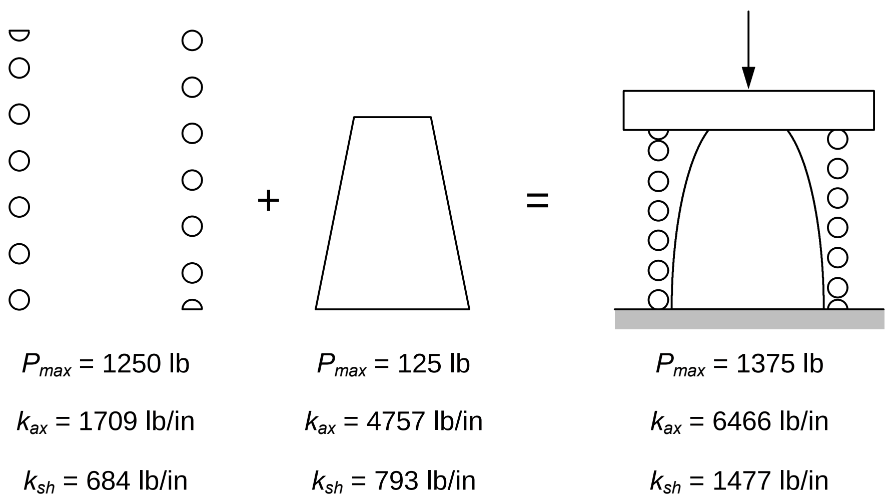


Figure 6: hybrid spring concept and properties at 100 Hz.

The resulting hybrid spring would have a load capacity P_{max} = 1375 lbs, a dynamic stiffness k_{ax} = 1709 + 4757 = 6466 lb/in at 100 Hz, and $d_{max}= P_{max}/ k_{ax}$= 0.2126", about 8 times larger than the original VITON spring. Improvement in vertical isolation (4-stage stack) could be as high as 8^4, or approximately 4000. Actual designs using those hybrid springs are listed in Table 1 (designs 5 and 6). Again, the largest conceivable improvement of 4000 is not achieved by those designs because of discreteness. In some stages, the springs are loaded to only 45% of their capacity. Note also that the number of springs in some stages had to be pushed up for stability reasons (indicated by * in the table, see [3]). Actual improvements compared to baseline are about 746 and 859 for the 3 and 4 leg designs, respectively. If such designs were to be pursued, acoustic transmission through the steel coil and mechanical resonance of the coil spring would be concerns.

The shear stiffness of rectangular cross section springs cannot be easily predicted analytically and is not available from manufacturers (it is also dependent on the applied axial load and deflection). We performed 3 dimensional MATLAB simulations assuming a ratio k_{sh}/k_{ax} of 0.4, consistent with experimental results obtained on similar springs[4]. The results are compared to baseline in Fig. 7.

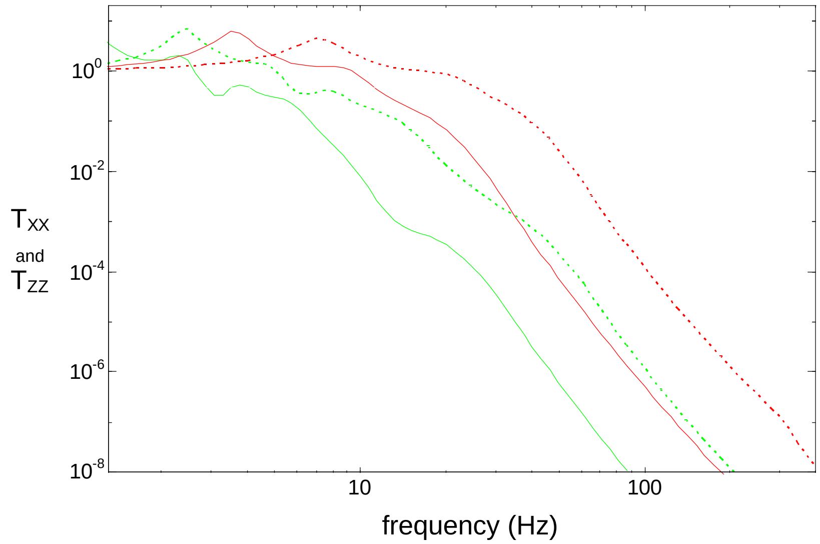


Figure 7: 3D MATLAB predictions of transmissibilities for 4 leg design with hybrid springs (design 5, T_{xx}, T_{zz}) compared to baseline (design 1, ---T_{xx}, ---T_{zz}); k_{sh}/k_{ax} assumed equal to 0.4 for the coil spring.

2.4 Undamped Coil Springs

To get an idea of realistic improvements that could be achieved with metal springs, we selected off-the-shelf helical steel springs about the same size as the VITON springs and with the largest possible d_{max}. Those springs have no damping and therefore do not constitute viable design options. They do however provide a realistic limit on the isolation level that may be achievable with specially designed damped metal coil springs. The springs (catalog number 103-712) were selected from the die spring catalog from Associated Spring/Raymond[5]. They have a rectangular cross-section wire, an inside diameter of ¾", outside diameter of 1½", free length of 3", maximum operating deflection of 1.2" under a 403.2 lb load (minimum working length 1.8"), and an axial stiffness of 336 lb/in. With a static deflection of 1.2" (45 times more than the VITON springs), the largest expected performance improvement at 100 Hz compared to baseline is about 45^4=4.1 10^6. Again, 3 and 4 leg designs were generated using these springs. They are listed in Table 1 (designs 7 and 8). Note that these designs almost achieve the expected improvement of 4 10^6. This is because the coil springs have a moderate load capacity so that each stage uses a relatively large number of springs and the effect of discreteness is reduced. This is confirmed by the high working loads obtained with those designs (87 to 100% of capacity). Three dimensional simulations were performed assuming again a ratio k_{sh}/k_{ax} of 0.4. The results are plotted in Fig. 8.

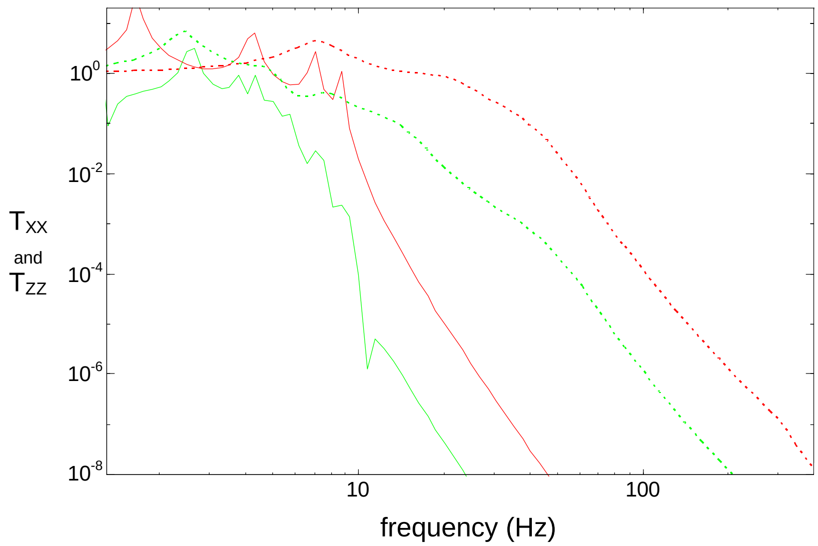


Figure 8: 3D MATLAB predictions of transmissibilities of 4 leg design with undamped coil springs (design 7, T_{xx}, T_{zz}) compared to baseline (design 1, ---T_{xx}, ---T_{zz}); k_{sh}/k_{ax} assumed equal to 0.4.

To gain some insight into the effect of the shear stiffness, we repeated the 3D simulations with ratios k_{sh}/k_{ax} equal to 0.2, 0.4, 0.6, and 1.0. The axial stiffness was kept constant (336 lb/in). Those results are shown in Fig. 9. Note that T_{zz} is independant of k_{sh}/k_{ax} for symmetric stacks. It is interesting to observe that because of variable coupling of the various horizontal/rocking modes of the stack, the isolation at high frequency is not monotonous in the shear stiffness (compare T_{xx} for k_{sh}/k_{ax}=0.2 and 0.4). These effects however are relatively small and there is a general tendency for T_{xx} to improve for smaller values of k_{sh}.

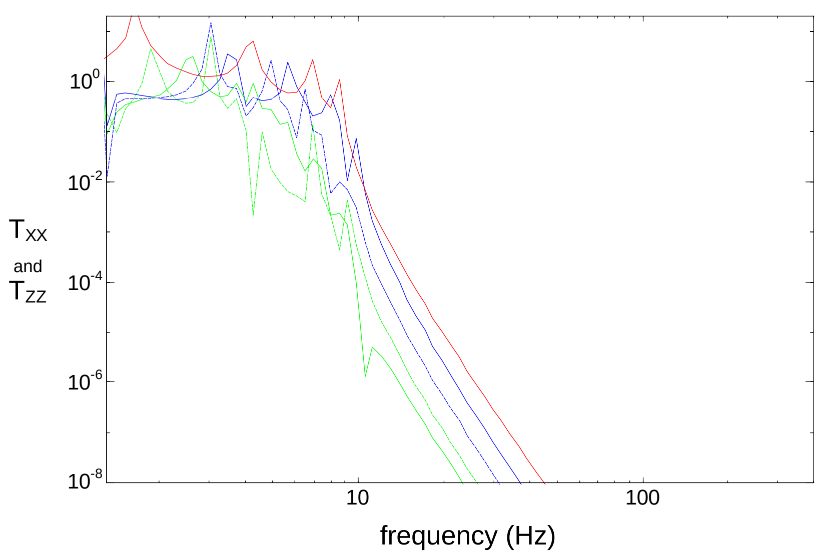


Figure 9: Effect of shear stiffness: comparison of horizontal transmissibility T_{xx} of 4 leg design with undamped coil springs for different values of k_{sh}/k_{ax} (--- 0.2, 0.4, --- 0.6, 1.0); vertical transmissibility (independant of k_{sh}/k_{ax}) is also shown (T_{zz}).

#	Springs		Stack						1 dim.		3 dim.			
	k_{ax} *[lb/in]*	P_{max} *[lb]*	n_{leg}	**stg 1** m_1 M_1 *[lb]* % P_{max}	**stg 2** m_2 M_2 *[lb]* % P_{max}	**stg 3** m_3 M_3 *[lb]* % P_{max}	**stg 4** m_4 M_4 *[lb]* % P_{max}	**Total** # spgs *M [lb]*	**log T_{zz}**	**PI_{zz}**	**log T_{zz}**	**PI_{zz}**	**log T_{xx}**	**PI_{xx}**
1	4757	125	4	27 *1064* 100	19 *749* 97	13 *512* 96	9 *4200/4* 93	272 *13499* -	-4.18	1	-3.95	1	-5.92	1
2	''	''	3	36 *1431* 100	25 *993* 98	17 *676* 98	12 *4200/3* 93	270 *13499* -	-4.19	1	-3.96	1	-6.28	2
3	9514	500	4	7 *1050* 94	5 *750* 90	3 *450* 100	3 *4200/4* 70	72 *13197* -	-5.20	10	-5.07	13	-7.00	12
4	''	''	3	9 *1350* 94	6 *900* 97	4 *600* 100	3 *4200/3* 93	66 *12749*	-5.32	14	-5.20	18	-7.38	28
5	6466	1375	4	3 *776* 82	3* *776* 63	3* *776* 45	1 *4200/4* 76	40 *13512* -	-7.05	746	-6.31	230	-8.39	292
6	''	''	3	4 *1242* 82	3 *931* 79	3* *931* 57	2 *4200/3* 51	36 *13512* -	-7.12	859	-6.38	271	-9.02	1259
7	336	403	4	9 *1103* 93	6 *735* 94	4 *490* 95	3 *4200/4* 87	88 *13512* -	-10.74	3.6 10^{6}	-10.74	6.1 10^{6}	-13.16	1.7 10^{7}
8	''	''	3	11 *1355* 98	8 *986* 93	5 *616* 100	4 *4200/3* 87	84 *13070* -	-10.75	3.7 10^{6}	-10.74	6.2 10^{6}	-12.55	4.3 10^{6}

Table 1: LIGO-BSC stack designs with 4200 lbs top structure mass using VITON springs (1,2), scaled up VITON springs (3,4), hybrid springs (5,6), and coil springs (7,8). The * indicates stages where the number of springs had to be pushed to 3 for stability reasons.

3. Designs with Lighter 1500 lbs Downtube Structure

Preliminary results from the downtube structure design effort indicate that the weight of the downtube structure may be reduced to about 1500 lbs. The total mass of the top structure (downtube + payload) would then be about 2000 lbs instead of 4200 lbs in the initial estimate (52% reduction). A series of stack designs were again generated using 3 or 4 legs and VITON, scaled up VITON, hybrid, or coil springs. These designs are listed in Table 2. The 1D T_{ZZ} performance of each design in Table 2 was adjusted[3] to match that of the corresponding design of Table 1. The primary goal is to examine the propagation of the 52% weight reduction into the total system weight (bold in Table 2).

The number in green give the % savings in the total system weight compared to the corresponding designs in Table 1. Note that in many cases the savings does not fully propagate (if at all, see design 13) into the total system weight. This is again a result of the discrete number of springs. Designs using springs with small static load limits give the best results (designs 9 and 10 for example). In contrast, when the spring load capacity is too high, the total mass of the system does not follow the mass of the upper structure. In some cases (design 13) the design with a lightened top structure is actually slightly heavier than original (design 5).

#	Springs		Stack						1 dim.	
	k_{ax} [lb/in]	P_{max} [lb]	n_{leg}	stg 1 m_1 M_1 [lb] % P_{max}	stg 2 m_2 M_2 [lb] % P_{max}	stg 3 m_3 M_3 [lb] % P_{max}	stg 4 m_4 M_4 [lb] % P_{max}	Total # spgs *M [lb]* *% saved*	log T_{zz}	PI_{zz}
9	4757	125	4	13 *501* 97	9 *347* 96	6 *231* 97	4 *2000/4* 100	128 ***6313*** 53	-4.18	1
10	"	"	3	18 *727* 98	12 *485* 98	8 *323* 99	6 *2000/3* 89	132 ***6604*** 51	-4.19	1
11	9514	500	4	4 *533* 92	3 *399* 87	3* *399* 60	1 *2000/4* 100	44 ***7326*** 44	-5.20	10
12	"	"	3	6 *1007* 95	4 *671* 92	3 *503* 78	2 *2000/3* 67	45 ***8543*** 33	-5.32	14
13	6466	1375	4	3 *994* 84	3* *994* 60	3* *994* 36	1 *2000/4* 36	40 ***13924*** -3	-7.05	746
14	"	"	3	3 *946* 85	3* *946* 62	3* *946* 39	1 *2000/3* 48	30 ***10517*** 22	-7.12	859
15	336	403	4	6 *822* 94	4 *548* 90	3* *411* 75	2 *2000/4* 62	60 ***9128*** 32	-10.74	$3.6\ 10^6$
16	"	"	3	6 *751* 95	4 *501* 96	3 *376* 86	2 *2000/3* 83	45 ***6883*** 47	-10.75	$3.7\ 10^6$

Table 2: LIGO-BSC stack designs with 2000 lbs top structure mass (52% reduction) using VITON springs (9,10), scaled up VITON springs (11,12), hybrid springs (13,14), and coil springs (15,16). The number in green show % weight reductions compared to corresponding designs in Table 1.

4. Observations

Effect of discreteness: for optimal performance, all springs must to be loaded close to 100% capacity. When this is not the case, the excess stiffness leads to sub-optimal decoupled natural frequencies and performance. When the springs are small (small P_{max}), each stage uses a relatively large number of them and the effect of discreteness is small: by adjusting the number of springs in each stage, all springs can be made to carry almost 100% of their axial load capacity. This is not always the case with larger springs. In particular, a minimum of 3 springs per leg is required at all stages to insure stability of the leg elements. With large springs, this can lead to largely sub-optimal stages. Also, when springs are too large, weight savings at the top structure level do not translate into proportional weight savings in the whole system (at equal performance) because the number of springs is determined by stability conditions instead of load carrying capability.

Three or four legs: A somewhat non-intuitive result is the fact that at equal performance, 3 legged designs do not weigh less than 4 legged designs. Also, for a given total mass, 3 legged designs use heavier leg elements. Because the diameter of the leg elements may be limited by the available space (54 cm maximum in this study), 3 legged designs may be taller than 4 legged ones. In addition, four legged designs may have the advantage of bringing the leg support points closer to the cross beams, potentially leading to a stiffer support structure. Conceptual arrangement drawings are needed to confirm these observations.

Accuracy of Tzz predictions: as expected, the 1 dimensional asymptotic approximation proved very accurate and is clearly sufficient for preliminary design purposes. Predictions of horizontal transmissibilities on the other hand require 3 dimensional models. We observe however that - especially with a given spring concept (i.e. fixed ratio k_{sh}/k_{ax}) - the horizontal performance follows the same trends as the vertical (similar trends in PI_{xx} and PI_{zz}, Table 1), suggesting that the effect of 3 dimensional coupling is mild.

5. Design Goals for Damped Metal Springs

Axial Load Capacity: the effects of discreteness decrease with decreasing load capacity (smaller springs). For very small capacities however, the number of springs in the bottom stages may be prohibitive. In contrast, for large capacities, the number of springs required in some stages to support the static load may be less than 3. We then have to increase that number to 3 for stability reasons, leading to incompletely loaded springs and excess stiffness. A reasonable compromise consists for example of designing the springs so that the upper stage of the stack requires between 2 and 5 springs per leg (arbitrary numbers). For a stack with 4 legs and a 2000 lbs top structure this reasoning leads approximately to **100 lbs < P_{max} < 500 lbs.**

Static Deflection Capacity: As shown in [3], the axial deflection capacity d_{max} of the springs is directly related to the optimal performance of a stack. Maximum performance requires the **largest possible d_{max}**. Note that axial stiffness is not directly involved although it is implicitly defined (for a linear spring) by P_{max} and d_{max}.

Shear stiffness: Although horizontal transmissibilities can only be predicted with 3 dimensional models, our results confirm intuition that springs softer in shear tend to produce better horizontal isolation than stiffer ones. Since horizontal isolation requirements for the LIGO stacks are more stringent than those for vertical isolation, the spring designs efforts should attempt to **minimize the ratio of shear to axial stiffness**. Care must be taken however to not induce shear instabilities in the stack.

Damping: Damping is required to limit the stack's resonant response to low frequency disturbances. Viscous damping cannot be used because it greatly reduces performance at higher frequencies. Viscoelastic damping on the other hand does not significantly affect high frequency isolation. The prototype VITON springs have loss factors of about 0.3 at frequencies around 10 Hz. We propose that a **minimum loss factor of 0.1 at all frequencies below 30 Hz** is an acceptable goal.

Geometry: taller springs lead to taller isolation stacks. However, the taller the springs, the easier it is to achieve the large static deflection capacities d_{max} required for high performance. Also, the outside diameter of the springs should be kept small enough that there is room to fit the springs of the lower stages on the leg elements. **Upper limits of about 4   for the length and 4   for the diameter** (room for 13 springs in a circle on a 54 cm leg element) appear reasonable.

6. References

1. J. Worden, A. Sibley, A. Lazzarini, and M. Coles, *Specification for the LIGO Vacuum Equipment*, LIGO project, California Institute of Technology, document #LIGO-E940002-02-V, Rev. 2., August 18, 1995.
2. *Baseline Isolation and Vacuum Chamber Configuration*, LIGO draft document #LIGO-T95000x-xxx, October 1995.
3. E. Ponslet, *Isolation Stack Design Methodology*, HYTEC Inc., Los Alamos, NM, document #HYTEC-TN-LIGO-02, February 21, 1996.
4. E. Ponslet, M. Eldred, and C. Fulcher, *Discrete Optimization of Isolator Locations for Vibration Isolation Systems,* Sandia Technical Report, to be released, 1996.
5. Associated Spring/Raymond, Barnes Group Inc., 1705 Indian Wood Circle, Suite 210, Maumee, OH 43537-4046, (800) 872-7732.

Note 1, Linda Turner, 09/03/99 11:27:54 AM
LIGO-T960034-00-D