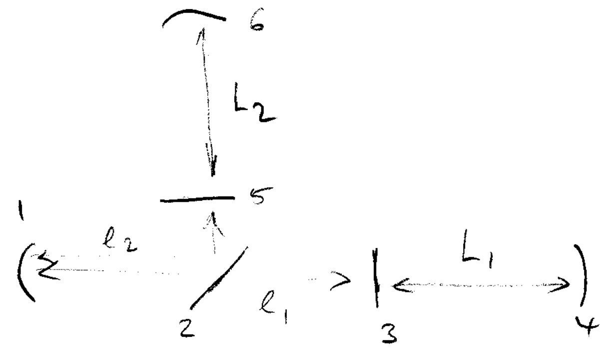
New Folder Name Mirror Positions



Mirror positions in an unbalanced-Michelson LIGO.

MR920419

Consider only the first upper and lower sidebands. Assume all mirrors are positioned so that the carrier is "resonating" (constructive and destructive interference where desired). We still have the freedom to move the mirrors by integral multiples of $\lambda/2$ without effect on the carrier

Criteria for the sidebands:

i) small fraction of sideband power incident on recycling mirror from outside reflected back towards laser. The best shot noise sensitivity to grav'l waves is achieved for minimum (zero) reflection of sidebands, but this suppresses

(920715) Not true. different conditions: opt. cplg wrt. rec. mirror vs opt. cplg wrt antisym port signals otherwise available from the light reflected from the recycling mirror.

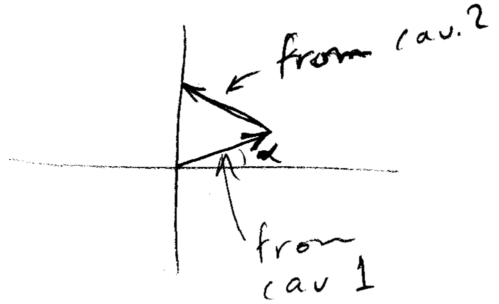
ii) phase of sideband reflected from inside of recycling mirror same as phase of sideband entering recycling cavity from laser. An equal and opposite phase difference for the two sidebands is probably okay, but complicates the analysis and adds little design flexibility.

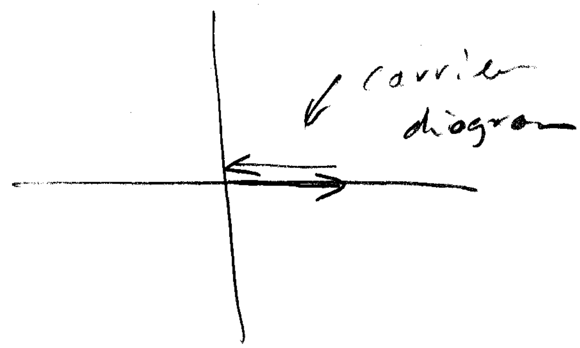
~~(iii) amplitude of two sidebands the same (between BS and rec. mirror)~~

iii) sidebands well away from resonance in arm cavities. Quite arbitrarily, choose a length at least 10 cavity linewidths away from the nearest resonance.

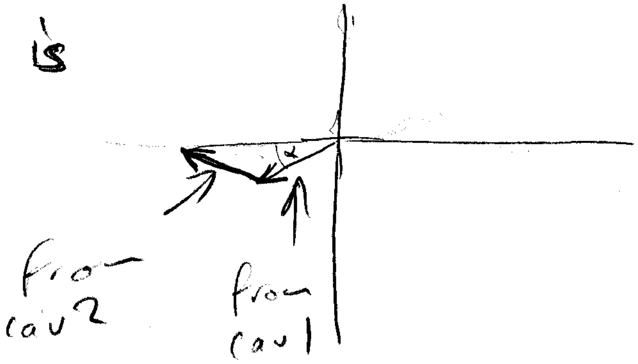
Arm imbalance

For $l_1 > l_2$, the piece of upper sideband reflected from cavity 1 has some extra phase shift (α) relative to the carrier, and the piece reflected from cav. 2 has an equal and opposite phase deficit. At the antisymmetric port of the BS, the phasor diagram is





After reflection from the recycling mirror, the phasor diagram is



(sign changed because BS reflectivity defined to be negative on one side)

⇒) sideband is attenuated by a factor $\cos\alpha$ because of leakage out of antisymmetric port (or equivalently because the two pieces reflected from the two cavities do not ~~the~~ quite have the same phase).

sideband bounce number

$$\frac{1}{1 - r_1\sqrt{1-L}\cos\alpha}$$

↑ rec. mirror

where L is the total additional loss (typically dominated by pickoff in recycling cavity, say 1%)

Sideband (amplitude) transmission to antisymmetric port of BS:

$$\frac{t_1 \sin\alpha}{1 - r_1\sqrt{1-L}\cos\alpha}\text{ ; } \text{max for}$$

$$\cos\alpha = r_1\sqrt{1-L}$$

(see plot 920421/10:30)

$$(\text{or } \sin^2\alpha \approx L + t_1^2)$$

Sideband reflection from recycling mirror: $\dfrac{r_1 - \cancel{(r_1^2+t_1^2)}^{\,1}\cos\alpha\sqrt{1-L}}{1 - r_1\sqrt{1-L}\cos\alpha}$

see plot 920421/10:40

Recycling cavity length

Sidebands resonant in recycling cavity

$\Rightarrow$ average recycling cav length $= \frac{\lambda_m}{4}$

$$\frac{r_2^2 l_2 + t_2^2 l_1}{r_2^2 + t_2^2} = \frac{\lambda_m}{4}$$

$$\simeq> \quad r_2^2 l_2 + t_2^2 l_1 = \frac{\lambda_m}{4}$$

$$l_2 = \frac{\lambda_m}{4 r_2^2} - \frac{t_2^2}{r_2^2} l_1$$

This implies that adjustability of l_1 and l_2 should be on the order of expected uncertainty in $\frac{t_2^2}{r_2^2}$ times l_1 $\left(\simeq \frac{\lambda_m}{4}\right)$

$$\left[\frac{d}{dx}\left(\frac{x}{1-x}\right) = \frac{1}{(1-x)^2} \text{ ; for } x = t_2^2, \ \frac{1}{(1-x)^2} \simeq 4 \right]$$

adjustibility $\Delta l \simeq 4 \, A(t_2^2) \frac{\lambda_m}{4}$

$$\simeq \lambda_m A(t_2^2)$$

Length selection

Use reflectivities and losses as agreed on with MIT. Use recycling cavity loss of 1% for sidebands (includes pickoff loss, contrast defect, transmission through cavities, etc.)

Pick $\alpha = 0.29$ (arbitrary)

$\Rightarrow$ sideband reflection 0.3
transmission 0.9

$$\Rightarrow l_2 - l_1 = 0.29 \cdot \frac{\lambda_m}{2\pi}$$

Assume TMC's can not be moved closer to BSC

Case 1 : 12.33 MHz , $\lambda_m = 24.33$ m

$\frac{\lambda_m}{2} = 12.17$ m

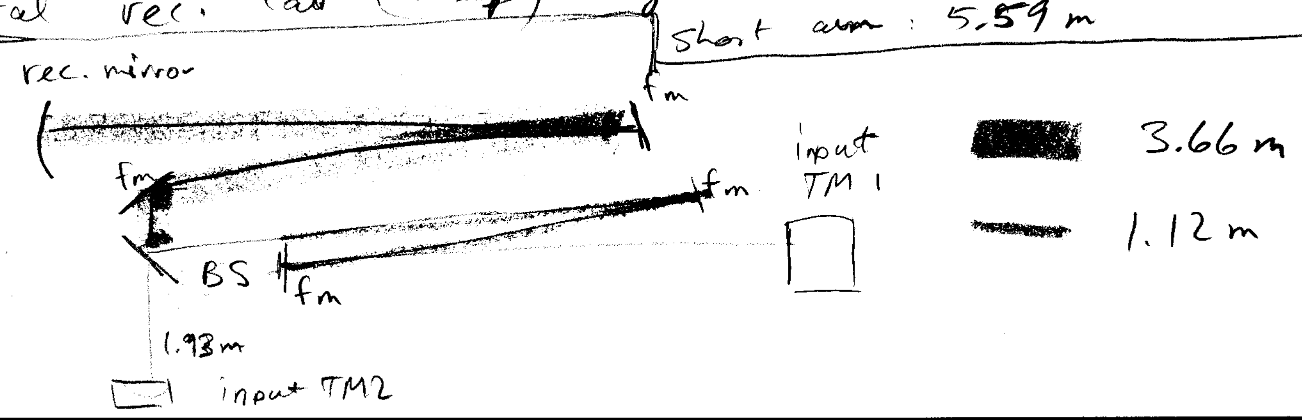
$l_2 - l_1 = \pm 1.12$ m

move input mirror 1 away from BS by an additional 1.12m (using folding mirrors). cav 1 length 38.35, or 1.85 m mod $\frac{\lambda_m}{2}$. Look up Michelson length (l_1) correction on plot 920420/10:14 :

$$\left(\frac{1.85m}{\frac{\lambda_m}{2}}\pi\right) + \frac{\pi}{2} = 2.05 \Rightarrow \text{correction is } -0.035 \text{ (rad)}$$

Since cavities have equal length (arbitrarily), the length correction is the same for cav 2. Both Michelson lengths appear shorter than they are because the cavities are shorter than the nearest $(n+\frac{1}{2})\frac{\lambda_m}{2}$. Consequently, recycling cavity needs to be longer by $\frac{\lambda_m}{2\pi}\left(\frac{1}{2}\right) 0.07 = 7$ cm

total rec. cav (average) length : **6.15 m**



cf 12m recycling Sha's Scheme

Case 2

17 MHz , $\lambda_m = 17.65$ m

$\frac{\lambda_m}{2} = 8.82$ m

$\frac{\lambda_m}{4} = 4.41$ m.

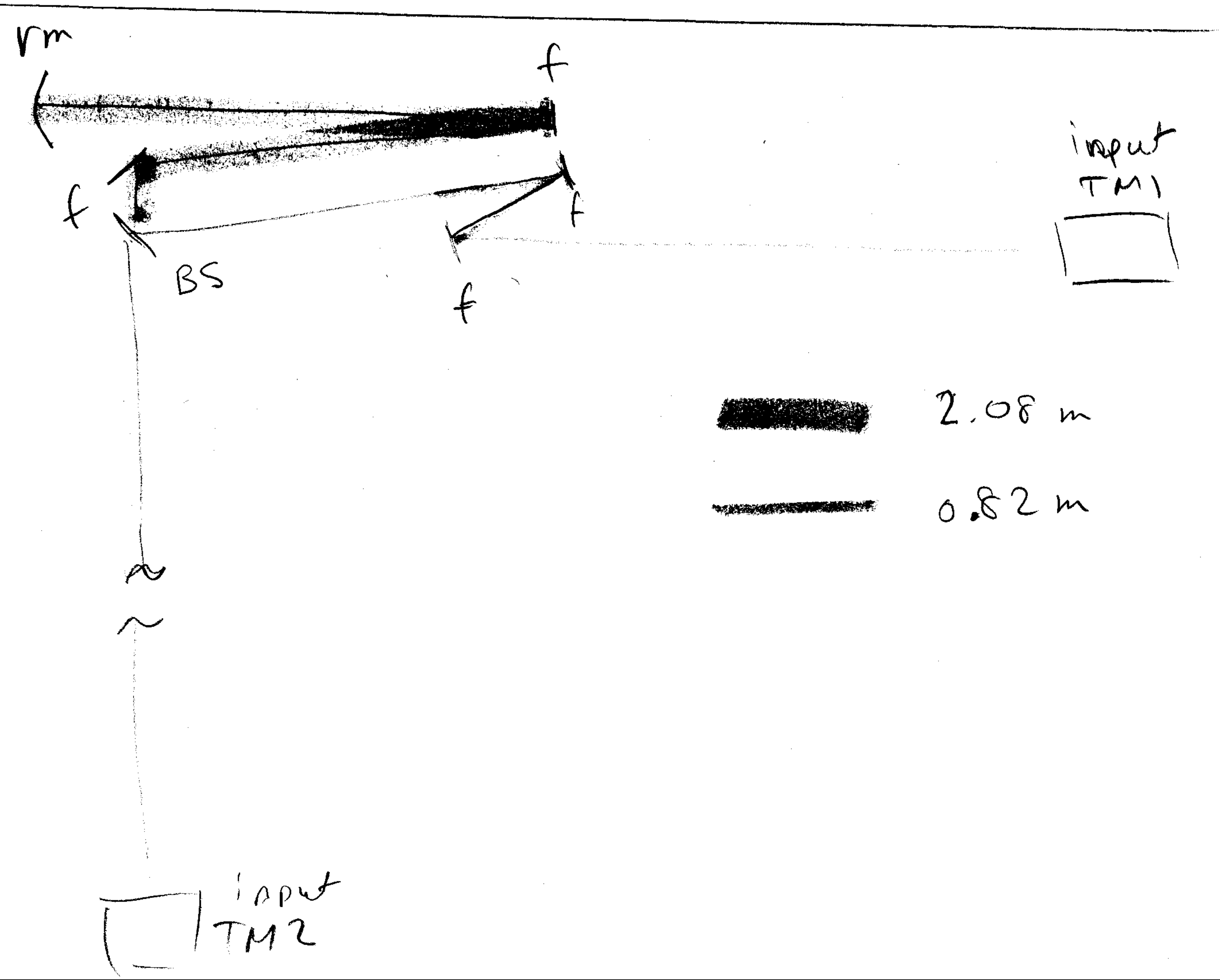
$l_2 - l_1 = 0.82$ m

cav length 38.35 $= 2\lambda_m + 3.05$

correction : $-\left(\frac{3.05}{\frac{\lambda_m}{2\pi}}\right) + \frac{\pi}{2} = 0.49 \Rightarrow$ correction $= -0.008$

average recycl. cav length: $4.41\text{m} + (0.015)\left(\frac{\lambda_m}{4\pi}\right) = 4.42\text{m}$

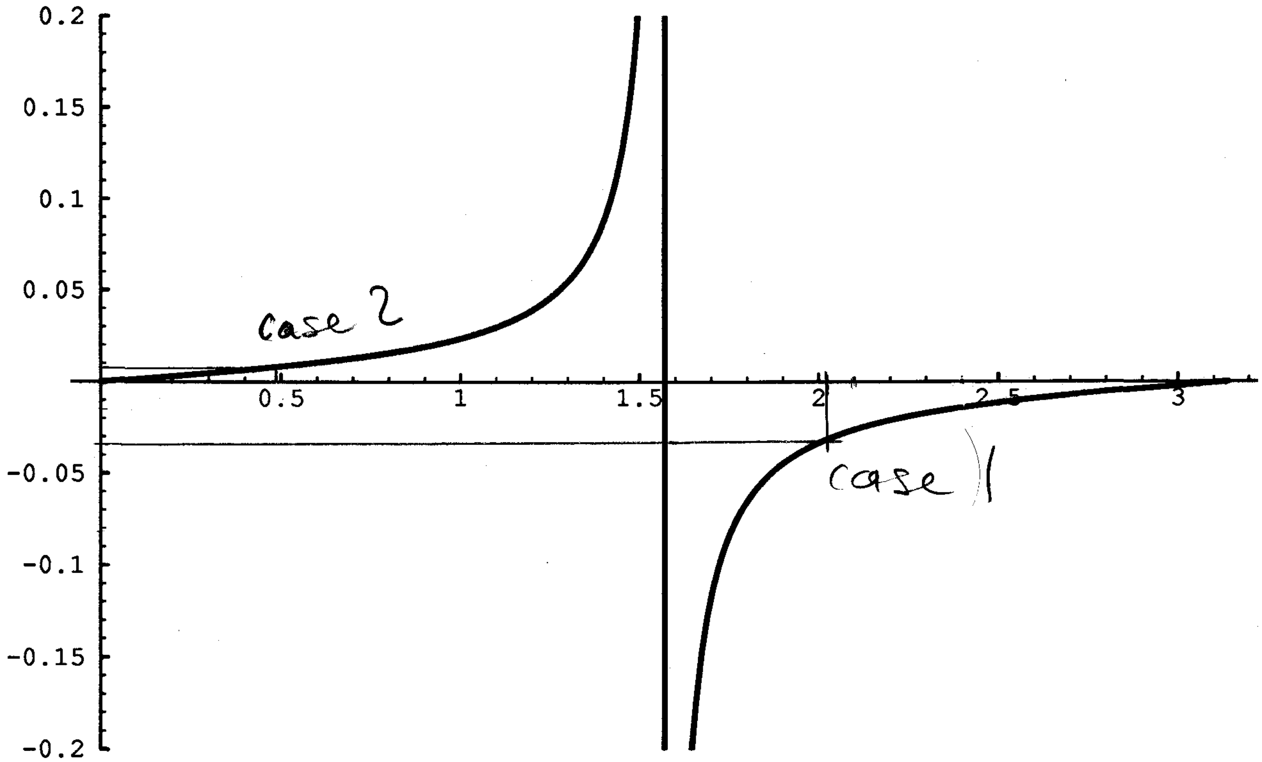
shorter Michelson arm: 4.01 m



MR920420 / 10:14

magnified copy of

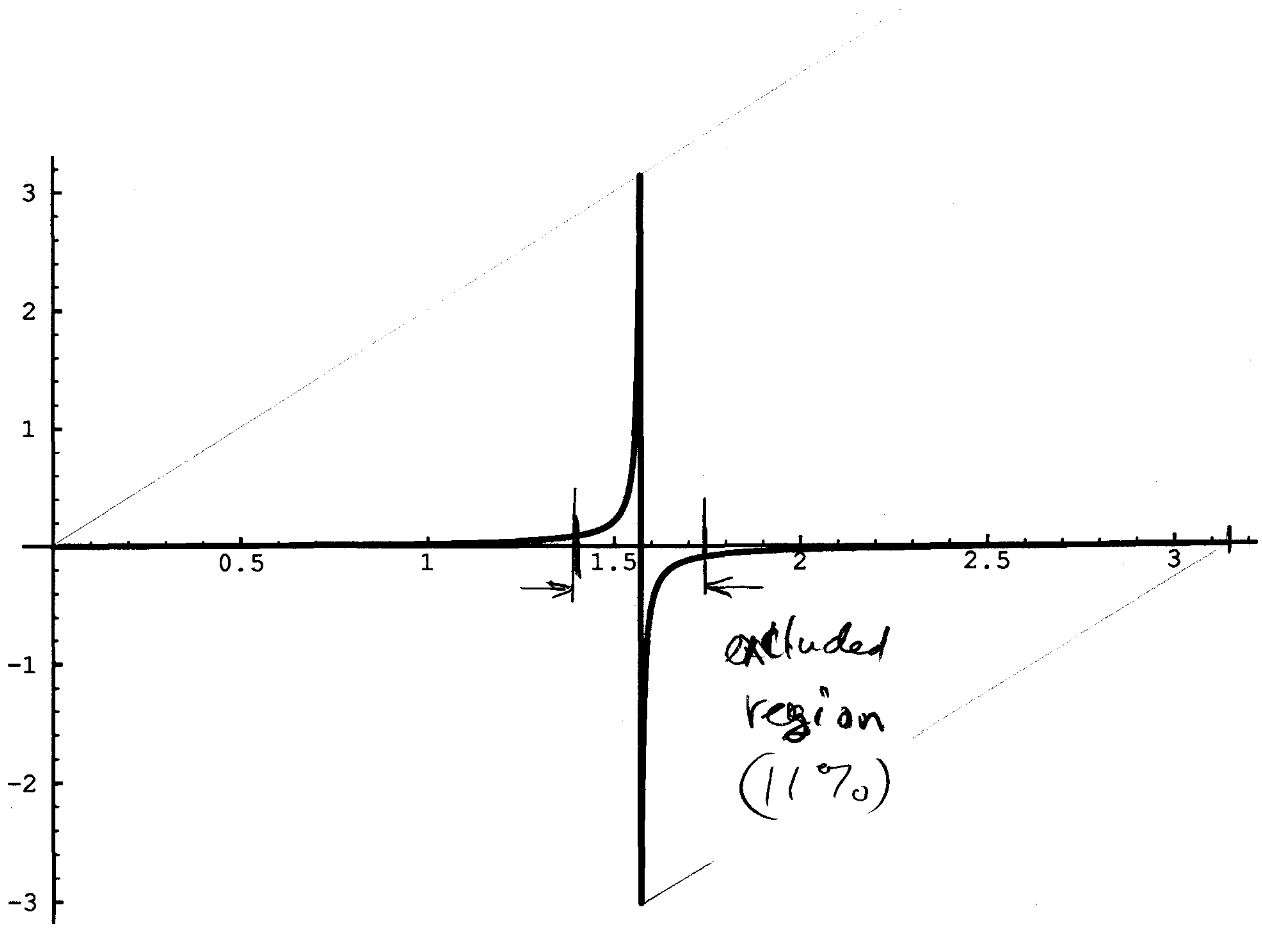
MR920419 / 21:30



MR920419 / 21:30

jacob / armlength / ps.n
/ p1

Arg{rc1} vs phi3



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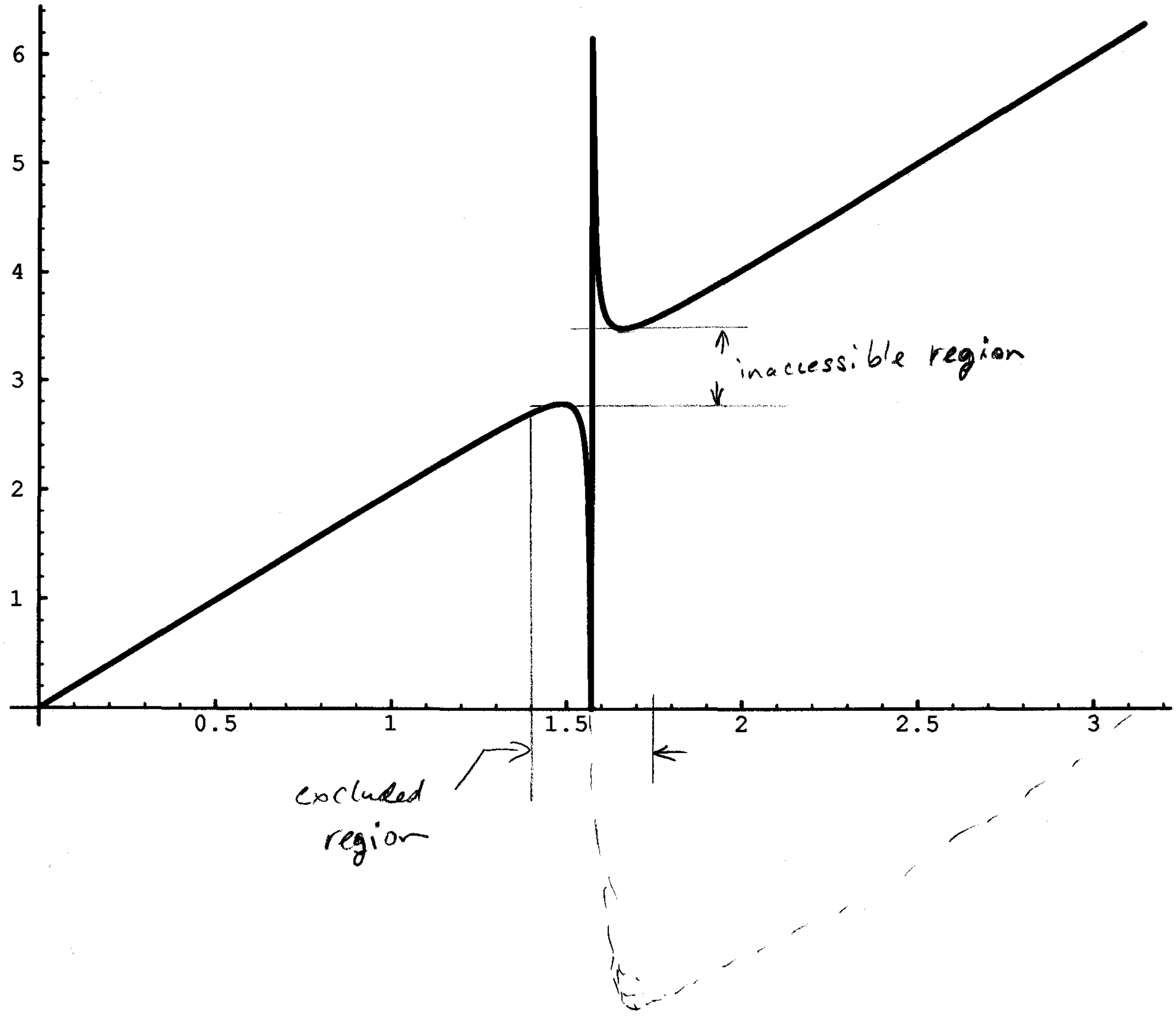
armlength 1ps.m,

1p1

phase on reflection as a function of input mirror position



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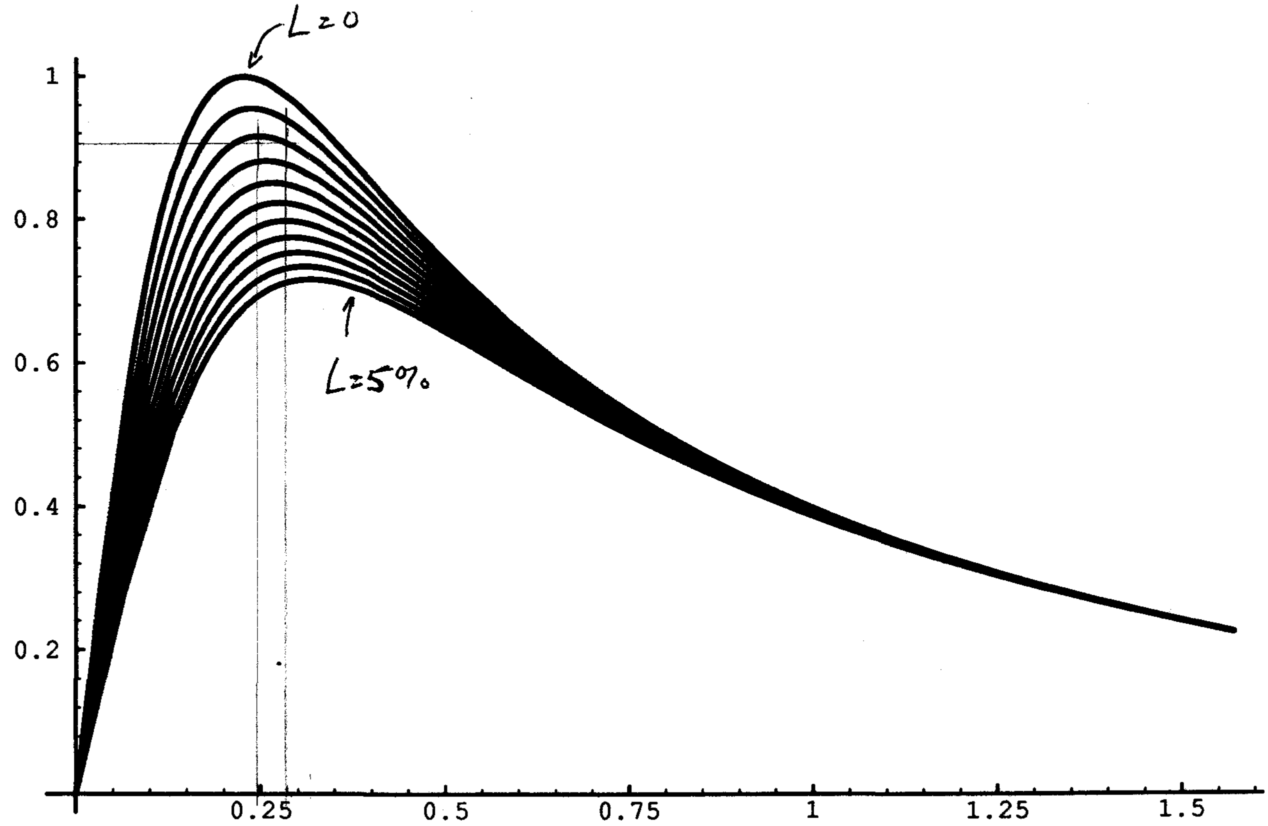
Sideband transmission vs unbalance

$$\frac{t_1 \sin\alpha}{1 - r_1\sqrt{1-L}\cos\alpha}$$



MR920421 / 10:40

Sideband reflection vs unbalance

$$\frac{r_1 - \cos\alpha\sqrt{1-L}}{1 - r_1\sqrt{1-L}\cos\alpha}$$