New Folder Name Effect of Asymmetry

Effect of Asymmetry on Contrast of Interferometer

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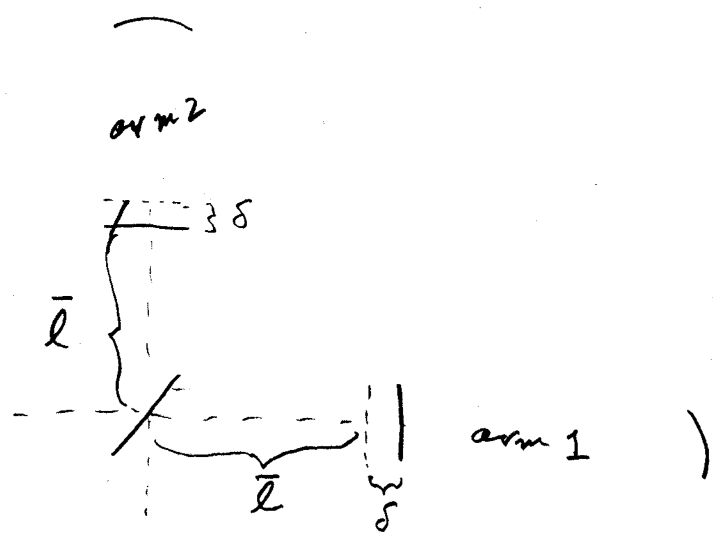
There are two fields to keep track of (in each arm), the field E_N directly reflected from the near mirror of the FP cavity and the field E_c which leaks out of the FP cavity. We have $|E_c| \simeq 2|E_N|$ for overcoupled cavities.

Consider the interferometer as shown in Figure 1.



The power leaving the beam splitter (antisymmetric port) is

$$P_A = 2\pi \int_0^\infty \rho\, d\rho \,\left| E_{c1}(\rho,0) + E_{N1}(\rho,0) - E_{c2}(\rho,0) - E_{N2}(\rho,0) \right|^2 . \qquad (1)$$

Here ρ is the transverse distance from the beam center, measured in units of the beam waist, and 0 refers to the origin of the longitudinal coordinate at the beam splitter. The indices 1 and 2 refer to the respective arms of the interferometer. We can expand the expression in equation (1) to obtain

$$\begin{aligned} P_A = \; & \overset{2}{\langle E_{c1} | E_{c1} \rangle} + \overset{\frac{1}{2}}{\langle E_{N1} | E_{N1} \rangle} + \overset{2}{\langle E_{c2} | E_{c2} \rangle} + \overset{\frac{1}{2}}{\langle E_{N2} | E_{N2} \rangle} \\ & - 2\,\mathrm{Re}\left\{ \langle E_{c1} | E_{c2} \rangle + \langle E_{N1} | E_{N2} \rangle + \langle E_{N1} | E_{c2} \rangle + \langle E_{N2} | E_{c1} \rangle \right\} \\ & + 2\,\mathrm{Re}\left\{ \langle E_{N1} | E_{c1} \rangle + \langle E_{N2} | E_{c2} \rangle \right\} . \end{aligned} \tag{2}$$

Here we have introduced the notation

$$\langle E_A | E_B \rangle = 2\pi \int_0^\infty \rho\, d\rho \; E_A^*(\rho, 0)\, E_B(\rho, 0) \tag{3}$$

In the vicinity of a beam waist we may write the normalized TEM_{00} field pattern as

$$u_0(\rho, z \ll z_0) = \left[\frac{2}{\pi} \Gamma(z) \right]^{1/2} \exp\left\{ -\rho^2 \left[\Gamma(z) + i\Phi(z) \right] \right\} \tag{4}$$

where z_0 is the confocal parameter, $\Phi(z) = \frac{z}{z_0}$, and $\Gamma(z) = (1 - \Phi^2)$. u_0 is normalized such that

$$\langle u_0(\rho, z) | u_0(\rho, z) \rangle = 1 \tag{5}$$

We assume the waists for the fields E_{N1} and E_{N2} are a distance $\bar{l}$ from the beam splitter and for E_{c1} and E_{c2} the waists are at the input mirrors of the respective cavities. Table 1 gives the field amplitudes at the beam splitter and Table 2 gives the appropriate integrals for substitution into equation (2).

E_{c1}	$-\sqrt{2}\,\mu_0(\rho, \bar{z}+\delta)$
E_{c2}	$-\sqrt{2}\,\mu_0(\rho, \bar{z}-\delta)$
E_{N1}	$\frac{1}{\sqrt{2}}\,\mu_0(\rho, \bar{z}+2\delta)$
E_{N2}	$\frac{1}{\sqrt{2}}\,\mu_0(\rho, \bar{z}-2\delta)$

Table 1: Field amplitudes at beam splitter.

$\langle E_{c1} \vert E_{c1} \rangle$	2
$\langle E_{c2} \vert E_{c2} \rangle$	2
$\langle E_{N1} \vert E_{N1} \rangle$	$\frac{1}{2}$
$\langle E_{N2} \vert E_{N2} \rangle$	$\frac{1}{2}$
$2\,\mathrm{Re}\{\langle E_{c1} \vert E_{c2} \rangle\}$	$4\left[1-\left(\frac{\delta}{z_0}\right)^2\right]$
$2\,\mathrm{Re}\{\langle E_{N1} \vert E_{N2} \rangle\}$	$1\left[1-4\left(\frac{\delta}{z_0}\right)^2\right]$
$2\,\mathrm{Re}\{\langle E_{N1} \vert E_{c2} \rangle\}$	$-2\left[1-\frac{9}{4}\left(\frac{\delta}{z_0}\right)^2\right]$
$2\,\mathrm{Re}\{\langle E_{N2} \vert E_{c1} \rangle\}$	$-2\left[1-\frac{9}{4}\left(\delta/z_0\right)^2\right]$
$2\,\mathrm{Re}\{\langle E_{N1} \vert E_{c1} \rangle\}$	$-2\left[1-\left(\delta/z_0\right)^2\right]$
$2\,\mathrm{Re}\{\langle E_{N2} \vert E_{c2} \rangle\}$	$-2\left[1-\left(\delta/z_0\right)^2\right]$

Table 2: Field integrals

We then obtain

$$P_A \simeq 3\left(\delta/z_0\right)^2 \qquad (6)$$

For the 40-m interferometer we have

$$w_0 = 2.15 \times 10^{-3}\ m \qquad (7)$$

$$z_0 = 28\ m \qquad (8)$$

$$P_A \simeq 3.4 \times 10^{-4}\left(\frac{\delta}{30\,cm}\right)^2 \qquad (9)$$

Math Notes

$$2\pi \int_0^\infty \rho\, d\rho\; u_0^* u_0 = \frac{2}{\pi}\Gamma(z)\cdot 2\pi \int_0^\infty \rho\, d\rho\, e^{-2\rho^2\Gamma(z)}$$

$$= 4\Gamma(z)\int_0^\infty \frac{1}{2}\cdot 2\rho\, d\rho\, e^{-2\Gamma(z)\rho^2}$$

$$= 2\Gamma(z)\int_0^\infty dv\, e^{-2\Gamma(z)v}$$

$$= \frac{2\Gamma(z)}{2\Gamma(z)}\left[1\right]$$

$$E_{C1} = \sqrt{2}\, u_0(\rho, \bar{\ell}+\delta) \qquad E_{N1} = \frac{1}{\sqrt{2}} u_0(\rho, \bar{\ell}+2\delta)$$

$$E_{C2} = \sqrt{2}\, u_0(\rho, \bar{\ell}-\delta) \qquad E_{N2} = \frac{1}{\sqrt{2}} u_0(\rho, \bar{\ell}-2\delta)$$

$$\boxed{\begin{aligned} \langle E_{C1}|E_{C1}\rangle &= \langle E_{C2}|E_{C2}\rangle = \cancel{4}2 \\ \langle E_{N1}|E_{N1}\rangle &= \langle E_{N2}|E_{N2}\rangle = 1/2 \end{aligned}}$$

$$\Gamma(\bar{\ell}+\delta) + \Gamma(\bar{\ell}-\delta) = 2 - \left(\frac{\bar{\ell}+\delta}{z_0}\right)^2 - \left(\frac{\bar{\ell}-\delta}{z_0}\right)^2$$

$$= 2 - 2\left(\frac{\bar{\ell}}{z_0}\right)^2 - 2\left(\frac{\delta}{z_0}\right)^2$$

$$\langle E_{c1}|E_{c2}\rangle = 2\cdot\langle u_0(\rho,\bar{l}+\delta)\,|\,u_0(\rho,\bar{l}-\delta)\rangle$$

$$2\,\mathrm{Re}\{\langle E_{c1}|E_{c2}\rangle\} = \frac{8\left[\Gamma(\bar{l}+\delta)\Gamma(\bar{l}-\delta)\right]^{1/2}}{\Gamma(\bar{l}+\delta)+\Gamma(\bar{l}-\delta)}\cdot\frac{1}{1+\dfrac{\left(\frac{2\delta}{z_0}\right)^2}{\left[2-\left(\frac{\bar{l}+\delta}{z_0}\right)^2-\left(\frac{\bar{l}-\delta}{z_0}\right)^2\right]^2}}$$

$$\simeq \frac{8\,\Gamma(\bar{l}+\delta)\Gamma(\bar{l}-\delta)}{\Gamma(\bar{l}+\delta)+\Gamma(\bar{l}-\delta)}\left\{1-\frac{1}{4}\left(\frac{2\delta}{z_0}\right)^2\right\}$$

$$\simeq 4\left\{1-O\left\{\left(\frac{\delta}{z_0}\right)^4\right\}\right\}\left\{1-\left(\frac{\delta}{z_0}\right)^2\right\}$$

$$\boxed{2\,\mathrm{Re}\{\langle E_{c1}|E_{c2}\rangle\} \simeq 4\cdot\left[1-\left(\frac{\delta}{z_0}\right)^2\right]}$$

$$\langle E_{N1}|E_{N2}\rangle = \langle u_0(\rho,\bar{l}+2\delta)\,|\,u_0(\rho,\bar{l}-2\delta)\rangle$$

$$\boxed{\begin{aligned}2\,\mathrm{Re}\{\langle E_{N1}|E_{N2}\rangle &\simeq \cancel{2}\left[1-\left(\frac{2\delta}{z_0}\right)^2\right]\\ &\simeq \cancel{2}\left[1-4\left(\frac{\delta}{z_0}\right)^2\right]\end{aligned}}$$

$$(-)\quad \langle E_{N1}|E_{C2}\rangle = -\cancel{8}\,\langle \mu_0(\rho,\bar{\ell}+2\delta)\,|\,\mu_0(\rho,\bar{\ell}-\delta)\rangle$$

$$(-)\quad \langle E_{N2}|E_{C1}\rangle = -\cancel{8}\,\langle \mu_0(\rho,\bar{\ell}-2\delta)\,|\,\mu_0(\rho,\bar{\ell}+\delta)\rangle$$

$$(+)\quad \langle E_{N1}|E_{C1}\rangle = -\cancel{8}\,\langle \mu_0(\rho,\bar{\ell}+2\delta)\,|\,\mu_0(\bar{\ell}+\delta)\rangle$$

$$(+)\quad \langle E_{N2}|E_{C2}\rangle = -\cancel{8}\,\langle \mu_0(\rho,\bar{\ell}-2\delta)\,|\,\mu_0(\rho,\bar{\ell}-\delta)\rangle$$

$$\boxed{\begin{aligned}2\,\mathrm{Re}\{\langle E_{N1}|E_{C2}\rangle\} &= -\cancel{8}\cdot 4\cdot \tfrac{1}{2}\left\{1-\tfrac{1}{4}\left(\tfrac{3\delta}{z_0}\right)^2\right\}\\ &= -\cancel{4}2\left[1-\tfrac{9}{4}\left(\tfrac{\delta}{z_0}\right)^2\right] = 2\,\mathrm{Re}\{\langle E_{N2}|E_{C1}\rangle\}\end{aligned}}$$

$$\boxed{2\,\mathrm{Re}\{\langle E_{N1}|E_{C1}\rangle\} = -\cancel{4}2\left[1-\left(\tfrac{\delta}{z_0}\right)^2\right] = 2\,\mathrm{Re}\{\langle E_{N2}|E_{C2}\rangle\}}$$

$$\Rightarrow P_A = \overset{5}{\cancel{16}} - \overset{4}{\cancel{8}}\left[1-(\delta/z_0)^2\right] - \cancel{2}\left[1-4(\delta/z_0)^2\right]$$

$$+\overset{4}{\cancel{8}}\left[1-\tfrac{9}{4}(\delta/z_0)^2\right] - \overset{4}{\cancel{8}}\left[1-(\delta/z_0)^2\right]$$

$$= [4+4-9+4]\,(\delta/z_0)^2 = 3\,(\delta/z_0)^2$$