New Folder Name Simplified Analysis of the Shot Noise Limited Sensitivity of an Externally Modulated Interferometer

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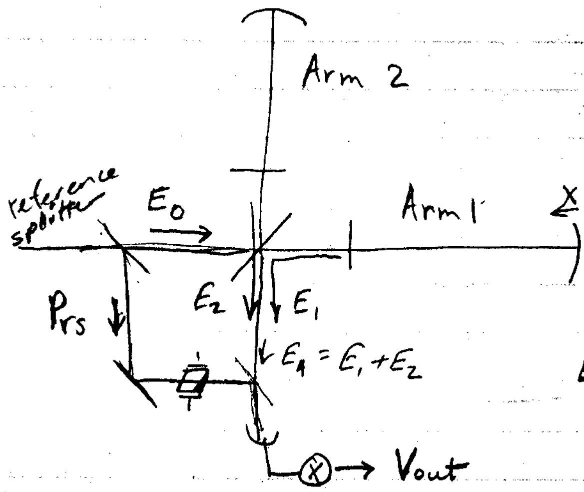
David, here is my "simplified analysis of the shot noise limited sensitivity of an externally modulated interferometer." It exceeds by a factor of three the one-page limit I had specified. To try to atone for this, I'm willing to talk you through it. I think you will find that most of the space is consumed by diagrams and words of explanation, rather than cryptic equations.

— Martin.

Simplified Analysis of the Shot-Noise Limited Sensitivity of an Externally Modulated Interferometer.



E_0 - field incident on BS
E_1 - field reflected onto PD from arm 1

Assume length of Arm 2 fixed. Displacement of Arm 1 end mirror is x.

$$|E_1| = \frac{1}{2}|E_0| \quad (1 \text{ transmission}, 1 \text{ refl. of BS})$$

$$\frac{dE_1}{dx} = 4k \frac{1}{1-r_{in}} i E_1$$

r_{in} - FP input mirror amplitude reflectivity $= \sqrt{1-0.03} = 0.985$

carrier at antisymmetric port in absence of reference splitter

$$E_A(x) = E_1(x) + E_2$$

$$\frac{dE_A}{dx} = \frac{4k}{1-r_{in}} i\left(\frac{1}{2}|E_0|\right)$$

factor 4 because $\frac{d\phi}{dx} = 2kx$ and because phase changes twice as fast outside cavity

Let $E_{rs1} = \sqrt{2P_{rs}/\epsilon_0}\, J_1(\Gamma_{opt})$ $\quad$ P_{rs} - power picked off

$\Gamma_{opt} \cong 1.84$

then, for $x=0$, field on photodiode is:

$$E_{rs1} e^{i\omega_m t} - E_{rs1} e^{-i\omega_m t}$$

Instantaneous Power on photodiode, in presence of carrier field $E_A(x)$:

$$P_{PD}(t) = \frac{1}{2}\epsilon_0 \left| E_{rs1} e^{i\omega_m t} + E_A(x) - E_{rs1} e^{-i\omega_m t} \right|$$

Note $|z_1 + z_2 + z_3|^2 = |z_1|^2 + |z_2|^2 + |z_3|^2 + \sum_{j>i} 2\,\mathrm{Re}\{z_i z_j^*\}$

$$P_{PD}(t) = \tfrac{1}{2}\epsilon_0 \Big[2\,\mathrm{Re}\Big\{ E_A(x)\, \underbrace{E_{rs_1}}_{\in \mathbb{R}} \underbrace{\left(e^{-i\omega_m t} - e^{i\omega_m t}\right)}_{-2i\sin\omega_m t} \Big\}$$

$$+ \text{ DC \& } 2\omega_m \text{ terms} \Big]$$

$$= 2\epsilon_0\, \mathrm{Im}\{E_A(x)\}\, E_{rs_1} \sin\omega_m t \quad + \text{ DC \& } 2\omega_m \text{ terms}$$

Suppose photo diode / demod do the following:

$$V_{out} = \frac{1}{T}\int_{t-T}^{t} P(t)\sin\omega_m t\, dt$$

then $V_{out}(x) = \epsilon_0 \mathrm{Im}\{E_A(x)\}\, E_{rs_1}$

$$\frac{d V_{out}(x)}{dx} = \frac{2k}{1-r_{in}} \epsilon_0 |E_0|\, E_{rs_1}$$

$$= \frac{4k}{1-r_{in}} \underbrace{P_{laser}}_{\text{incident laser power}} \Big(\sqrt{1-\underbrace{R_{rs}}_{\text{reference splitter (power) reflectivi}}}\Big)\Big(\sqrt{R_{rs}}\, J_1(\Gamma_{opt})\Big)$$

Shot noise:

$$S^{1/2}_{P_{PD}}(f) = \sqrt{2 h\nu P_{rs}}$$

$$S^{1/2}_{V_{out}}(f) = \sqrt{h\nu P_{rs}}$$ since half of shot noise power goes into quadrature demod phase

Finally

$$S_x^{1/2}(f) = \frac{S^{1/2}_{V_{out}}(f)}{\frac{dV_{out}}{dx}} = \frac{\sqrt{h\nu P_{laser} R_{rs}}}{\frac{4k}{1-r_{in}} P_{laser} \sqrt{1-\cancel{R_{rs}}} \sqrt{R_{rs}}\, J_1(\Gamma_{opt})}$$

$$S_x^{1/2}(f) = \sqrt{\frac{h\nu}{P_{laser}}} \quad \frac{1-r_{in}}{4k} \quad \frac{1}{J_1(\Gamma_{opt})}$$

$$S_h^{1/2}(f) = S_x^{1/2}(f) / 4000\,m$$

Numerically, for

rec. gain

$P_{laser} = 1.54\,W \cdot 37.9$

$J_1(\Gamma_{opt}) = 0.582$

$h\nu = 3.87 \times 10^{-19}\ J$

$k = 1.22 \times 10^{7}\ m^{-1}$

$r_{in} = 0.985$

$S_x^{1/2} = 4.33 \times 10^{-20}$

$S_h^{1/2} = 1.08 \times 10^{-23}$