New Folder Name Strain Effects

June 2, 1993

DC Strain Effects in Anelastic Oscillators

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Abstract

Possible effects of an externally applied DC strain on the damping of a mechanical oscillator are discussed. A special beam oscillator was designed to check for such effects. We describe experimental results obtained with a steel spring.

LIGO-T930025-00-R

I. Introduction

To reduce thermal noise in a interferometric gravitational wave detector, pendulums of high quality factor Q are used to suspend the test masses. Construction of such pendulums involves using fine suspension wires in order to reduce loss from internal friction in the wires. In addition, test masses as heavy as allowed by the strength of the wires should be chosen to maximize the fraction of frictionless gravitational restoring force. Then, a simple model predicts that the Q of the pendulum should exceed that of an ordinary oscillator made from the same wire material by a factor of the ratio of the gravitational spring constant mg/ℓ to the spring constant of the pendulum wire.[1]

Implicit in this prediction is the assumption that the internal friction in the pendulum wire is the same as it would be if the wire were not under stress. However, in such pendulums the static stress in the wires due to the weight of the test masses will be large, causing an appreciable DC strain. This naturally poses the question: will this DC strain influence the Q of the pendulum? And if so, by what mechanisms might this occur?

A variety of experiments are necessary to understand the best way to design suspensions for gravitational test masses. Here, we discuss a method of addressing the general question of the strain dependence of internal friction in an especially clean way. We report a set of experiments carried out on a specially designed beam oscillator, which has the property that it can be operated so that the DC strain due to the weight of the inertia element can be altered drastically, without any other effect on the operation of the oscillator. We can thus examine whether DC strain, by itself, necessarily causes changes in the internal friction of the spring.

Studies of DC stress effects on internal friction in metals have a long history. One well-known mechanism is the motion of stress-induced dislocations, known as the Bordoni relaxation.[2] Experiments to study this effect have typically used externally applied DC torques to torsional pendulums; measured Q's were then compared with those without DC bias torque. Despite the difference between torsional and ordinary pendulums, we believe that the basic Bordoni relaxation mechanism should be applicable to both cases.

The Bordoni effect is usually associated with previous cold work of the spring. However, the Bordoni relaxation is not the only mechanism that could produce a DC strain effect on Q. We are especially interested in the possibility of any effects that might be unavoidably associated with anelasticity itself, even in materials that have not been cold-worked. This is the reason we have chosen to design our special-purpose beam oscillator.

The following section outlines some elementary ways in which the DC force of gravity can affect the Q of a mass-spring oscillator. Section III describes the design of our angle-adjustable beam oscillator. Results of our measurements are discussed in Section IV. Section V describes some important systematic errors. The final section contains a few concluding remarks.

II. DC Force Effect On Damping

A. *The Simple Mass-Spring Oscillator*

Let's start by considing a simple oscillator with a mass M and a spring of constant k as shown in Figure 1. In (a) the oscillator lies on a smooth frictionless horizontal surface, so the weight Mg is irrelevant. On the other hand, in (b) the oscillator is hanging from a "roof"; the weight of the mass thus produces a stretch of the spring given by

$$\Delta X_0 = Mg/k. \tag{1}$$

Despite such a DC strain, it is an elementary exercise to show that the two oscillators in Figure 1 have the same dynamics, as long as the internal friction is neglected. This can be seen from the fact that the two pendulums have the same kinetic and potential energies, up to an additive constant related to the elastic energy associated with the DC stretch and to the position of the mass in the gravitational field.

Internal friction may be introduced by replacing the real elastic constant k of an ideal Hooke's Law spring by a complex number with an imaginary part, such that[4]

$$k = k_0(1 + i\phi_0). \tag{2}$$

The angle ϕ_0 is called the loss factor; it represents energy dissipation associated with internal relaxations of the material. It is in general a quite complicated function of frequency, temperature, geometric shape, state of defects, DC and dynamic stess and strain, and so on.

In the Bordoni relaxation theory, the motion of dislocation loops is responsible for the internal friction. However, in order for this kind of relaxation to be significant, Paré's condition has to be satisfied to provide two energetically equivalent configurations for a dislocation loop to jump between during relaxation.[4] This condition can be fulfilled either by the presence of a substantial internal stress induced by a large density of defects, or by a substantial externally applied DC stress in case the internal stress is not big enough, such as in annealed metals. Figure 2 (reproduced from reference[2]) illustrates this effect in a torsional pendulum.

A direct detection of the DC strain effect on the mechanical loss of the simple oscillator in Figure 1 would require comparing the Q's associated with the two configurations (a) and (b). This in turn would require an ideal frictionless surface in configuration (a). Such a requirement makes the detection essentially impossible in a normal laboratory on the Earth.

B. *Angle-Adjustable Oscillating Beam*

Our angle-adjustable oscillating beam is composed of a short, thin flexure spring which was connected to a long, thick and heavy rigid beam. The spring was clamped to a base as shown in Figures 3 and 4. If the spring and beam were initially set to point directly upward, it would become an inverted pendulum, as described in reference.[5] However, there are four properties of this beam oscillator which distinguishes it from the other, to serve our special purpose here:

1. We can adjust the angle α between the vertical direction and the orientation of the clamped end of the spring.
2. The spring is a flat strip, i.e. $w \gg d$, where w and d are the width and the thickness of the spring, respectively.
3. There are no adjustable masses attached to the beam. The moment of inertia I of the spring/beam system is essentially a constant for all angular positions.
4. The whole system of spring, rigid beam and clamp can be turned 90 degrees to obtain the configuration shown in Figure 5. This configuration is our equivalent of the ideal horizontal oscillator of Figure 1(a), while

the orientation of Figure 4 is like that of Figure 1(b). We achieve this with a single apparatus, simply by changing its orientation, without the need to try to construct a frictionless surface to support the mass.

Under the force of gravity, the spring and beam in the orientation of Figure 4 won't keep straight at its natural angular position α. Instead, it will sag by an amout of $\Delta\alpha$. If κ is the angular spring constant of the ideal loss free spring (with dimension of force×length), then the sag angle $\Delta\alpha$ can be found from equation:

$$\Delta\alpha = \frac{MgL}{2\kappa} sin(\alpha + \Delta\alpha), \tag{3}$$

where M and L are the mass and the length of the rigid rod, respectively. (In obtaining Eq. 3, small terms involving mass and length of the spring strip have been neglected, and will be throughout the following derivations.)

For small oscillations $\theta(t)$ around the equilibrium angle $\alpha + \Delta\alpha$, the potential energy V of the system can be written as a sum of flexure elastic energy $V_{flex} = \kappa(\Delta\alpha + \theta)^2/2$, and gravitational potential energy $V_{grav} = MgL\cos(\alpha + \Delta\alpha + \theta)/2$. After a bit of algebra, this simplifies to

$$V = \frac{1}{2}\left(\kappa - \frac{MgL}{2} cos(\alpha + \Delta\alpha)\right)\theta^2. \tag{4}$$

It is now easy to derive the equation of motion

$$I\ddot{\theta} + \left(\kappa - \frac{MgL}{2} cos(\alpha + \Delta\alpha)\right)\theta = N_{ext}, \tag{5}$$

where N_{ext} is an externally applied torque.

Internal friction of the spring is introduced the same way as before by defining a complex elastic constant

$$\kappa = \kappa_0(1 + i\phi_0), \tag{6}$$

where κ_0 and ϕ_0 are real functions of frequency. The spring loss factor $\phi_0 = \phi_0(\omega, \Delta\alpha)$ is in general a function both of the frequency and DC stretch.

The free oscillation solution of Eq. 5 can be written as

$$\theta(t) = \theta_0 exp\left(-\frac{\Phi}{2}\omega t\right) exp(i\omega t), \tag{7}$$

where

$$\omega^2 = \frac{1}{I}\left(\kappa_0 - \frac{MgL}{2} cos(\alpha + \Delta\alpha)\right) \tag{8}$$

is the resonant frequency at the beam angle α. The angle Φ is the loss factor of the beam oscillator. It is related to the loss factor of the spring as follows:

$$Q^{-1} \equiv \Phi = \frac{\omega_0^2}{\omega^2}\phi_0(\omega, \Delta\alpha), \tag{9}$$

where $\omega_0^2 \equiv \kappa_0/I$ is the natural resonance frequency.

It is worth mentioning that when $\alpha = 0$, we obtain an inverted pendulum as in reference.[5] From Eq. 3, $\Delta\alpha = 0$ too there, so there is no DC sag at this angle. The resonant frequency $\omega^2 \equiv \omega_i^2$ is a minimum at this position, and

$$\Phi = \Phi_i \equiv \phi_0(\omega_i, \Delta\alpha = 0)\ \frac{\omega_0^2}{\omega_i^2} \tag{10}$$

has its largest gravitational correction ω_0^2/ω_i^2. Here the subscript "i" refers to the "inverted pendulum" configuration.

Equations 8 and 9 show clearly how the DC strain influences the oscillator Q through its appearence in ω explicitly and in ϕ_0 implicitly. Since ϕ_0 itself is in general also an implicit function of ω, the DC effect is very complicated for an arbitrary angular position. However, there is a particular angle $\alpha = \alpha_0$ with DC sag $\Delta\alpha = \Delta\alpha_0$ such that

$$\alpha_0 + \Delta\alpha_0 = \frac{\pi}{2}, \tag{11}$$

at which the situation is much simplified. From Eq. 8 it is easy to see that $\omega^2 = \omega_0^2$, and thus

$$\Phi = \Phi_v \equiv \phi_0(\omega_0, \Delta\alpha_0). \tag{12}$$

Here the subscript "v" emphasizes the fact that oscillation is now exactly along the vertical direction. The loss angle Φ_v measures the loss factor at the natural resonant frequency, under the influence of a substantial DC strain, but with no explicit gravitational effect on the dynamics. It is thus equivalent to the simple oscillator of Figure 1 (b), without gravitational corrections to the resonant frequency. We assume here that there is no appreciable change in κ as a result of the DC strain.

But can we also provide a way to measure its counterpart $\phi_0(\omega_0, \Delta\alpha = 0)$ under no DC strain, without the difficulty of creating a frictionless surface? The answer is yes. If we turn the whole apparatus in Figure 4 by 90 degrees we will get the configuration shown in Figure 5. The oscillator will now oscillate horizontally. The frequency of oscillation will be equal to the natural resonant frequency ω_0. Vertical motion is negligible for such a flat strip spring. Even though the DC gravitational force will still be the same as before, its effect can be neglected because the following reasons :

1. The force is perpendicular to the oscillation, and
2. The vertical DC sag is negligible due to the large aspect ratio w/d of the spring.

This horizontal oscillation thus provides us with the way suitable for measuring the loss factor at the natural resonant frequency without the DC strain effect :

$$\Phi = \Phi_h \equiv \phi_0(\omega_0, \Delta\alpha = 0), \tag{13}$$

with the subscript "h" denoting the horizontal direction of the oscillation. This has been achieved without the problem of creating a frictionless surface.

III. Apparatus Description

Our angle-adjustable oscillator was constructed from a piece of blue tempered spring steel strip, AISI grade 1095, 6.35 cm (2.5 inches) in width and 0.5 mm (0.02 inches) in thickness. No pretreatment of the spring was performed before the experiment. One end of the spring was clamped to a rigid aluminum beam of the same width, but 2.5 cm (1 inch) in thickness and 25 cm (10 inches) in length. The other end of the spring was clamped between two surfaces of a pair of half cylinders also made of aluminum. The whole cylindrical clamp was then held firmly within an outer aluminum clamp in the shape of a cube. The structure was in turn bolted onto a table base (See Figure 4). The net length of the strip spring between the clamps at each end was 2.0 cm (0.8 inches). By rotating the cylinder clamp, the angle α could be easily adjusted to satisfy Eq. 11, so that the whole length of the rigid beam would lie in a horizontal orientation. Our procedure was to adjust the angle α_0, then fasten the cubical clamp. Then, by turning the cubical clamp 90 degrees we could switch between the two configurations at will. For our system, the value of $\Delta\alpha$ was about 14.5 degrees.

We studied this system by observing its free oscillations. The oscillator was excited by an electrostatic force from a conducting plate placed near the beam. Motion of the oscillator was registered by a split photodiode illuminated with an infrared LED. Signal currents from the diode were then sent to a pair of current-to-voltage pre-amplifiers, followed by a differential amplifier. A PC-based data acquisition system recorded the whole time series. The envelope of the free decay was fitted with an exponential, to extract the loss factor Φ.

IV. Results

Besides Φ_v and Φ_h, we have also measured Φ_i, loss factor of the inverted pendulum. The measurements were performed at both atmospheric pressure $P = 1$ atm and at a vacuum of $P = 10^{-6}$ torr. Our main results are summarized in Table 1, where the second subscrpts "a" and "v" represent air and vacuum, respectively. In Figure 6 a typical free damping signal and its exponential envelope fit are shown.

From the table, a set of facts can be summarized as follows:

1. Resonant frequencies f_{hv} and f_{vv} (or f_{ha} and f_{va}) are almost equal. Their deviation is less than 0.2%. This demonstrates the validity of Eq. 8, $\omega = \omega_0$, when $\alpha + \Delta\alpha = \pi/2$.

2. In both atmospheric and high vacuum pressures, loss factors associated with free damping under the influence of DC strain are larger than those measured when $\Delta\alpha = 0$. In vacuum Φ_v exceeds Φ_h by 18%. This indicates that a DC strain effect exists in our experiment.

3. Air friction contributes as much as 40% to the total damping. The vacuum condition is essential for this experiment.

4. In previous work the dependence of the spring loss factor $\phi_0(\omega)$ on frequency was mainly parametrized by two models[1]: frequency independent damping (ϕ a constant), and velocity damping (ϕ proportional

to ω.) Here we can test these models by comparing the measured Φ_{iv} with that calculated according to Eq. 10 by means of these models from the measured natural frequency f_{hv}, the natural loss factor Φ_{hv} and the inverted pendulum frequency f_{iv}. For frequency independent model we have

$$\Phi_{iv} = \frac{f_{hv}^2}{f_{iv}^2}\Phi_{hv} = 4.2 \times 10^{-4}.$$

On the other hand, if velocity damping is assumed, then

$$\Phi_{iv} = \frac{f_{hv}}{f_{iv}}\Phi_{hv} = 3.7 \times 10^{-4}.$$

Comparing them with the measured $\Phi_{iv} = 3.9 \times 10^{-4}$, we see that it prefers neither the velocity nor the frequency independent model, but something in between. But note that, since the frequency span over which the comparison was made is not great, the significance of this result is of limited value.

5. Using formulas from reference,[3] we can calculate the contribution to loss from the thermoelastic relaxation:

$$\phi_{th} = \frac{E\alpha^2 T_0}{c_\sigma}\frac{\omega\tau}{1+(\omega\tau)^2},$$

and

$$\tau = \frac{b^2}{\pi^2}\frac{c_\sigma}{\kappa_{th}},$$

where E, α, T_0, c_σ, b and κ_{th} are the Young's modulus, coefficient of thermal expansion, room temperature, specific heat at constant stress, thickness of the spring and thermal conductivity, respectively. Plugging in all the numbers relevant to the experiment, we find:

$$\frac{E\alpha^2 T_0}{C_\sigma} \approx 3.0 \times 10^{-3} \qquad \text{and} \qquad \tau \approx 1.9 \times 10^{-3}\ \text{s}.$$

For the horizontal and the vertical oscillations

$$\phi_{th} \approx 0.9 \times 10^{-4},$$

and for inverted pendulum:

$$\phi_{th} \approx 0.8 \times 10^{-4}.$$

Thermoelastic loss occupies as much as 20 to 30% of total the mechanical losses.

V. Discussion

The validity of the above results, especially that of the magnitude of the measured DC stretch effect, depends on whether all external losses, such as from air friction, friction in the clamp, and recoil damping in the supporting structure, have been reduced to a degree small enough to be negligible. Unfortunately, quantitative estimation of clamp and recoil losses are not easy to perform for a real mechanical system like ours. Here we give a discussion of how these problems have been dealt with.

Air damping is easy to eliminate by putting everything into a vacuum chamber. The pressure reached in our experiments was typically 10^{-6} to 10^{-7} torr. However, in most cases no appreciable difference was observed even if much poorer vacuum ($\sim 10^{-2}$ torr) was used. Therefore we are confident that air damping in our high vacuum measurements can be neglected.

Recoil damping can occur if there exist other resonant frequencies of the system (oscillator and the supporting structure) close to the main resonant frequency. This can be seen from a formula derived in reference[1]:

$$Q^{-1}_{\text{recoil}}(f_0) = Q^{-1}_{\text{no recoil}}(f_0) + Q^{-1}_{\text{ext}}(f_1)\mu\frac{f_1 f_0^3}{(f_1^2 - f_0^2)^2}, \qquad (14)$$

where f_0 and f_1 are the main and its nearby resonant frequencies, respectively; Q_{ext} is the quality factor of the nearby external resonance, and μ is the ratio of the oscillator mass to the modal mass of the recoiling system. A quick check of the power spectra of the horizontal and vertical oscillations in Figure 7 (with the system driven by ambient seismic noise) tells us that there are several resonances nearby the 2.5 Hz main frequency in the horizontal case. But the resonances are essentially absent in the vertical situation. From this observation we can infer that the recoil effect, if any, would have increased the value of Φ_{hv} more than Φ_{vv}.

The magnitude of extra damping in the horizontal oscillation caused by recoil from the first nearby resonance peak can be roughly estimated if we assume a value of μ relevant for the system. From Figure 7(a) we find $f_1 \approx 6.8$ Hz. $Q^{-1}(f_1)$ can also be estimated by the ratio between Δf, the full width of the peak at the half-power points, and f_1. In this way we find $Q^{-1}(f_1)$ ≈ 0.01. The mass of the oscillating beam is about 1 kg, and the mass of the whole system is ≥ 100 kg. Given the possibility that not all of the parts of the system have participated in the recoil, we assum $\mu \approx 1/20$. Putting all the numbers into the second term in equation 14, we obtain

$$Q_{\rm ext}^{-1}(f_1)\mu\frac{f_1 f_0^3}{(f_1^2 - f_0^2)^2} \approx 3 \times 10^{-5},$$

which is about 10% of Φ_{hv}.

Friction in the clamp did have a significant effect in an early stage of our experiment. At first the spring was clamped between the smooth flat surfaces of the two aluminum half cylinders. It turned out to give very substantial damping, for the horizontal oscillation in particular. As shown in Figure 8, this clamp produced as much as four times more damping in horizontal oscillation than in the vertical orientation.

The reason behind this was not difficult to find. Since the surfaces were smooth, the area of contact between the clamp and the spring was large. The stress of the clamp was not strong enough to provide a firm hold of the spring. In the orientation used for vertical oscillations, gravity helped to preload the spring firmly against one side of the clamp; we did not have the benefit of this effect when the beam was arranged to oscillate horizontally, so the problem was evident then. The problem was solved by cutting a sharp edge, or "tooth", along the front edge of each of the two cylinder surfaces (tooth thickness < 1.5 mm). These two teeth bit into the spring tightly enough so that the previous substantial loss in horizontal oscillation was eliminated. The sign of the difference in loss factors now indicates a real DC strain effect.

In our measurements, both the recoil and clamp losses would tend to mask a DC strain effect, by increasing the horizontal losses more than the vertical losses. It is our belief that this bias is small; this is demonstrably so in the case of recoil loss, although in the case of friction in the clamp it is hard to be certain. It is a generic difficulty of experimental investigations of internal friction that the effect of mounting the sample can never be proven to be negligible in all cases. Often, the best one can do is recognize additional friction above the measurements with the highest Q.

VI. Conclusions

This experiment reveals that a DC strain can indeed increase the internal friction of a steel spring subjected to no pretreatment, although the observed effect was not large considering the large angle (nearly 15 degrees) through which the spring was bent. A thorough study of the situation in proposed designs for gravitational wave test mass suspensions would be prudent.

ACKNOWLEDGMENTS

This work was inspired by stimulating discussions with S. Whitcomb and M. Zucker. We would also like to thank G.González and A. Ivashchenko for helpful suggestions.

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Pressure		Horizontal	vertical	Inverted
P=1 atm.	Res. Freq.(Hz)	f_{ha}=2.5285	f_{va}=2.5250	f_{ia}=2.2312
	$\Phi \times 10^{-4}$	$\Phi_{ha} = 5.4$	$\Phi_{va} = 5.7$	$\Phi_{ia} = 5.4$
P=10^{-6} Torr.	Res. Freq.(Hz)	$f_{hv} = 2.5206$	$f_{vv} = 2.5253$	$f_{iv} = 2.2336$
	$\Phi \times 10^{-4}$	$\Phi_{hv} = 3.3$	$\Phi_{vv} = 3.9$	$\Phi_{iv} = 3.9$

Table 1: Summation of measured data.

Figure Captions

Figure 1: Two configurations of the simplest pendulum. (a) No DC forced stretch. (b) DC forced stretch ΔX_0.

Figure 2: Effect of a static bias stress σ_s and vibration amplitude ϵ_m on the internal friction spectrum (f$\approx$ 1Hz) of aluminium neutron irradiated at 77K, deformed in torsion 0.8% and annealed at 273K. (From Fantozzi etc. 1982)

Figure 3: Schematic diagram of an angle adjustable beam pendulum.

Figure 4: Vertical oscillation of the beam pendulum

Figure 5: Horizontal oscillation of the beam pendulum

Figure 6: A typical free damping signal and its envolop fitting. The detailed sinusoid expenantial decaying wave is squeezed too much to be seen.

Figure 7: Power spectrums of (a): horizontal oscillation; and (b): vertical oscillation.

Figure 8: Damping signals with bad clamp. (a)Vertical oscillation; (b)Horizontal oscillation.

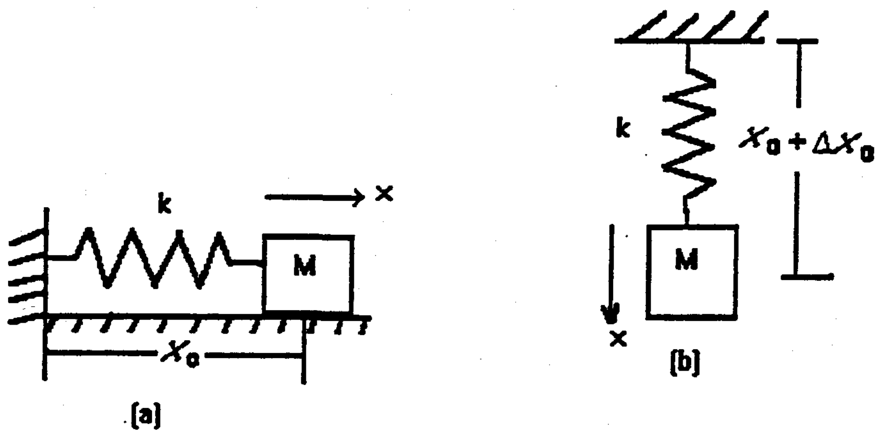
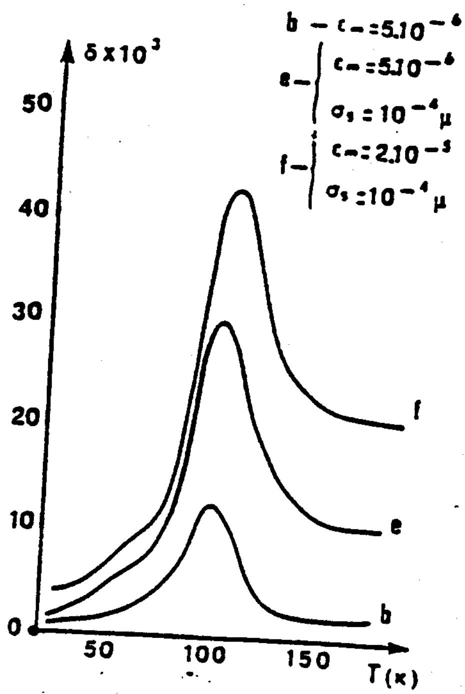


Figure 1



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b — ε∞ = 5.10⁻⁶
e — ε∞ = 5.10⁻⁶
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f — ε∞ = 2.10⁻⁵
σs = 10⁻⁴ μ
f
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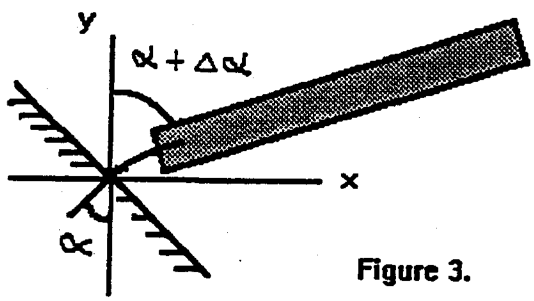


Figure 3.

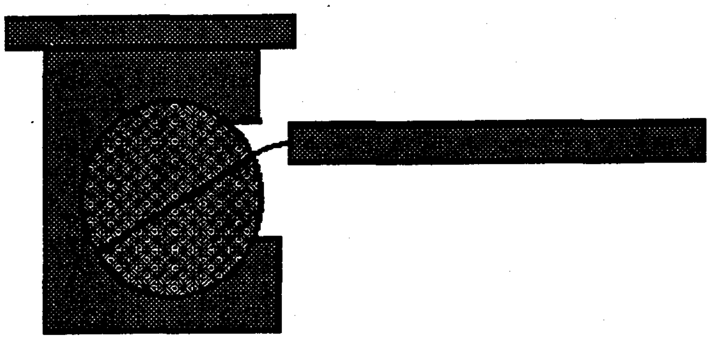


Figure 4.

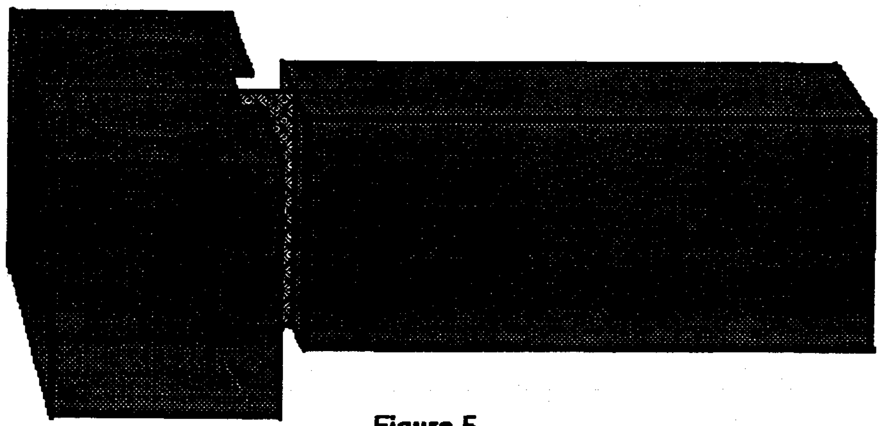
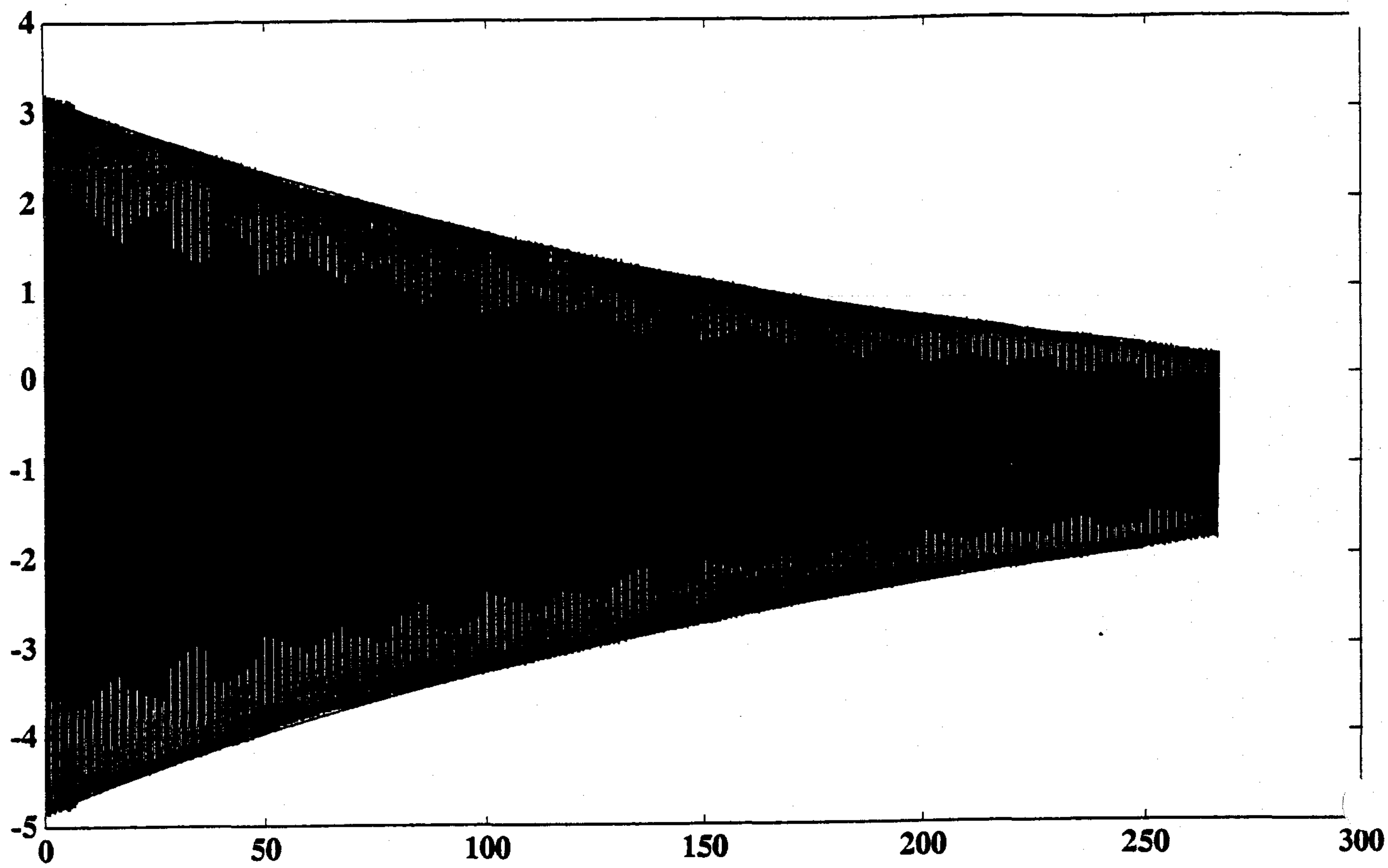


Figure 5.



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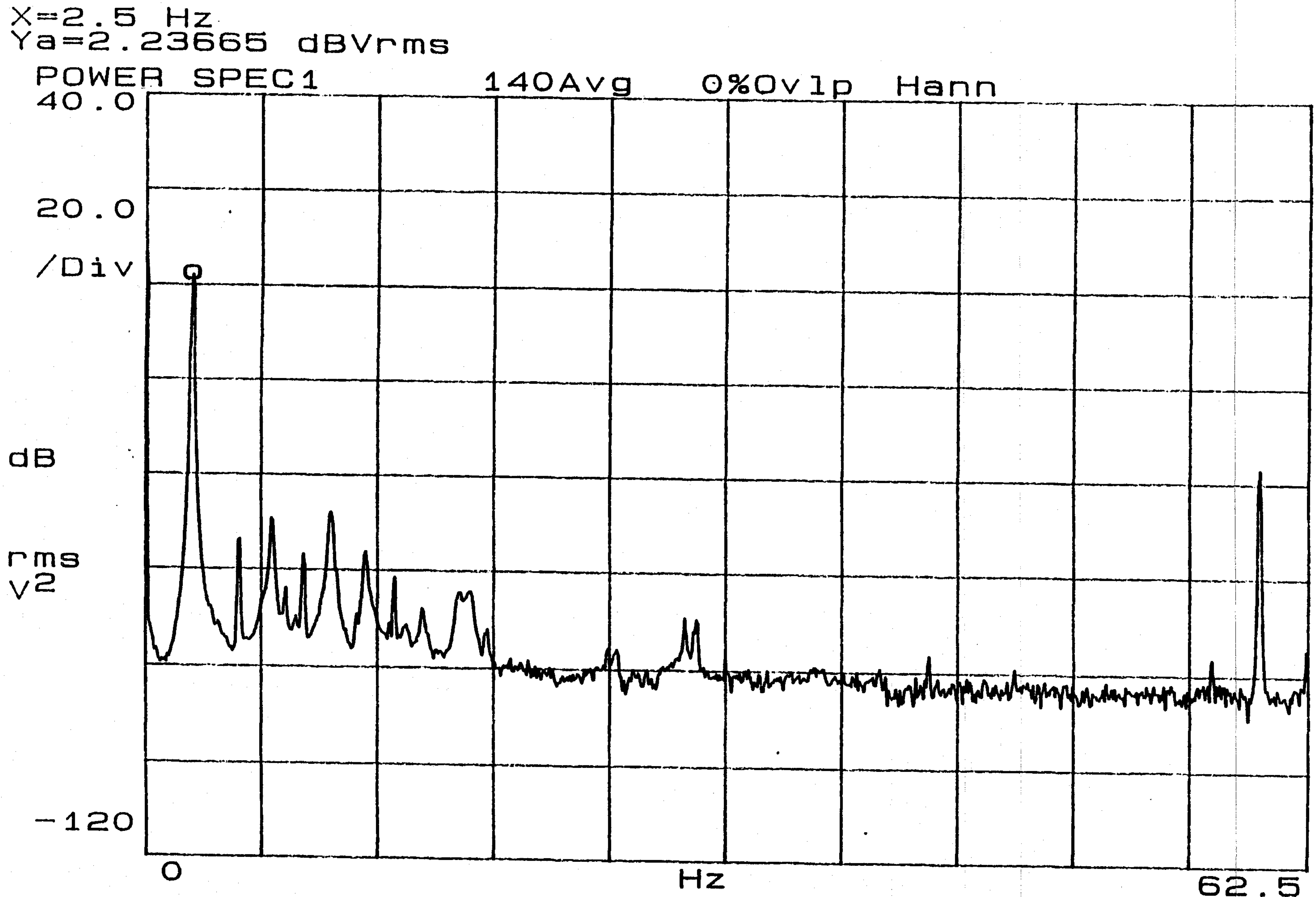
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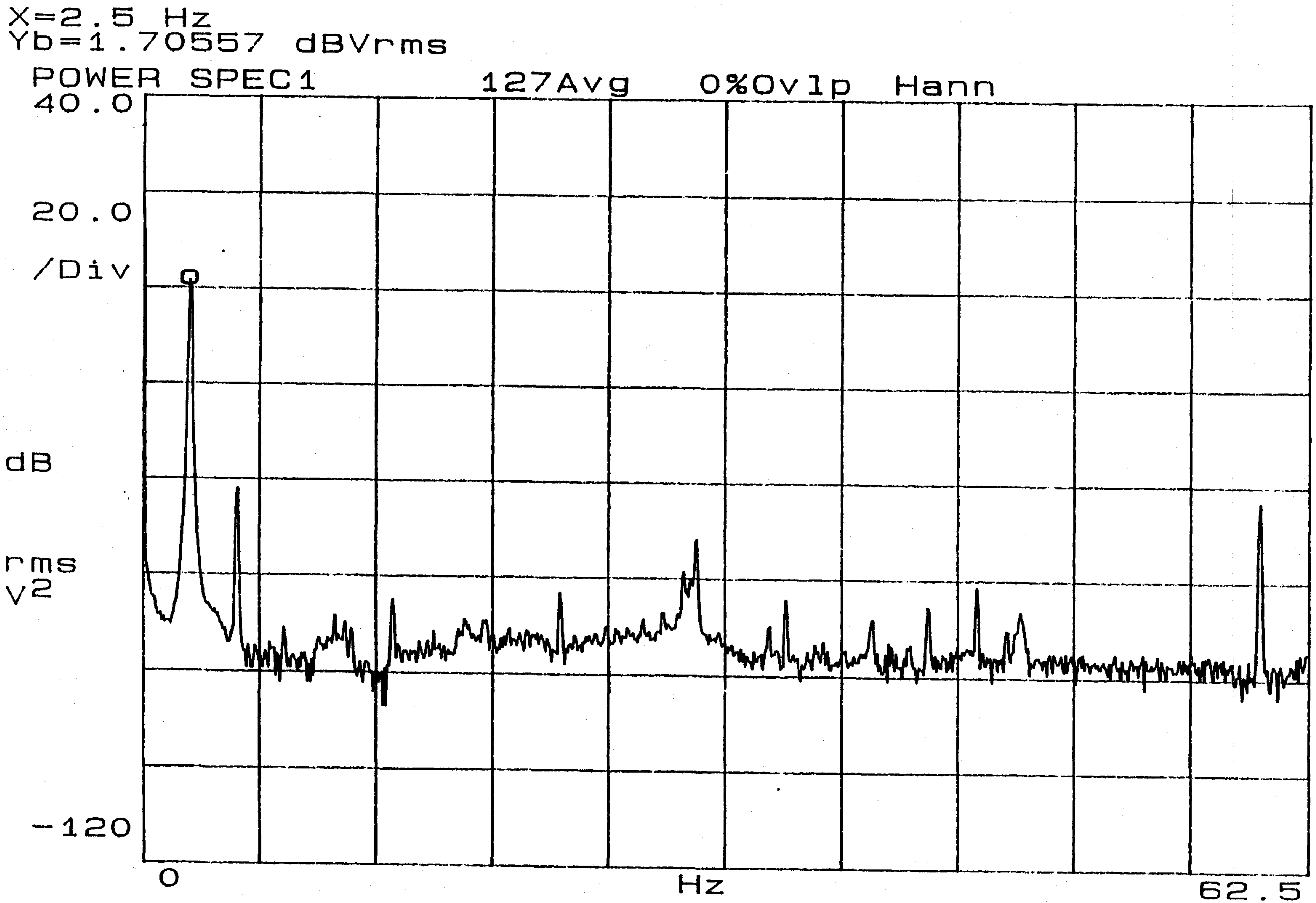
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