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LIGO FACILITIES STUDY REPORT  
(Foundation Response and  
Local Optical Lever Performance)

performed as part of the LIGO

ALIGNMENT SENSING /CONTROL DESIGN TASK

by

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# EXECUTIVE SUMMARY

## Introduction

The LIGO facilities study was undertaken to assist in quantifying the suitability of the currently envisioned corner and end station foundations for operation of local optical lever (LOL) systems capable of initial lock acquisition of the fabry-perot cavities. To acquire lock, a reasonable value for the relative stability of the components composing the LOL is the order of  $1\text{E-}7$  rad. For the LOL to handle operational alignment of the LIGO the required stability would be nearly an order of magnitude better and involve global as well as local pointing parameters.

No detailed information exists regarding the particulars of the building or foundation designs, other than the foundation possessing dimensions of approximately  $50\text{ m} \times 50\text{ m} \times 1\text{ m}$ . Therefore, many assumptions were made to enable an order of magnitude estimate of the foundation's response to tidal, thermal, and mechanical loadings. The results are generally expressed in terms of the relative displacements and rotations of locations on the foundation where an alignment laser, a test mass, and a quadrant detector are arranged as an isosceles triangle of height  $40\text{ m}$  (coincident with the beam tube and the test mass located at the vertex) and base  $4\text{ m}$ . The results are indicative of the behavior of the LOL only. Caution should be observed in attempting to extrapolate the results to accurately estimate the performance of a longer baseline alignment system referenced to the stability of the individual foundations (e.g. rigid body motions of the foundation, resulting from tidal forces or vibrations, do not degrade LOL performance but would affect alignment of any system not coexisting on the foundation).

## Results

The tidal effects analysis indicated that crust strains of the order  $2\text{E-}7$  m/m, induce rigid-body motions of the foundation, and hence are not deleterious to LOL performance.

The thermal effects were found to be separable into a static, upward, central bow of the foundation, resulting from the descending thermal gradient from its top to bottom surface. This effect is approximately  $3\text{ mm}$  in magnitude and will be offset by the nearly inverse gravitational sag of the foundation of approximately  $6\text{ mm}$  with the residual countered by a static alignment. A time varying thermal effect attributable to solar flux heating the surrounding soil, which was assumed to effectively transmit heat and force to the foundation, produces an upward bow. Depending on the exact boundary conditions chosen, the resulting stability of the LOL would typically be  $1\text{E-}8$  rad for the order of a few minutes,  $1\text{E-}7$  rad for the order of tens of minutes, and  $1\text{E-}5$  rad of relative misalignment over a  $4\text{ hr}$  period could be expected. Provided that care is taken in constructing the foundation and facility, such that heat and force are not effectively transmitted from the surrounding soil and wall footings to the foundation, the thermal effects would be reduced to a static offset and some small time varying component associated with the accuracy of the air conditioning controller that was not treated here.

The vibration response of the foundation resting on an elastic half-space, the soil, was found to produce maximum relative LOL misalignments of approximately  $3\text{E-}9$  rad and

4E-9 rad at the fundamental and fourth frequencies of vibration of the foundation of 6.9 Hz and 8.2 Hz, respectively. These responses were determined using a forcing function characteristic of the LIGO Hanford, WA site ambient power spectrum.

### **Conclusions**

The conclusions enabled by the study follow:

1. A corner and end station foundation thickness of approximately 1 m is reasonable.
2. The currently envisioned foundation would be adequate to support LOL stability on the order of 1E-7 rad for a time scale of minutes to facilitate fabry-perot cavity lock acquisition.
3. The currently envisioned foundation would not support LIGO operational alignment, the order of 1E-8 rad, on time scales the order of an hour. The analysis indicates, however, that this level of stability could be expected on time scales of a few minutes.
4. The need for excellent thermal and mechanical decoupling of the foundation from its surroundings is clear.

### **Commentary**

It must be indicated that the dynamic response of the foundation/soil system was not the limiting factor in the above conclusions. This is principally because the model was excited using an input spectrum characteristic of a pristine, very quiet site. The operational vibration spectra of the LIGO corner and end stations will differ significantly in a frequency band from tens of hertz, where induction motors operate or an automobile passing at a 30 m distance may represent a long signal as much as 30 db above background, to hundreds of hertz, where air handling equipment will have acoustic emissions associated with rotor and blade motions. Therefore, minimizing the operational vibration input spectrum to the foundations should be considered as thoughtfully as minimizing the transmission of heat and force from the surrounding soil and wall footings.

# LIGO FACILITIES STUDY REPORT

## (Foundation Response and Local Optical Lever Performance)

### 1.0 INTRODUCTION

This report describes engineering analyses that were performed as part of the LIGO Alignment Sensing/Control Design Task. Although the title indicates a facilities study was performed, the analyses and this report address only a foundation resting upon soil characteristic of LIGO's Hanford, WA site. Foundation parameters similar to those approximated for the LIGO corner and end stations and a simplified local optical lever (LOL) configuration, consistent with the current beam tube design, were chosen for analysis purposes. Ground-borne vibration, thermal, and tidal responses of the foundation and the impact of these responses on the LOL's performance were estimated and will be reported.

### 1.1 OBJECTIVES

The objectives of this study were twofold and consisted of the following:

- Determine the mechanical stability of the LIGO corner and end station foundations with respect to their suitability for supporting LOL systems capable of initial lock acquisition of the fabry-perot cavities.
- Determine the mechanical viability of using the LOL systems as a long-term LIGO alignment tool for maintaining alignment of the cavities on the order of hours.

### 1.2 SOLUTION APPROACH

The finite element method was chosen as the solution approach because of its capability to address the varying classes of problems represented by the facilities study. The three classes of problems considered are eigen value, propagation, and steady-state (pseudo steady-state) problems.

The natural frequencies of vibration of structures and the modes of vibration associated with those frequencies are eigen values and eigen vectors, respectively, of the structure (modal information). The dynamic response of a system is calculated using the input to the system, the modal information, and damping. The time evolution of the temperature field that gives rise to thermal gradients and thermal expansion effects producing structural deformations is a propagation problem. A load that is applied so slowly, without lags that the system response can be approximated as steady-state over long intervals is a pseudo steady-state problem. The diurnal tidal expansion/contraction of the earth's crust can be modeled as steady-state where the interval possessing the maximum crust strain is that of interest.

### 1.3 MECHANICAL LOADING AND ASSUMPTIONS

The standard LIGO random noise vibration spectrum, characteristic of the Hanford, WA site, was used as the excitation function for the dynamic response calculations. Three orthogonal directions of uncorrelated input were applied to the foundation through the soil upon which it rests. Verification of this uncorrelated noise assumption is useful and is in

progress, however, solutions assuming correlated noise were executed and little impact on the results was observed. Input with periods greater than 10 s do not strongly couple to the foundation's response characteristics, hence they are not deleterious to LOL operation and were not considered in these calculations.

Solar radiation-induced thermal input was applied to the soil in the vicinity of the foundation. The analysis assumed that the soil convects to the outside air, the foundation convects to room temperature, the foundation lower surface temperature is fixed at the soil temperature at that location, and that the surrounding soil conducts heat and transmits forces to the foundation. A schematic drawing of this information can be found in Appendix I. Radiation heat transfer is small and is thus neglected.

## 2.0 THE FINITE ELEMENT METHOD

The finite element method is based upon segmenting a continuum into a discrete system of elements possessing a finite number of degrees of freedom whose state variable solution can be numerically managed. A system idealization is developed (a model), equilibrium equations are required to be satisfied on the element scale, element interconnection requirements assure that continuity equations are satisfied across element boundaries, and simultaneous equations are solved for state variables subject to element equilibrium and boundary conditions.

## 2.1 THE FINITE ELEMENT MODELS

Several two dimensional (2D) and three dimensional (3D) models were created to facilitate the analysis. A description of these models and the utility of each follows.

- Eigen Value Problem (Modes and Frequencies)

A 2D model composed of 9-node, isoparametric shell elements describing the foundation and spring elements describing the soil was used to estimate the natural frequencies of vibration, the mode shapes associated with those frequencies, and the dynamic response of the soil/structure model to an input power spectrum applied to the base nodes of the springs. A 3D model, using 20-node elements for both the foundation and soil, was developed to verify the modes and frequencies and hence, validate use of the simpler 2D model for response calculations. The 50 m x 50 m x 1m foundation was modelled as resting on 200 m x 200 m x 10 m of soil. Response calculations using 3D models of soil and structure is prohibitively time consuming. The boundary conditions applied to the 2D model were fixing the vertical and transverse "soil" springs in 6 spatial degrees of freedom (DOF) at their soil ends. The 3D model soil elements were fixed in 3 translational DOF's at the outer perimeter.

- Propagation Problem (Heat Transfer)

A 2D model composed of 4-node, isoparametric shell elements describing the foundation and soil was developed to estimate the effects of solar flux-induced soil heating on the foundations stability. Time varying solar flux impinged on the outside soil, heating the soil which in turn convected heat to the outside air. The air conditioning system was assumed to provide air at a constant temperature to the surface of the foundation, which convected to the air on the top surface, but whose bottom surface was fixed at the soil temperature assumed to exist under the foundation. A 3D model composed of 8-node solid elements was developed to describe both the foundation and soil. The 50 m x 50 m x 1m foundation was modelled as resting on 200 m x 200 m x 10 m of soil. This model accurately describes the system and was of reasonable size to execute for time evolution of the temperature field. The boundary conditions applied to the 2D model was fixing the lower surface temperature at 10 C with the soil elements fixed in 6 spatial DOF's at their outer perimeter. Other parameters assumed for the analysis include convective film coefficients from the foundation to the internal air conditioned air and from the soil to the ambient air of 20 W/m<sup>2</sup>/K. The thermal conductivity between the soil and foundation edge, influenced by detailed footing design and resultant soil fragmentation, was assumed to be 0.1, 0.03,

or 0.001 W/m/K for various solutions. The boundary conditions and special parameters for the 3D cases were identical excepting soil elements were fixed in 3 translational DOF's at the outer perimeter.

- Steady-State Problem (Tidal Distortions)

A 3D model composed of 20-node solid elements was developed to describe the effects of tidal-induced strains of the earth's crust. The 50 m x 50 m x 1m foundation was modelled as resting on 200 m x 200 m x 10 m of soil. The rate of tidal strain is such that the maximum isotropic expansion of the crust may be modeled as isotropic thermal expansion reaching a maximum value at 12 hr. intervals. The boundary conditions applied to this model were to fix the soil elements in 3 translational DOF's at the outer perimeter.

## 2.2 FINITE ELEMENT MODEL MATERIALS PARAMETERS

The following parameters were assumed to describe the foundation, soil, LOL, and facility. The foundation is a monolithic, reinforced-concrete structure with dimensions 50 m x 50 m x 1 m. The LOL consists of an alignment laser, a test mass, and a quadrant detector arranged as an isosceles triangle of height 40 m (coincident with the beam tube) and base 4 m. The materials properties of the foundation and soil are:

PROPERTY	FOUNDATION	SOIL
Elastic modulus	31E9 Pa	230E6 Pa
Poisson ratio	0.18	0.37
Mass density	2.3E3 kg/m <sup>3</sup>	1.5E3 kg/m <sup>3</sup>
Thermal conductivity	0.93 W/m/K	0.33 W/m/K
Thermal expansion	12.6E-6 /K	12.6E-6 /K
Specific heat	650 J/kg/K	800 J/kg/k

The facility walls are thermally and structurally isolated from the foundation, such that wind loading and air handling equipment do not augment the LIGO input spectrum. Wall footings are assumed to transmit forces and heat to the foundation, with some resistance to heat flow assumed for fragmented soil conditions. The facility air conditioning system is assumed to operate continuously and maintain the air, in contact with the foundation's upper surface, at a constant temperature.

### 3.0 RESULTS

Many cases were executed using the various finite element models described in section 2.1. Sensitivity analyses were performed on parameters such as the thermal conductivity between the soil and foundation, generally a strong function of wall footing design and the temperature at the bottom surface of the foundation, a quantity requiring measurement. Because of the lack of design details, all results generated should be considered valid to within an order of magnitude not valid within a factor of 2, for example. For this reason, only those results which offer significant insight into the behavior of the foundation and soil system will be presented.

#### Eigen Value Problem

The modal results of the analysis indicated a rigid body rocking motion of the foundation is expressed at approximately 5.25 Hz. This motion has a negligible impact on the relative displacements and rotations of points on the foundation where LOL components would be fixed, but would negatively interact with objectives such as attempting to use the LOL for LIGO alignment. Because of the aspect ratio of the foundations length and width to its thickness, 10:1, the fundamental vibration characteristics expressed by the foundation are plate bending modes. The first four frequencies of these modes are 6.9 Hz, 7.3 Hz, 7.9 Hz, and 8.2 Hz, representing typical central, bipolar, quadrupolar, and diagonal plate bending mode shapes, respectively. These frequencies and deformed mode shapes can be found in Appendix I. The fundamental and fourth modes were found to possess more than average participatory mass and deform the foundation in a manner most deleterious to LOL alignment stability. The relative displacements of points on the foundation where LOL components would be fixed are negligible, however the relative rotations of these points are of the order  $3E-9$  rad. This is based upon an input excitation spectrum characteristic of the Hanford, WA site and modal damping of 2% of critical, based upon literature cited in the references section of this report. The acceleration transfer functions,  $AZ(f)$  and  $AY(f)$ , associated with nodal locations of LOL components, nodes 62 (laser), 82 (quad detector), and 72 (test mass) indicate gains of approximately 35 and 25 for the modes at frequencies of 6.8 Hz and 8.2 Hz. Plots of the squares of these transfer functions can be found in Appendix I. Response spectra denoted  $AZ(f)$ ,  $UZ(f)$ ,  $RX(f)$ , and  $RY(f)$  in terms of acceleration, displacement, and angular power spectral densities in units of  $\text{microns}^2/\text{s}^4/\text{Hz}$ ,  $\text{microns}^2/\text{Hz}$ , and  $\text{microrad}^2/\text{Hz}$ , respectively can also be found in that Appendix.

#### Propagation Problem

The thermally induced deformations of the foundation result from the thermal gradient that exists from the top surface, influenced by the air conditioning system, and the bottom surface, influenced by the soil temperature underneath the foundation, typically cooler than the soil heated by solar irradiation. Various cases were run using room air temperatures from 20 C to 23 C and using bottom surface temperatures from 10 C to 15.5 C. The central bow of the foundation induced by this gradient is approximately a maximum of 3 mm. This upward bow is offset by the gravitational sag of the foundation which assumes a nearly inverse deformation pattern with a maximum downward



displacement of approximately 6 mm. The residual is expected to be countered as an initial alignment.

A worst case analysis that assumed significant shadowing of the solar flux via the station walls was performed. This is a reasonable assumption owing to the fact that the thermal conductivity of soil is low and a uniform temperature distribution around the perimeter of the station, such as is assumed in other cases analyzed, would only result from direct exposure. In the absence of details of the facilities design, however, this case which assumes complete shadowing is probably too stringent. The resultant in an asymmetric temperature field producing temporally varying relative rotation of nodal locations of LOL components, nodes 1445 (laser), 1370 (quad detector), and 5 (test mass) of XXX E-XX rad over a period less than XXXXX min.

The most representative result obtained assumed time varying solar flux distributed isotropically around the perimeter of the facility. This scenario maintained a relative stability of order 1E-8 rad for a few minutes, 1E-7 rad for tens of minutes, and 1E-5 rad over a period of approximately 4 hours. Plots of the temperature field associated with time step 8, the step of maximum distortion, can be found in Appendix I.

The most favorable case executed assumed that the foundation is nearly perfectly thermally and structurally isolated from its surroundings, excepting the soil upon which it rests. A close approximation of this condition is sought and can only be the result of intensive design and analysis performed by the facilities design team. For this case, the static bow of the foundation was similar to the previous cases but the temporal stability was much improved. Alignment stability of 1E-7 rad was held for the order of XXX min. and 1E-8 rad stability was realized for the order of XXX of mins.

#### Steady-State Problem

The tidal force-induced strains of the earth's crust were modeled as a steady state thermal expansion leading to a maximum strain of 2E-7 m/m. This value was taken from literature presented in section 5.0. Because the modulus of elasticity of the foundation is approximately 135 that of the soil, the expanding or contracting soil does not extend or bend the foundation. Because the area and mass of soil underneath and surrounding the foundation is far greater than that of the foundation, the soil is capable of moving the foundation as a rigid body, however, and does so. The relative displacements and rotations of points on the foundation where LOL components would be fixed are negligible. The deformed configuration of the soil and foundation can be found in Appendix I. The absolute value for the heave of the foundation resulting from this effect was neither well defined by this analysis nor in the literature, but is expected to be on the order of millimeters.

#### 4.0 CONCLUSIONS AND COMMENTARY

This "order of magnitude" facilities study was based upon very little information about the corner or end station facilities. Many reasonable assumptions were introduced to produce solutions that should not be regarded as accurate estimates of a particular facility's behavior but rather should be viewed as a general indicator of what can be expected from typical facilities if careful design and construction practices are not followed. At the onset of this study, a couple of very basic questions were posed. These questions were answered and those answers are repeated in numbers 1 through 3 below. Number 4 of the following list is a summation of the qualitative results of this work.

1. A corner and end station foundation thickness of approximately 1 m is reasonable.
2. The currently envisioned foundation would be adequate to support LOL stability on the order of  $1\text{E-}7$  rad for a time scale of minutes to facilitate fabry-perot cavity lock acquisition.
3. The currently envisioned foundation would not support LIGO operational alignment, the order of  $1\text{E-}8$  rad, on time scales the order of an hour. The analysis indicates, however, that this level of stability could be expected on time scales of a few minutes.
4. The need for excellent thermal and mechanical decoupling of the foundation from its surroundings is clear.

It must be indicated that the dynamic response of the foundation/soil system was not the limiting factor in the above conclusions. This is principally because the model was excited using an input spectrum characteristic of a pristine, very quiet site. The operational vibration spectra of the LIGO corner and end stations will differ significantly in a frequency band from tens of hertz, where induction motors operate or an automobile passing at a 30 m distance may represent a long signal as much as 30 db above background, to hundreds of hertz, where air handling equipment will have acoustic emissions associated with rotor and blade motions. Therefore, minimizing the operational vibration input spectrum to the foundations should be considered as thoughtfully as minimizing the transmission of heat and force from the surrounding soil and wall footings.

## 5.0 REFERENCE MATERIALS

- Eigen Value Problem (Modes and Frequencies)
- 1. Dames and Moore, "Report of Geotechnical Survey, LIGO Project, Hanford WA," 1993.
- 2. R.V. Whitman and F.E. Richart, "Design Procedures for Dynamically Loaded Foundations," 1967.
- 3. H.B. Seed et al., "Soil Moduli and Damping Factors for Dynamic Response Analyses," 1970.
- 4. K.J. Bathe, "Finite Element Procedures in Engineering Analysis," 1982.
- Propagation Problem (Heat Transfer)
- 1. F. Kreith, "Principles of Heat Transfer," 1967.
- 2. R.V. Dunkle and J.T. Gier, "Selected Spectral Characteristics of Solar Collectors," Transactions of the Tucson Conference on Applied Solar Energy, 1957.
- 3. R. Weiss, memorandum to W.E. Althouse, memo no. THERMAL121788.tex, 1988.
- 4. 1993 ASHRAE Handbook-Fundamentals, SI edition, 1993.
- 5. R. Sugahara et al., "Measurement of the Seismic Motion and the Displacement of the Floor in the TRISTAN Ring," 1993.
- Steady-State Problem (Tidal Distortions)
- 1. J. Berger and J. Levine, "The Spectrum of the Earth Strain from 10E-8 to 10E-2," 1974.
- 2. J.N. Brune and J. Oliver, "The Seismic Noise of the Earth's Surface," 1967.
- 3. R. Weiss, memorandum to W.E. Althouse, memo no. THERMAL121788.tex, 1988.
- 4. V. Braginsky, memoranda to R. Vogt and S. Whitcomb and private communications, 1994.

# APPENDIX I

(Vibration Modes and Frequencies)

EIGEN VALUE PROBLEM

## INPUT / OUTPUT QUANTITIES

### INPUT POWER SPECTRUM

- Hanford site ambient ground spectrum
- Displacement power
- Acceleration power
- Standard power units e.g. [microns<sup>2</sup>/Hz]

### TRANSFER FUNCTION

- ABSOLUTE acceleration frequency response function,  $H(f)$
- Plotted values are  $H(f)/2$

### OUTPUT POWER SPECTRA

- RELATIVE displacement power {  $S_{zz}(f) = S_{yy}(f) - S_{xx}(f)$  }
- Standard units [microns<sup>2</sup>/Hz]





















