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**LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY**  
**- LIGO -**

**CALIFORNIA INSTITUTE OF TECHNOLOGY**  
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**DRAFT**

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## DRAFT

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## BASIS OF THE OPTICAL WAVEFRONT SPECIFICATIONS

## INTRODUCTION AND OVERVIEW

This document is intended to accompany *Surface Specifications for the LIGO Arm Cavity Mirrors* November 1, 1993.

**Basic Assumptions:** The large aperture optics (the recycling mirror, beam splitter and arm cavity mirrors) constitute the major untested extrapolation we are making from the laboratory scale interferometers to the LIGO. These optics will not be integrated into a gravitational wave interferometer until the initial LIGO interferometer is assembled in the LIGO facilities. The strategy we have adopted to minimize the risk of failure is to couple metrology of the optics to physical optical models of the interferometer and stray light models of the LIGO beam tubes. A variety of analytic and computer models of the interferometer have been developed for this purpose as well as analytic and computational stray light propagation models of the LIGO beam tubes.

Although it may be uncomfortable to rely so on models, no compelling arguments have been presented for intermediate length tests of the full scale optics in either the current prototypes or in ancillary intermediate baseline facilities such as the X-ray collimation tube at NASA, Huntsville, Alabama. The metrology is expected to be good enough to predict the optical performance on the spatial and angular scales relevant to the LIGO.

**Basis of the specifications:** The specifications for the large aperture optics of the initial LIGO interferometer are based on a combination of technical and scientific criteria strongly weighted by our expectations of the near term capabilities of the optics industry.

The goal for the initial interferometer is to achieve an rms strain sensitivity of  $10^{-21}$  in a band of a few hundred Hz near a hundred Hz. We have chosen to use existing ion lasers with single mode output power of a few watts. Having set this goal and made the choice of the laser source, the interferometer must be a recycled system and the mirror properties become derived requirements.

The interferometer phase sensitivity is determined by the interferometer transfer function relating optical phase,  $\phi(f)$ , at the antisymmetric port to the gravitational wave strain,  $h(f)$ , incident on the interferometer.

$$\frac{\phi(f)}{h(f)} = \frac{4\pi\tau_{st}}{\tau_{opt}\sqrt{(1+(4\pi\tau_{st}f)^2)}}$$

$$\tau_{st} = \frac{2L}{c(T_{arminput} + L_{singlepass})}$$

The optical loss in an arm cavity is

$$L_{arm} = \frac{4L_{singlepass}}{T_{arminput}}$$

The phase noise due to quantum fluctuations in the light and photodetection with power recycling is given approximately by (a more complete expression including the RF modulation is given in *Analysis of an Externally Modulated Recycled Interferometer* D. Shoemaker and R. Weiss):

$$\phi_n^2(f) \approx \frac{h\nu L_{\text{total}}}{C\eta P_{\text{input}}(1 - L_{\text{arm}})} \quad L_{\text{total}} = L_{\text{arm}} + L_{\text{cyl}} + (1 - C)$$

where  $C$  is the fringe contrast at the antisymmetric port.

The equivalent gravitational wave strain noise becomes

$$h_n(f) = \frac{h(f)}{\phi(f)} \phi_n(f)$$

### Sample Parameters

$T_{\text{arminput}}$	$3 \times 10^{-2}$
$\tau_{\text{st}}$	$9 \times 10^{-4}$ sec
$\tau_{\text{opt}} = \lambda/c$	$1.7 \times 10^{-15}$ sec
$L_{\text{arm}}$	$< 3 \times 10^{-2}$
$L_{\text{singlepass}}$	$< 2.1 \times 10^{-4}$
$L_{\text{total}} = T_{\text{cyl}}$	$< 4 \times 10^{-2}$
$1 - C$	$< 3 \times 10^{-3}$
$\eta P_{\text{input}}$	2 watts
$\phi_n(f)$	$9 \times 10^{-11}$ radians/ $\sqrt{\text{Hz}}$
$h(100\text{Hz})$	$2 \times 10^{-23} 1/\sqrt{\text{Hz}}$

The arm cavity geometric parameters are set by the cavity  $g$  factors which determine the Guoy phase and with the wavelength determine the Gaussian spot radii.

$$g = 1 - \frac{L}{R}$$

where  $L$  is the cavity length and  $R$  is the radius of curvature of the curved mirror.

The Gaussian beam size on the flat mirror of the flat/spherical cavity is given by

$$w_0 = \sqrt{\frac{L\lambda}{\pi} \left( \frac{g}{1-g} \right)^{1/4}}$$

and on the curved mirror by

$$w = \sqrt{\frac{L\lambda}{\pi} \left( \frac{1}{g(1-g)} \right)^{1/4}}$$

In the spherical/spherical cavity the spot size at either mirror is

$$w = \sqrt{\frac{L\lambda}{\pi} \left( \frac{1}{1-g^2} \right)^{1/4}}$$

The Gaussian spot radii for the LIGO cavities are shown in figure 1 as a function of the curved mirror  $g$  factor. The diffraction power loss of the lower order Laguerre-Gauss cavity modes as a function of the ratio of aperture radius,  $r$ , to Gaussian spot size,  $w$ , is shown in figure 2. The power excitation of higher order Laguerre-Gauss modes by the diffraction of a finite aperture illuminated by a  $TE_{0,0}$  mode is shown in figure 3.

The Guoy phase for the  $TE_{0,0}$  mode is

$$\psi_{0,0} = \cos^{-1}(\sqrt{g_1 g_2})$$

and the difference between the Guoy phase of any other Laguerre-Gauss mode with radial index  $p$  and angular index  $m$  and the  $TE_{0,0}$  mode is

$$\Delta\psi_{pm} = (2p + m)\psi_{0,0}$$

The cavity  $g$  factors are chosen so that the Guoy phase difference of modes with low values of  $p$  and  $m$ , modes that have small diffraction loss in the cavities, will not be multiples of  $n2\pi$  and therefore resonant in the cavity simultaneously with the  $TE_{0,0}$  mode.

The requirement on the contrast defect is not only based on the shot noise estimate for the signal to noise which varies slowly with the contrast defect, as shown in figure 4 (the figure includes the optimization of the modulation index to maintain the shot noise minimum), but is also determined by the allowed power on the photo detector at the antisymmetric port assumed to be less than 300 milliwatts, a qualitative estimate for the effect on the wavefront alignment system and a hedge against extrinsic amplitude noise of the laser. The minimum coating diameter is chosen to maintain the allowed contrast defect and arm cavity loss.

The adopted cavity and interferometer parameters are summarized in Table 1 and Table 2.

The wavefront distortion specifications assume that the initial interferometer will not use an output mode filter and that the spatial mode degeneracy of the recycling cavity composed of the recycling and two front cavity mirrors does not make a serious contribution to the loss of the RF sidebands in the recycling cavity.

The use of an output mode filter is a backup in the event that we have difficulty in achieving the wavefront specifications. The deleterious effects of the recycling cavity degeneracy are currently under investigation. Should further optical modeling indicate that the wavefront distortion of the RF sidebands is significant and plays an important role in compromising the wavefront sensing alignment system, the preference is not to increase demands on the mirror specifications. Rather, to remove the degeneracy by figured optics in the recycling cavity or to abandon the asymmetric interferometer configuration.

The requirements for cavity loss, contrast defect and phase noise due to scattering, are in many regards satisfied by optics used in small space based low scatter optical telescopes and microelectronics optical masking machines. The primary differences arise from the need to control wavefront distortion on passing through thick substrates and from the transmission and reflection by large aperture multilayer dielectric coatings.

TABLE 1  
OVERALL INITIAL OPTICAL PARAMETERS

Optical wavelength:  $\lambda = 5.145 \times 10^{-5}$  cm

### Cavity geometric parameters:

Arm cavity length:  $L_{\text{arm}} = 4 \times 10^5$  cm

Recycling cavity length:  $L_{\text{recyl}} = 1.2 \times 10^3$  cm

Radius of arm cavity front mirror:  $R_{\text{front arm}} = R_1 = \infty$  (flat)

Radius of arm cavity back mirror:  $R_{\text{back arm}} = R_2 = 6 \times 10^5$  cm

Arm cavity g factor 1:  $g_1 = 1.0$

Arm Cavity g factor 2:  $g_2 = 0.333$

Gaussian spot radius at front:  $\omega_0 = \omega_1 = 2.15$  cm

Gaussian spot radius at back:  $\omega_2 = 3.73$  cm

Gouy phase of  $TE_{0,0}$  mode in arm cavity:  $\psi_{0,0} = 9.56 \times 10^{-1}$  radians

Closest mode  $p \leq 5$ :  $\Delta\psi_{s,3} = -0.14$  radians

Rayleigh range:  $z_r = 2.83 \times 10^5$  cm

Radius of recycling mirror:  $R_{\text{recyl}} = 6.64 \times 10^7$  cm (flat)

Recycling cavity g factor:  $g = 1 - 1.8 \times 10^{-5}$  (almost unstable cavity)

### Optical properties (scattering and losses):

Scattering and absorption loss of surfaces:  $A \leq 1.0 \times 10^{-4}$

BRDF of surfaces:  $\frac{dP_{\text{refl}}}{d\Omega * P_{\text{inc}}} \leq \frac{1 \times 10^{-6}}{\theta^2} \text{ sr}^{-1}$ ,  $\theta \leq 6 \times 10^{-3}$  radians

(Value used in tube scattering model)

Loss coefficient of bulk material:  $\alpha \leq 5 \times 10^{-6} \text{ cm}^{-1}$

Approximate rms surface error:  $\frac{\sigma_{\text{rms}}}{\lambda} \leq \frac{1}{400}$

### Optical Properties (reflectivity and transmission):

Reflectivity of recycling mirror:  $R_{\text{recyl}} = 0.96 - A$

Reflectivity of front arm cavity mirror:  $R_{\text{front arm}} = 0.97 - A$

Transmission of back arm cavity mirror:  $T_{\text{back arm}} = 1 \times 10^{-5}$

Reflectivity of back arm cavity mirror:  $R_{\text{back arm}} = 1.0 - A - T_{\text{back arm}}$

Reflectivity of beam splitter:  $R_{\text{beam split}} = 0.5 - A/2$

Transmission of beam splitter:  $T_{\text{beam split}} = 0.5 - A/2$

### Cavity and Interferometer Performance Parameters

Arm cavity loss:  $L_{\text{arm}} \leq 2.7 \times 10^{-2}$

Contrast defect at antisymmetric port:  $1 - C \leq 3 \times 10^{-3}$

Recycling cavity loss ( $A_{\text{recyl}}$ , AR coatings, bulk loss):  $L_{\text{rec}} \leq 2 \times 10^{-3}$

Recycling power gain =  $(L_{\text{arm}} + (1 - C) + L_{\text{rec}})^{-1}$ :  $\geq 30$

Power on antisymmetric port photodetector:  $P_{\text{det}} \leq 300 \text{ mW}$

**Optics dimensions:**

Arm cavity mirror diameter:  $D = 25$  cm

Arm cavity mirror thickness:  $t = 10$  cm

Minimum coating diameter front cavity mirror:  $D_{ct1} = 12$  cm

Minimum coating diameter rear cavity mirror:  $D_{ct2} = 20$  cm

Minimum coating diameter sph/sph arm cavity:  $D_{ct} = 14.5$  cm



**TABLE 2**  
**OPTICAL POWER AND INTENSITY AT VARIOUS COMPONENTS**

component	$\omega$ (cm)	Power (W)	Intensity (W/cm <sup>2</sup> )
$\phi$ modulator	$7 \times 10^{-2}$	4	$4 \times 10^2$
isolator	$5 \times 10^{-2}$	4	$8 \times 10^2$
mode filter (flat)	$1 \times 10^{-1}$	$4 \times 10^3$	$2 \times 10^5$
mode filter (curved)	$2 \times 10^{-1}$	$4 \times 10^3$	$6 \times 10^4$
telescope out. mir.	2.1	3	$4 \times 10^{-1}$
recycling mir.	2.1	$8 \times 10^1$	$1 \times 10^1$
beam splitter	2.1	$8 \times 10^1$	$1 \times 10^1$
arm cavity input mir.	2.1	$4.5 \times 10^3$	$6.7 \times 10^2$
arm cavity far mir.	3.8	$4.5 \times 10^3$	$2 \times 10^2$
main frame laser	$5 \times 10^{-2}$	$1 \times 10^2$	$3 \times 10^4$

**Extension from experience in the prototypes:** There are several critical areas in the specification of the large aperture optics for the initial LIGO interferometer which change priorities from the experience with the current prototypes. The initial LIGO interferometer cavities have much lower finesse than those now in the 40 meter system and the interferometer will be optically recombined. Even though we intend to power recycle the interferometer, the planned recycling power gain is modest. These factors taken together make optical loss from all sources of comparable importance to contrast defect at the antisymmetric port of the interferometer and the scattering by the mirror, which can contribute to phase noise from the tube walls. The other change is the increase in Gaussian spot radius from mm to cm scales. The new issue affecting the interferometer performance becomes the more difficult to control "figure errors" rather than the small scale roughness of the surfaces, substrates and coatings. The problem is compounded by the experience that the spatial power spectrum of optical wavefront phase distortions have a  $\frac{1}{\nu^n}$  dependence with  $n$  varying between 1 to 3 depending on the spatial frequency  $\nu$  and fabrication procedures.

### Wavefront characterizations

**Power spectrum:** Figure 5 shows a composite hypothetical one dimensional power spectrum of surface perturbations for a high quality small telescope mirror based on data from Hughes Danbury at low spatial frequencies and typical surface roughness measurements in the literature at high spatial frequencies. The spectrum is given in units of waves<sup>2</sup> (5145A) of optical distortion per wavenumber ( $\text{cm}^{-1}$ ) of spatial frequency. The integral of the power spectrum over spatial frequency gives the mean square fluctuations of the wavefront in units of optical waves when surface correlations become negligible: typically, above  $3 \text{ cm}^{-1}$ . The high frequency power spectrum shown represents a high quality but not superpolished surface: microroughness of several Angstroms rather than fractions of an Angstrom. The one dimensional power spectrum above about  $0.3 \text{ cm}^{-1}$  varies as  $\frac{1}{\nu}$ . The two dimensional power spectrum varies as  $\frac{1}{\nu^2}$  so that the differential scattering (BRDF) would vary as  $\frac{1}{\theta^2}$  where  $\theta$  is the scattering angle from the mirror normal. The sample spectrum would give a mirror with 130 ppm loss due to irregularities with spatial frequency larger than  $3 \text{ cm}^{-1}$ . The low spatial frequency errors would produce an interferometer contrast defect of  $\approx 10^{-4}$  (front mirror) and  $\approx 10^{-3}$  (back mirror) and equivalent overall cavity loss when such a mirror is put into a LIGO arm cavity. Over the Gaussian spot diameter the sample mirror would have a small spatial frequency rms of  $\approx \frac{\lambda}{500}$ .

The role of the different spatial frequencies of the wavefront perturbations is broadly indicated in the figure. Power at all spatial frequencies contribute to optical loss. The power at spatial frequencies between 0.1 to  $3 \text{ cm}^{-1}$  is particularly important in establishing the interferometer contrast defect. While the power between 3 to  $125 \text{ cm}^{-1}$  inurs scattering into angles that encounter the LIGO beam tube and baffles producing phase noise due to scattered light. The power in spatial frequencies larger than  $125 \text{ cm}^{-1}$  is scattered into large angles which is strongly attenuated by the beamtubes and also has a small probability of recombination with the main beam. This is the domain of microroughness which has been a major concern in the prototypes but is less important in the initial LIGO interferometer.

Figure 5 also shows the spatial frequency band and typical sensitivities of the available metrological techniques.

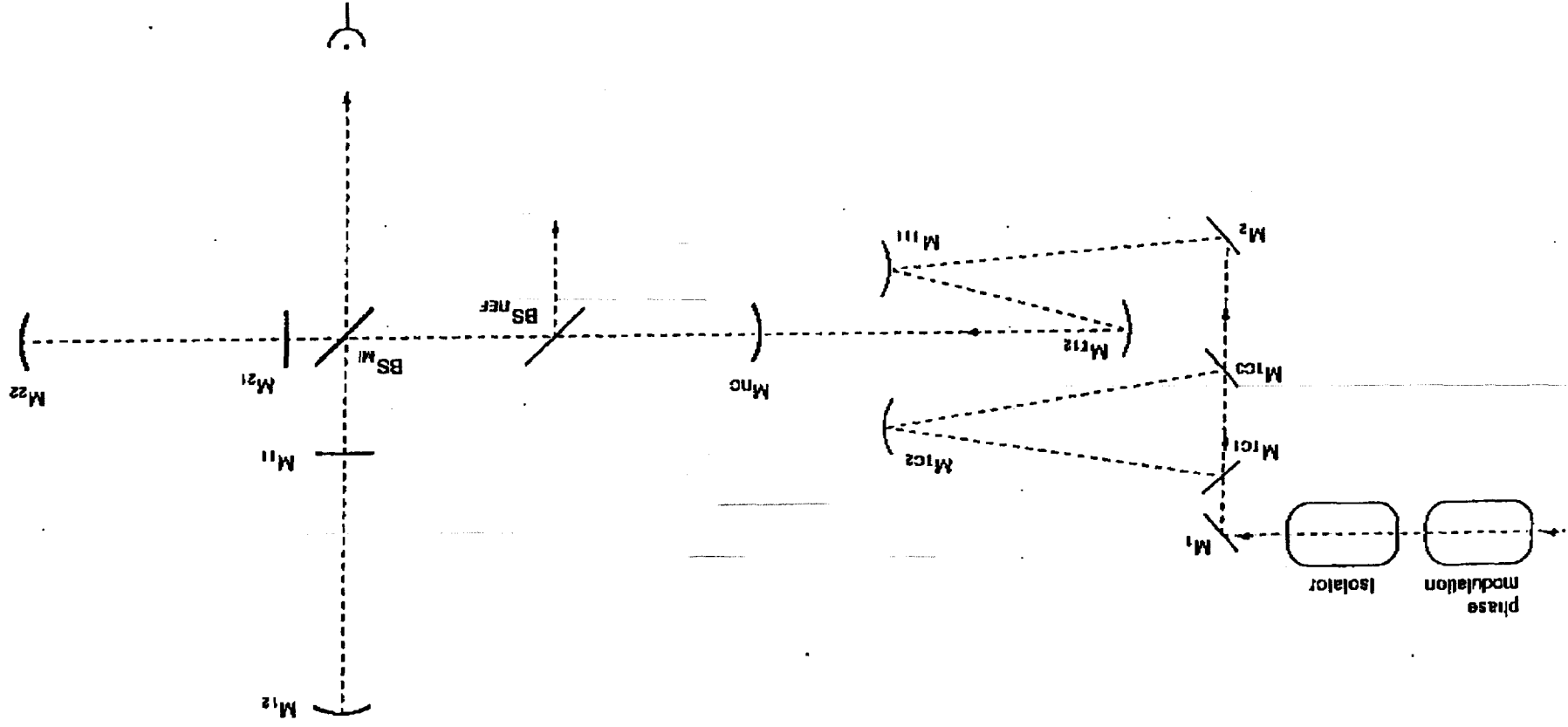
**Orthogonal cylindrical functions:** The spatial power spectrum is a useful quantitative measure for the differential scattering and the integrated scattering loss when one can neglect the spatial coherence of the exciting light and the correlations on the mirror surface: in our application for spatial frequencies larger than about  $3 \text{ cm}^{-1}$ . At spatial frequencies below  $3 \text{ cm}^{-1}$ , to estimate the performance of the cavities and the interferometer, it is more useful to expand the wavefront perturbations in terms of orthogonal cylindrical functions. We have chosen to express the low spatial frequency part of the mirror specifications in terms of Zernike and Laguerre - Gauss functions.

**Zernike functions:** The Zernike functions have become a standard in the optical industry. The lower order radial and angular Zernike functions are directly related to specific aberrations in the wavefront and optical shops have developed intuition for reducing their amplitude by adjusting polishing techniques.

There are several difficulties with using the Zernike functions, however. The first, trivial but confusing, is that the normalization of the functions has not been standardized so that one has to be careful to know the definitions used in the software associated with the wavefront decompositions. More relevant is that they have a poor weighting for our application.

The scale parameter for the Zernike functions is the aperture radius. The critical part of the wavefront in our application lies within about 3 Gaussian spot radii. A full mirror aperture Zernike decomposition, therefore, places the important terms at high radial order. This can in part be alleviated by reducing the aperture radius. The cavity optical modeling has shown that even with subapertures, the dynamic range in radial order required to properly characterize the interferometer contrast defect is still larger than that available in most commercial software packages. In addition, the radial functions themselves, especially the higher order functions, have large derivatives at the aperture edges so that with a finite number of pixels in the wavefront map, the orthogonality of the functions is not maintained in practice. This results in the large amplitude terms corrupting the estimates for the small amplitude ones. Standard computational techniques for orthogonalizing a subset of the functions evaluated on a given pixel grid have been developed but are also not part of most of the software packages used in optical shops.

**Laguerre- Gauss Functions:** The Laguerre - Gauss functions are the cylindrical form of the Cartesian Hermite - Gauss functions. Both are solutions of the paraxial ray equations associated with the cavity modes of the interferometer and are thereby a natural basis to transform the wavefronts in our application. The key parameter becomes the Gaussian beam spot size and the wavefront distortions of the mirrors, expressed in terms of these transforms, are optimally weighted. Furthermore, the interferometer performance can be easily related analytically to the transform amplitudes when projected onto the input TE<sub>00</sub> mode. The trouble with these functions is that they are not standard in the optical industry.



Initial Interferometer optical schematic

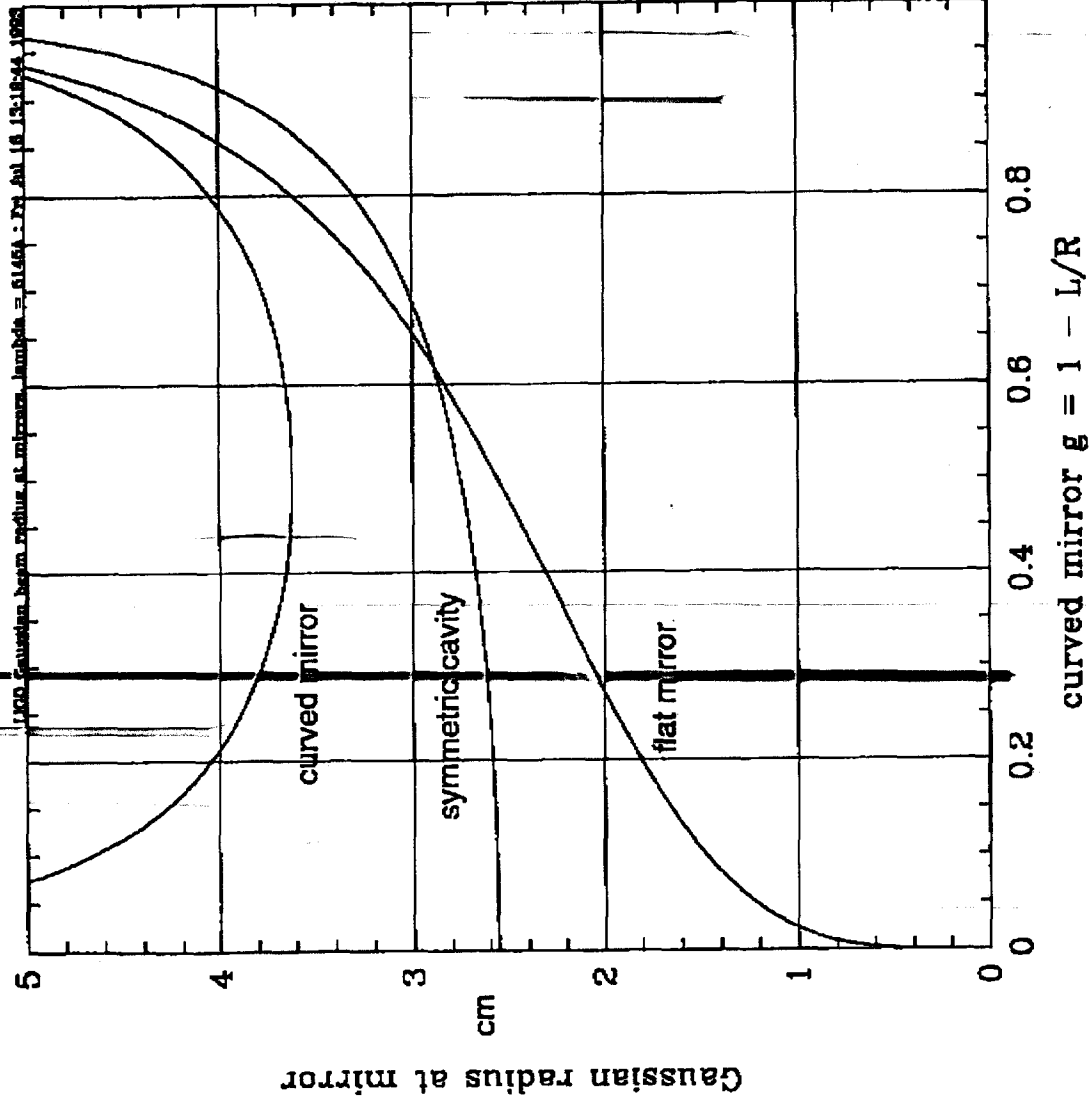
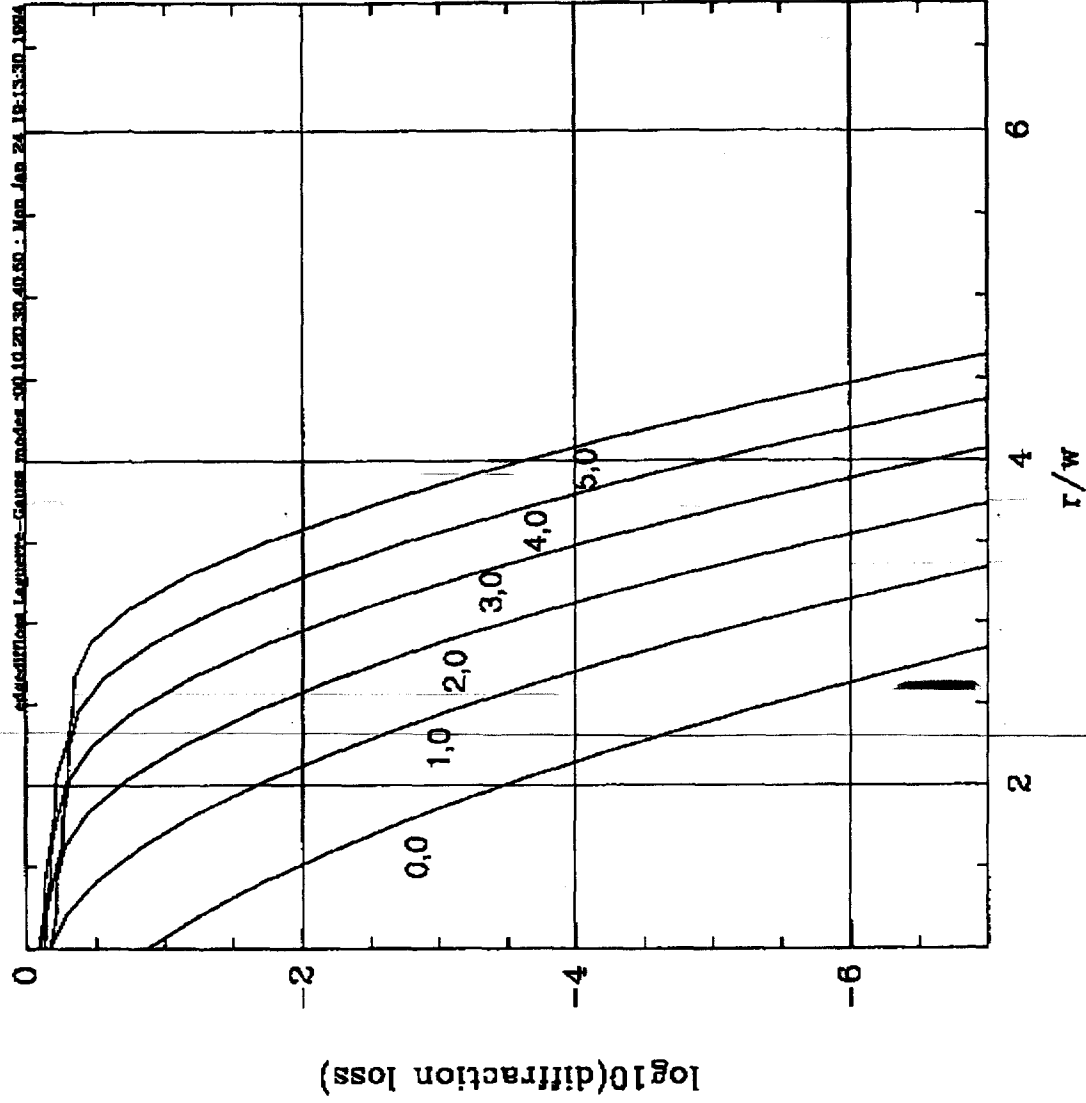


FIG 1



Gravitational strain amplitude spectrum,  $M_0$ , for phase noise for various constant defocus: Sun May 6 22:45:10 1992

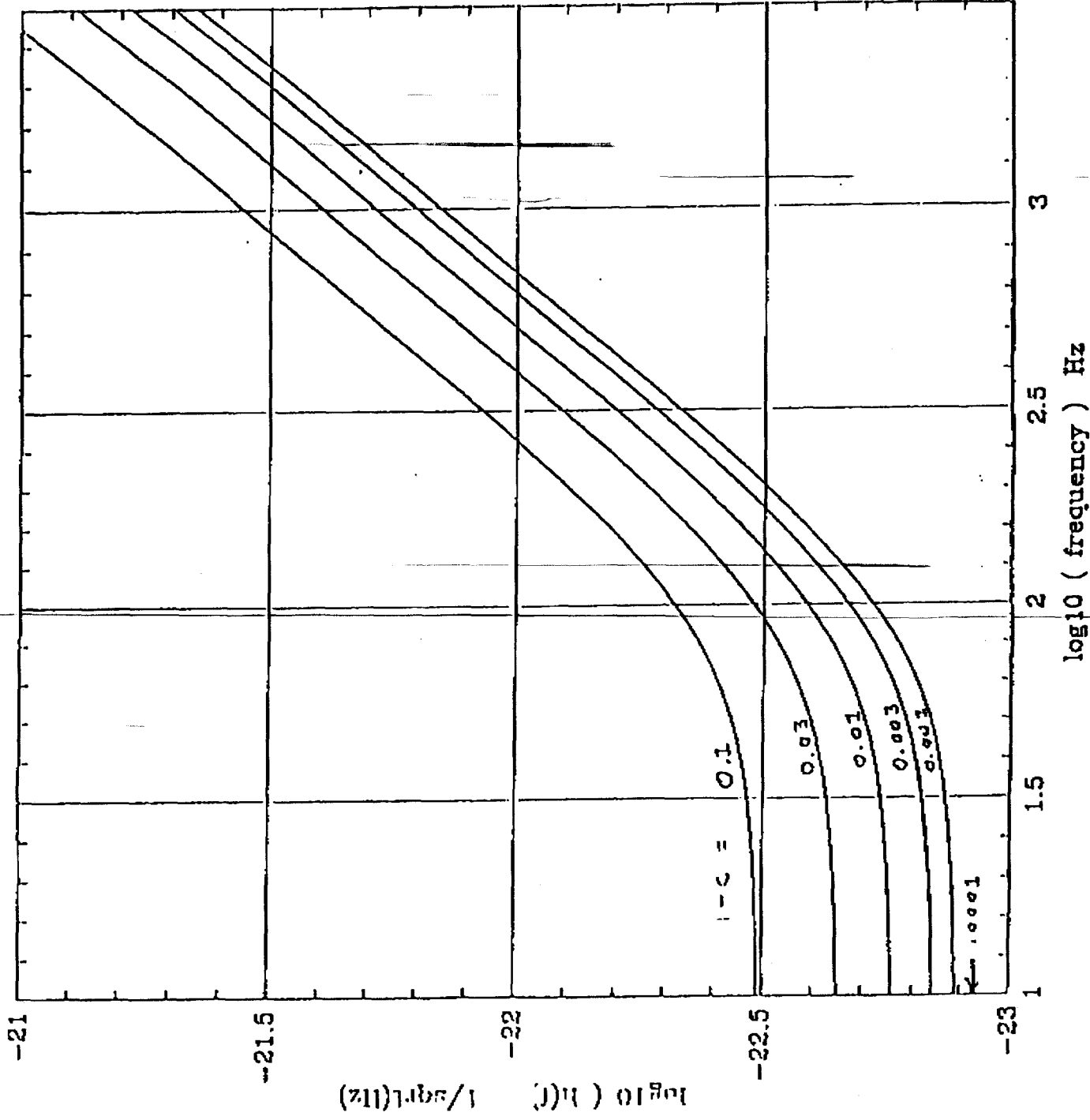
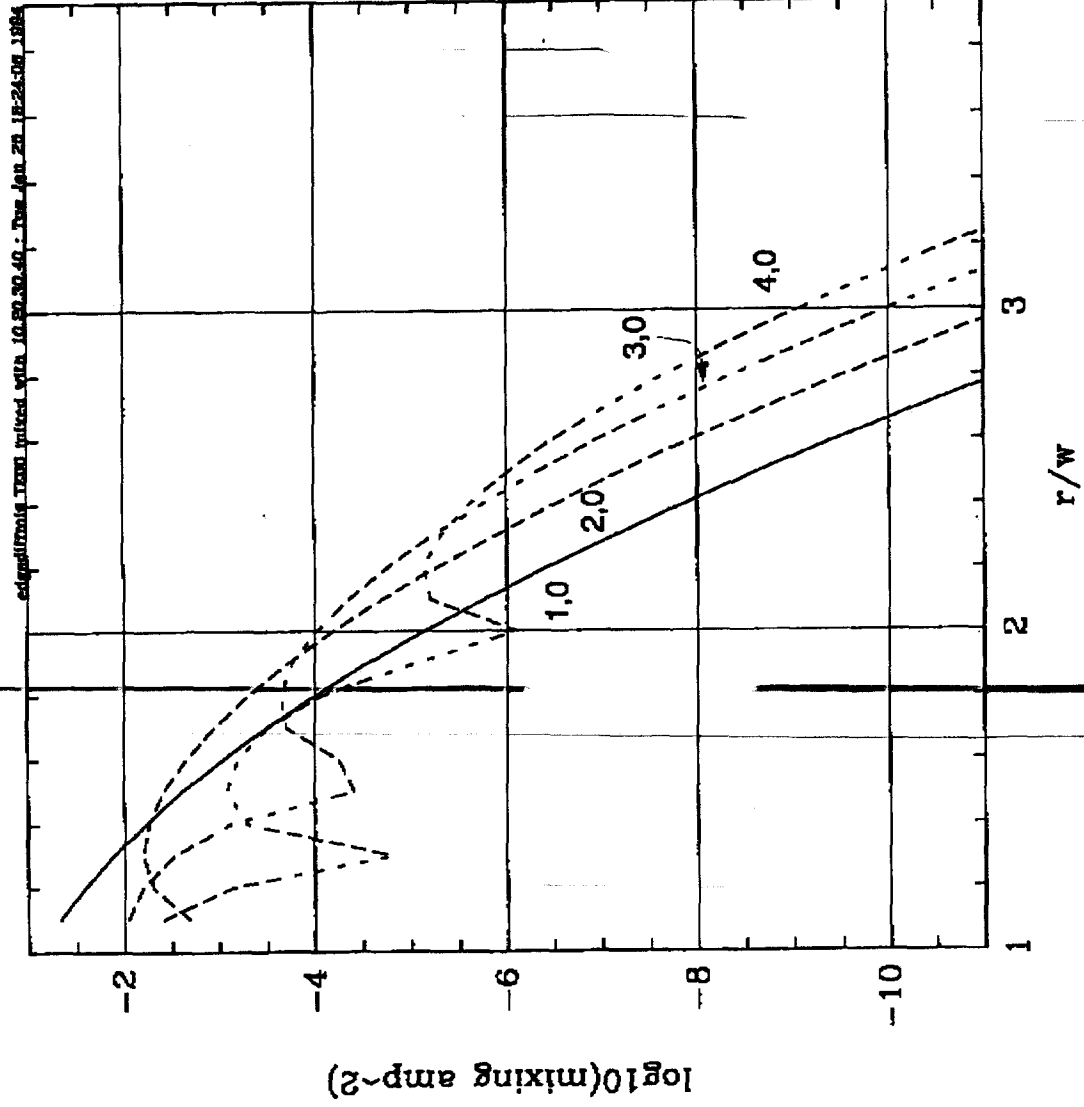


FIG 4





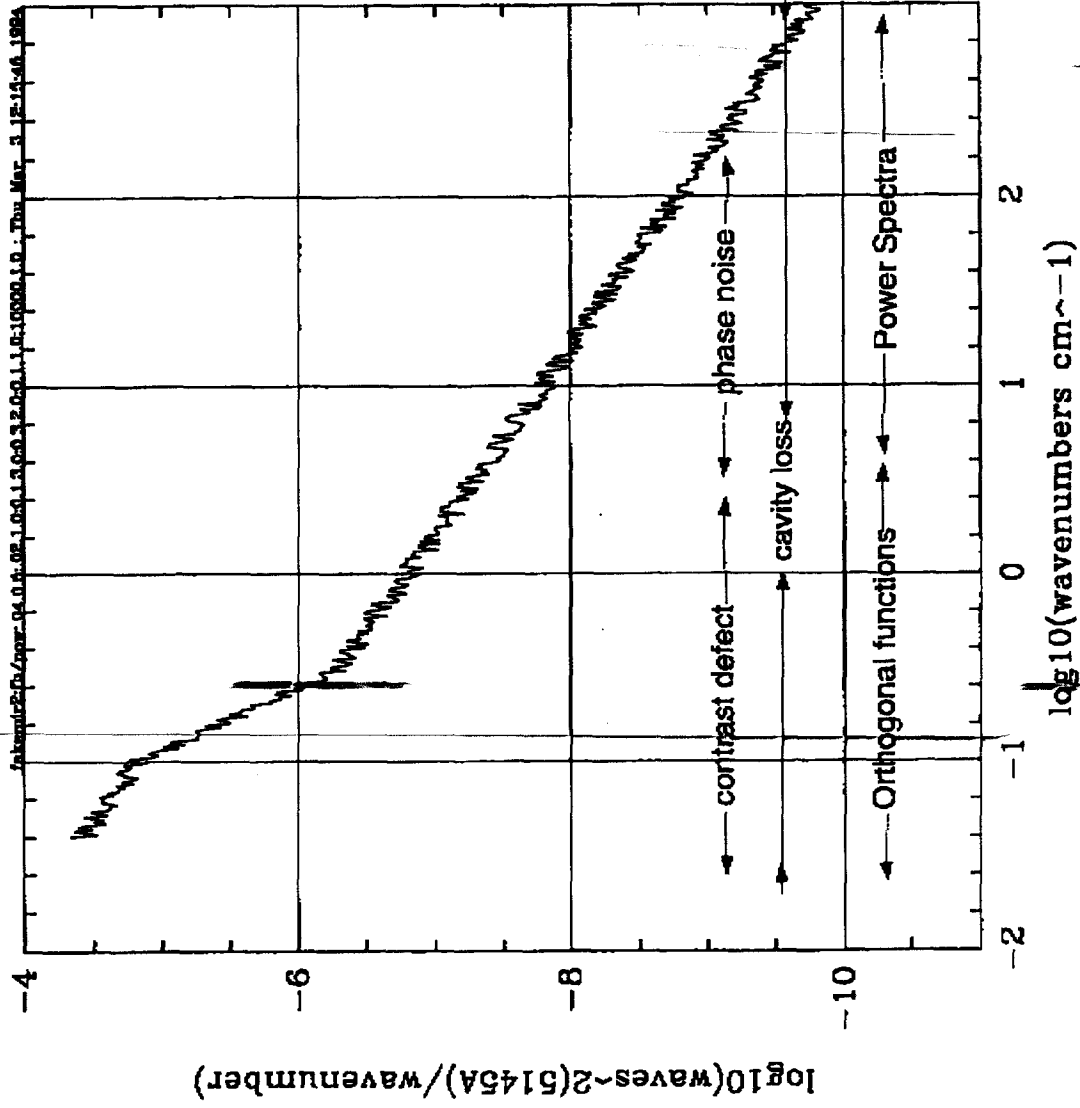


FIG 5

## TECHNIQUES TO MODEL THE CAVITIES AND THE INTERFEROMETER

**Introduction:** Two methods have been developed to calculate the cavity and interferometer performance from the measured wavefronts: the FFT paraxial ray propagation code and an approximate analytic technique that uses the orthogonal function transforms.

The FFT propagation code was developed by the VIRGO group and then improved by LIGO team members (P. Saha, Y. Hefetz and B. Bochner). The program calculates the complex optical field wavefronts directly by using the paraxial form of the Kirchoff diffraction integrals. The current pixel format is 128 by 128 so that the program is primarily useful for wavefront distortions at low spatial frequencies. Routines have been developed to converge on the field solutions in single cavities and a full recycled interferometer using guided iteration methods. The optical components are characterized by their phase maps in reflection (work is currently underway to include transmission phase maps). Once a field solution is determined by the main propagation program, auxiliary programs calculate the interferometer contrast and loss and modal decompositions of the fields, if these are desired.

The FFT propagation code is the best method we have for estimating the overall interferometer and cavity performance once the actual phase maps for the LIGO mirrors have been measured. The program will be the primary tool for making the final evaluation of the LIGO optics after metrology of the components.

The mirror specifications have been primarily determined by analytic perturbation techniques using single perturbed mirrors in modal expansions in a single cavity. The results were later spot checked for consistency with the FFT program. There are several reasons for this. The first is historical, the analytic techniques were developed before the FFT code became available. The second is that the computational efficiency of the analytic methods is much better than the FFT code, a typical Cray run with the FFT program takes about 15 minutes while an entire phase map can be analysed in a matter of seconds by the analytic methods. Finally, the analytic methods give some insight to the types of wavefront distortions significant in influencing the interferometer and cavity performance. The analytic methods, however, have not been developed to determine the performance of a full recycled interferometer.

One of the questions that was not addressed by the analytic methods was the overall contrast defect and loss with several perturbed mirrors in the interferometer. To establish the scaling relations for contrast defect and loss, the FFT program was used with perturbed but statistically similar mirrors at all locations in the interferometer. The results of these runs are that the contrast defect and loss grow linearly with the number of perturbed mirrors in the interferometer. The relation is only approximately true for the flat/spherical cavities where the beam size on the spherical mirror is larger than on the flat. In these cavities the spherical mirror is more critical than the flat to establish the overall cavity loss since the beam samples lower spatial frequencies on the mirror. For the contrast defect estimates this is partially offset by the spatial filtering of the cavity and the perturbation of the wavefront from the unfiltered reflection at the front flat mirror.