New Folder Name Asymmetry Shot Noise

Mach-Zehnder Asymmetry Shot Noise Comparison (M.R., 11/5/93)

This memo describes the formulae used in a pair of programs, `mzlist` and `aslist`, which were used to compare the shot noise sensitivity of the Mach-Zehnder and Asymmetry gravitational signal extraction schemes in August, 1993. Both programs currently reside in `~martin/recomb/shotn/asym_MZ`.

Mach-Zehnder Shot Noise

For the analysis of the Mach-Zehnder externally modulated interferometer, the formula used for the shot noise is

$$S_x^{\frac{1}{2}}(f) = \frac{1 - r_{in}}{4k}\sqrt{\frac{P_{DC} + P_{p0}}{P_B J_{1max}^2 P_{p0}}}$$

where r_{in} is the arm cavity input mirror (amplitude) reflectivity, P_{DC} is the waste power on the photodiodes (due to contrast defect at the main beam splitter and at the Mach-Zehnder beam splitter), P_B is the power incident on the beam splitter from the recycling mirror side, k is the wave number of the light, and J_{1max} is the maximum value of the Bessel function of order 1. This expression for the shot noise is derived under the assumptions that $1 - r_{in} \ll 1$ and that the visibility of the arm cavities is small. The expression below for shot noise in the asymmetry scheme is derived under the same assumptions. Power is measured in units of photons/sec. The "mode-matched modulation power" is

$$P_{p0} = C_{MZ} P_{in} R_p$$

Of the power picked off from inside the recycling cavity and phase modulated in the reference arm of the Mach-Zehnder, it is the part which has the right spatial distribution to interfere with the carrier field rejected from the interferometer when the latter is distorted by a gravitational wave. Thus this part of the picked off power contributes to both signal and noise, whereas the remainder, $(1 - C_{MZ})P_{in}R_p$, only contributes to the noise. Then

$$P_{DC} = P_B\frac{1 - C}{2} + (1 - C_{MZ})P_{in}R_p$$

where C is the contrast, C_{MZ} is the Mach-Zehnder contrast, R_p is the pick-off reflectivity, and P_{in} is the power just inside the recycling mirror, traveling towards the beam splitter. P_{in} differs from P_B only in that the latter is attenuated by transmission through the pick-off:

$$P_B = P_{in}(1 - R_p)$$

P_{in} is found in terms of known quantities by

$$P_{in} = \frac{P_{inc}}{\mathcal{L}_{cav} + \left(\frac{1-C}{2}\right) + 2R_p}$$

where P_{inc} is the power incident on the outside of the recycling mirror and $\mathcal{L}_{cav}$ is the loss on reflection from an arm cavity.

The program `mzlist` contains a list of sets of values for the parameters $1 - C$, $\mathcal{L}_{cav}$, r_{in} and C_{MZ}. For one set of values at a time, it adjusts R_p so as to minimize $S_x^{\frac{1}{2}}(f)$. This is then recorded as the shot noise for that set of parameter values, and the program moves on to the next set.

Asymmetry Shot Noise

For gravitational wave signal extraction by asymmetry, the expression used for the shot noise is

$$S_x^{\frac{1}{2}}(f) = \frac{1 - r_{in}}{4k}\sqrt{\frac{P_{DC} + \frac{3}{2}P_S}{P_B(P_{S0}/2)}}$$

where P_S is the power incident on the photodiode in the first RF sidebands (we assume that the second and higher order sidebands are reflected back toward the laser by the recycling mirror) and P_{S0} is the part of that power which has the correct spatial distribution to interfere with the carrier field rejected from the interferometer when the latter is distorted by a gravitational wave. P_{DC} is due to contrast defect at the main beam splitter:

$$P_{DC} = P_B\frac{1 - C}{2}$$

We assume negligible pick-off reflectivity, so that

$$P_B = P_{in}$$

with P_{in} defined as before. P_{inc} is the power in the carrier incident on the interferometer, so that

$$P_{inc} = J_0^2(\Gamma) P_{laser}$$

where $J_0(\Gamma)$ is the zeroth order Bessel function evaluated at the modulation index, and P_{laser} is the power incident on the phase modulator. We define a round-trip loss $\mathcal{L}_S$ to represent loss to the sidebands in the recycling cavity at the antisymmetric port due to sideband mode deformation caused by the degeneracy of the recycling cavity. Light which exits at the antisymmetric port due to this mechanism is assumed to have the wrong spatial distribution to interfere with the carrier and thus generates noise but no signal. Then the total noise-producing sideband power is given by:

$$P_S = P_{S0} + 2P_{laser} J_1^2(\Gamma) \frac{t_{rec}\sqrt{\mathcal{L}_S}}{1 - r_{rec}\sqrt{1-\mathcal{L}_S}}$$

where

$$t_{rec} = \sqrt{\mathcal{L}_{cav} + \frac{1-C}{2}}$$

is the recycling mirror (amplitude) transmission (if we assume that the carrier is optimally coupled at the recycling mirror), $r_{rec} = \sqrt{1 - t_{rec}^2}$ is the recycling mirror amplitude reflectivity (we neglect losses in this mirror), and $J_1(\Gamma)$ is the Bessel function of order one evaluated at the modulation index. Then we find

$$P_{S0} = 2P_{laser} J_1^2(\Gamma) \frac{t_{rec}\sin\alpha}{1 - r_{rec}\sqrt{1-\mathcal{L}_S}\cos\alpha}$$

and

$$\alpha = \frac{\omega_{mod}}{c}(l_2 - l_1)$$

is the asymmetry angle, where ω_{mod} is the angular modulation frequency, c is the speed of light, and $l_2 - l_1$ is the difference in the distances between the beam splitter and the second and first cavity input mirrors, respectively. The best shot noise performance occurs for that value of α,

$$\alpha_{opt} = \cos^{-1}\left(r_{rec}\sqrt{1 - \frac{1-C}{2}} \right)$$

which maximizes the transmission of sideband power to the interferometer antisymmetric port. We use $\alpha = \alpha_{opt}/2$ because of anticipated vacuum system and modulation frequency constraints.

The program `aslist` contains a list of sets of values for the same parameters as are used in `mzlist` (its results are obviously unaffected by the value of C_{MZ}, though) as well as for the parameter $\mathcal{L}_S$. For each set of parameter values it finds the optimum shot noise by adjusting the modulation index.