New Folder Name Interference Quality

CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

TO Science Team — DATE 13 May, 1992

FROM DHS — E-MAIL dhs

SUBJECT Interferometer contrast from Zygo data FILE ~ dhs/ifoconcept/tm_contrast.tex

Interference quality of a simple interferometer as inferred from the Zygo measurements of the 40m mirror substrates

Motivation: To help to specify the LIGO test masses. The goal is to make some connection between a index homogeneity specification for the LIGO test-mass substrates and a resulting limit on the quality of the contrast of a recombined-beam interferometer using these test masses.

Method: Measurements exist of the phase distortion resulting from passage of a beam through the 40m monolithic test masses (performed by Zygo on a Zygo interferometer). Here those measurements are used to calculate the relative intensity on the dark fringe of an interferometer made up of one 40m monolithic substrate, a perfect mirror, and a perfect beamsplitter. The interference pattern (and the 'alignment' of the interferometer) is weighted with Gaussian beam profiles of different diameters and positions. Measurements of the front and back surfaces of the substrates were also made, and contrast calculations as well; the contrast is limited by the index homogeneity rather than by the surface figure.

Limitations: The most important is that the contrast is calculated for a simple, *non-recycled* interferometer *without* Fabry-Perot cavities in the arms. In addition, a single measured mirror was used; using measured mirrors in both arms of the fictive interferometer would give more realism, but would probably not degrade the contrast by more than $\approx \sqrt{2}$. While birefringence data exists, no attempt has been made to use it.

Conclusions: For the two substrates for which calculations are performed, the relative minimum intensity on the dark fringe of a LIGO-sized beam would be $I_{min} \approx 2$ to 5×10^{-3}. This would be sufficient, *in a simple Michelson interferometer*, to achieve a goal of 30 recycles.

example of Figures 1 and 2. The total dark fringe intensity is normalized to the value for the bright fringe to give the desired result:

$$I_{min} = \frac{\sum_{x,y} I'_{min}(x,y)}{\sum_{x,y} G(x,y)}$$

This I_{min} is corrected by the ratio of the HeNe (632 nm) to Ar (514 nm) wavelength (after the fact by multiplying by the square of the ratio of the wavelengths) and by a factor of $\sqrt{2}$ to approximate the effect of using two mirrors with uncorrelated but comparable inhomogeneities.

Results: We have the following dark fringe relative intensities I_{min} (corrected for wavelength and including $\sqrt{2}$ for two mirrors):

Beam Position (cm)	Beam w (cm)	I_{min} for z02	I_{min} for z17
0,0	0.5	7.8×10^{-5}	4.1×10^{-5}
0,0	1	3.2×10^{-4}	4.8×10^{-5}
0,0	2	1.2×10^{-3}	3.4×10^{-4}
0,0	3	4.3×10^{-3}	1.8×10^{-3}
0,-2	1	5.8×10^{-4}	
3,0	1	1.7×10^{-3}	
0,0	2	7.6×10^{-5} (front surface)	
0,0	2	3.5×10^{-4} (no substrate)	

A beam of w=3 cm is the largest possible with the 4" substrates. This is, however, already comparable to the LIGO beam w. Note that I_{min} for a centered beam grows with roughly the area of the spot. This could be due to either a spectrum of surface irregularity which grows for longer spatial wavelengths, and/or the overall parabolic form of the homogeneity (due to the cooling process of the blank). If it is due to the latter, two similarly processed blanks would probably have similar parabolic homogeneity profiles, and some of the contrast degradation would be cancelled by common-mode effects. This makes our $\sqrt{2}$ for the second mirror a conservative approach to estimating the contrast.

Off center beams show greater I_{min}. This again may be due to the greater curvature from the overall parabolic cooling profile, or because of poorer quality at the edge.

We noted that this contrast is only sure to be correct for a simple non-recycled Michelson without cavities in the arms. The value for I_{min} may be indicative of how much power would not be coupled into a cavity, with such a mirror as an input coupler, due to distortions of the wavefront by the inhomogeneity. Since there would be a single pass (unlike the interferometer case), the phase error would be 1/2 of that for the interferometer, and thus the power that would be mismatched would be 1/4 of the figures above. We do not currently know of a reason why recycling would make the contrast of the Michelson *worse*, although the mode form in the recycling cavity may be strongly affected by the effects of

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Substrates: The substrates used in the calculations are the following:

Blank	Substrate	Mirror	Measurement files
C0AA-01-01-4.0	GOSU-22-01-4	PMS-19-01-4	z01-09
C0AA-01-02-4.0	GOSU-22-02-4	PMS-19-02-4	z16-21

These are 4" diameter, 3.5" thickness blanks from Corning of homogeneity '0AA' fused silica (the best standard available); they were superpolished at General Optics, and then tested in a Zygo interferometer by Zygo. The substrates have now been coated (by PMS), and new tests might give information on the effect of coating on the surface figure.

Zygo Measurements: Only measurements performed on the Zygo XP interferometer are used. See Burka, "Accessing the Zygo Data" and "Birefringence and Refractive Index Homogeneity in Fused Silica Test Masses", for descriptions of the setups and the data formats). The Zygo data files are turned into phase maps using scale factors supplied by Zygo. The units are in phase of a 632.8 HeNe laser wavelength. The rectangular phase maps are about 170 by 170 pixels (thus 0.6 mm per pixel), and the round substrate reaches the edge(s) of the map. The map typically shows tilts from the normal in both angles (fringes of equal inclination are used in the Zygo), and the average phase is not removed; see Figure 1, which shows a raw phase map. I have trusted Burka's and Gürsel's basic decoding of the Zygo files.

An estimate of the instrument noise (systematic and otherwise) comes from measurements without a substrate (data z06, z07). The I_{min} so calculated is a factor of 4 less than the I_{min} calculated with the substrate in place. The I_{min} calculated with the front or back surface (z08,z09) is as low (or lower) than that calculated for the empty system. Thus the homogeneity-limited contrast would dominate, and is measurable with this system.

Steps in the calculation: The tilt and level of the phase map which minimizes the phase difference between the phase map and a flat wavefront is found, with a (power) gaussian beam weighting. This ensures that if the phase map has e.g., a large defect at the edge, that the tilt is not taken to be the best compromise between the center and the edge, but rather the best value over the Gaussian beam used in the interferometer. The beam diameter w and position x_0, y_0 can be selected. Figure 2 shows the same sample as Figure 1, fitted to minimize the tilt and level with respect to a $w = 2$ cm radius Gaussian beam centered on the substrate.

The resulting phase map $\phi(x, y)$, corrected for the tilt and level found above, represents the phase deviations from zero phase expected at the dark fringe of an interferometer made of this substrate in one arm, a perfect (i.e., flat) mirror in the other arm, and a perfect beamsplitter; see Figure 3. The intensity $I'_{min}(x, y)$ at coordinates x, y in the plane for the dark fringe is

$$I'_{min}(x, y) = G(x, y)\frac{1}{2}\left(1 - \cos[\phi(x, y)]\right)$$

where $G(x, y) = A\exp\left([(x - x_0)^2 + (y - y_0)^2]\right)/w^2$ is the same gaussian weighting as used for the tilt and level removal above. This is shown in Figure 4 for the $w = 2$ cm beam example of Figures 1 and 2. The total dark fringe intensity is normalized to the value for the bright fringe to give the desired result:

$$I_{min} = \frac{\sum_{x,y} I'_{min}(x,y)}{\sum_{x,y} G(x,y)}$$

This I_{min} is corrected by the ratio of the HeNe (632 nm) to Ar (514 nm) wavelength (after the fact by multiplying by the square of the ratio of the wavelengths) and by a factor of $\sqrt{2}$ to approximate the effect of using two mirrors with uncorrelated but comparable inhomogeneities.

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Conclusions

These results, if there are no complications due to the cavities or the recycling, are consistent with a goal of (at least) 30 recycles. The losses on reflection from the arm cavities (due to losses in the mirror coatings) are expected to be about an order of magnitude more than the I_{min}s calculated here. The scaling and the importance of the overall parabolic curve in the homogeneity will be important to assess the LIGO case. This is the next interesting calculation to be performed with these data: to take two different substrates and 'interfere' the light of one with another.

The birefringence data has not been examined, and no conclusions can be drawn about the importance of this for the contrast or LIGO in general.

The need for a more sophisticated numerical analysis of more complete system (with recycling, and arm cavities) must be performed to obtain a firm specification for the mirror surfaces and homogeneity.

Appendix: Files and Programs

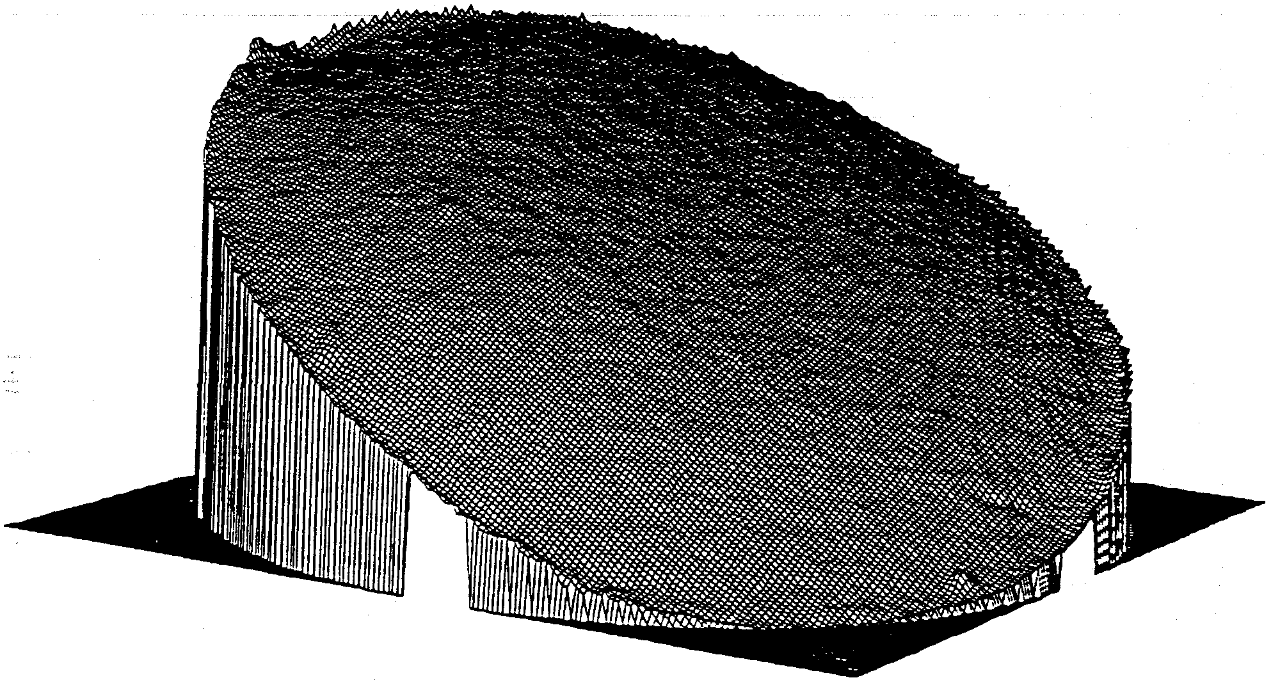
The raw data files are in ~ `yekta/zygo_data_and_analysis/data/` in compressed form (say 'uncompress z02' to restore). The programs are in `tristan:~ dhs/zygo/newsource/`. The fitting (removal of tilt) program is called `zxpfit` (takes a file z02 and makes file z02.fit). The program which calculates the contrast is called `zcontrast` (takes a file z02.fit and produces numbers and plots, and writes out a possibly-GLADV compatible output file). The Fortran sources and makefile are in the same directory.

What follows is the dialog for a sample session.

```
|data> ../newsource/zxpfit
enter zygo mark iv xp ascii filename (3 characters):  z02
phase width:  177 phase height:  180
part number:  1
serial number:  COAA-01-01-4.0
plotting the phase surface before tilt and level removal:
enter angle in degrees in the x-y plane to the line of sight:
(counterclockwise from the positive x axis)
30.
enter angle in degrees from the x-y plane to the line of sight:
(positive angles are above the middle z)
10.
enter a line of comments (80 characters maximum):
raw phase map
hardcopy?  (n):  n
Enter the diameter (inches) of the substrate
4.00
Enter x, y offset, omega (cm) for the beam weighting:
0.,0.,2.0
size, icent, jcent are 5.69188E-02 88 90
minimum before powell = 82.8585
powell has made 1 iterations...
powell has made 2 iterations...
powell has made 3 iterations...
powell has made 4 iterations...
number of iterations = 4
minimum after powell = 0.990012
fit parameters x, y, offset:  -2.00710E-02 4.36578E-02 0.372394
residual average is -0.148197
average phase = -0.148197 radians
variance = 3.49248E-02 radians^2
standard deviation = 0.186882 radians
maximum peak-to-valley phase [max(phase) - min(phase)] = 0.784975 radians
plotting the phase surface after tilt and level removal:
```

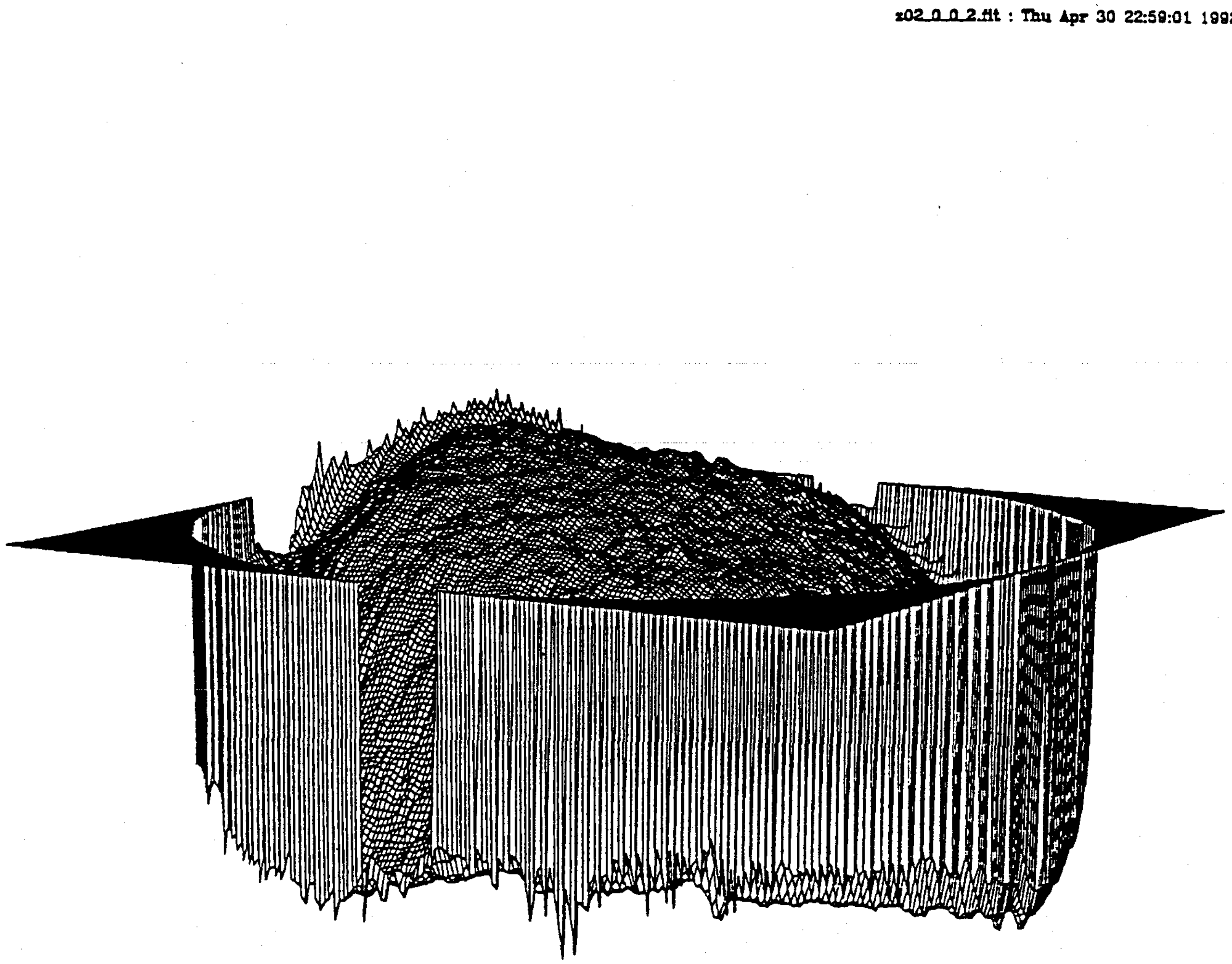
```
enter angle in degrees in the x-y plane to the line of sight:
(counterclockwise from the positive x axis)
30.
enter angle in degrees from the x-y plane to the line of sight:
(positive angles are above the middle z)
10.
max and min in radians are 1.49947 -7.33856
enter a line of comments (80 characters maximum):
after tilt and level fit
enter offset in x,y and dimension to plot
<carriage return>
max and min for weighted plot are 3.49965E-02 -2.63427E-02
hardcopy? (n): n
replot? (n): n
|data> ../newsource/zcontrast
enter the file name:
z02.fit
average = -0.148197 radians
variance = 3.49248E-02 radians^2
standard deviation = 0.186882 radians
maximum peak-to-valley phase [max(phase) - min(phase)] = 0.784975 radians
max and min are 0.158940 -0.626036
enter in-plane, and elevation, angles in degrees:
30.,10.
enter a line of comments (80 characters maximum):
after tilt and level fit
hardcopy? (n): n
size, icent, jcent are 5.69188E-02 88 90
normalized intensity is 5.94983E-04
enter offset in x,y and dimension to plot
<carriage return>
max and min are 5.26959E-04 0.
enter in-plane, and elevation, angles in degrees:
30.,10.
enter a line of comments (80 characters maximum):
dark fringe intensity
hardcopy? (n): n
replot? (n): n
|data>
```

z17 : Thu Apr 30 23:11:55 1992



Raw phase map (z17)

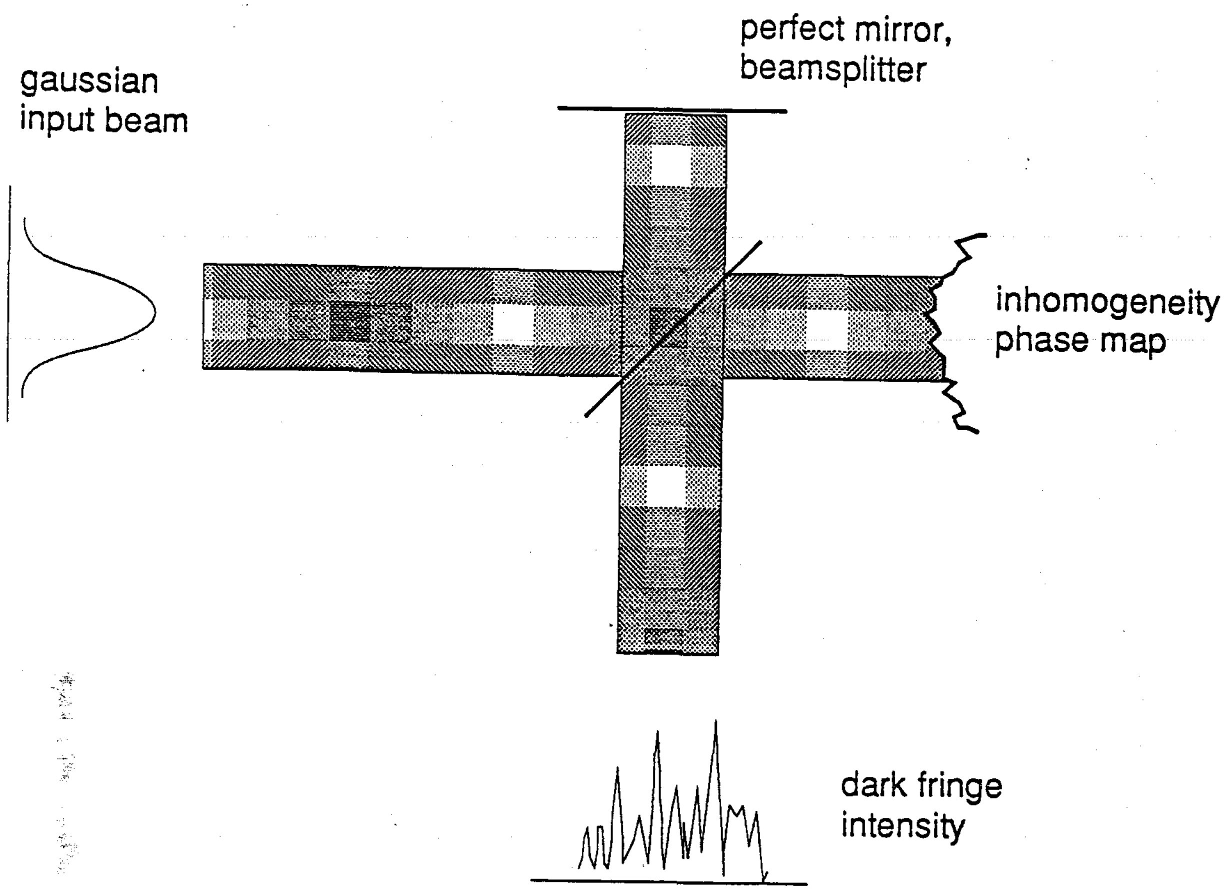
Figure 1



Phase map with tilt and level removed; beam at 0,0,2

Peak to peak phase: 0.78 radian

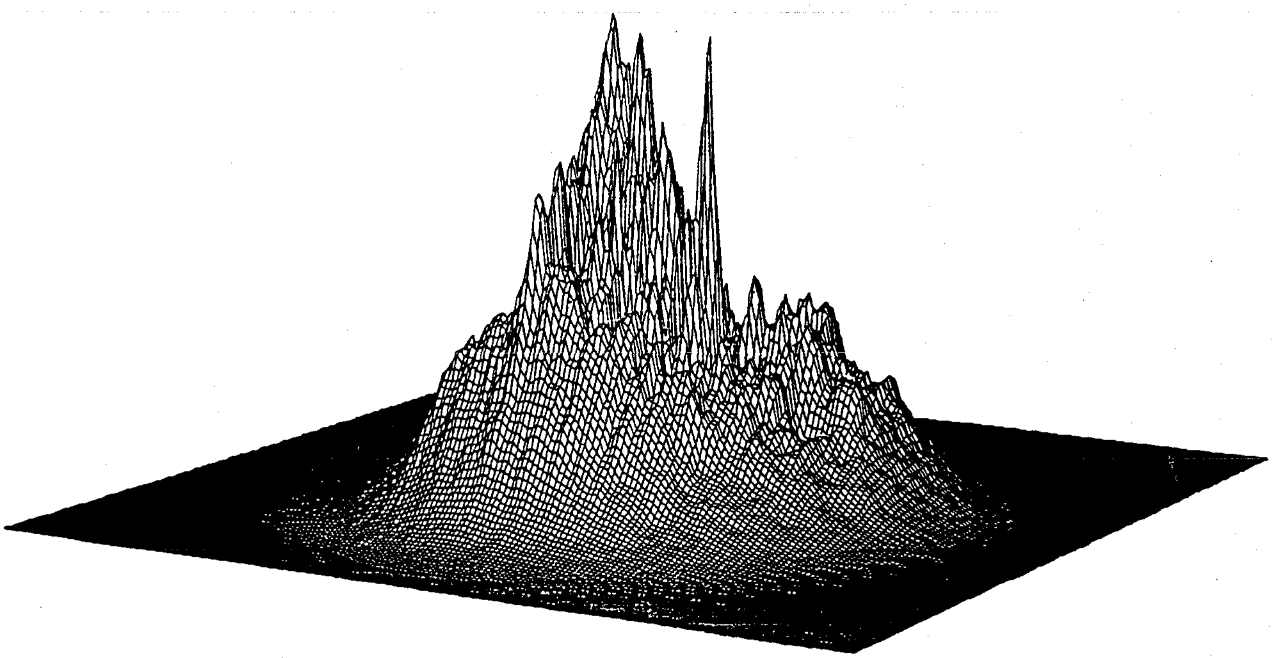
Figure 2



Setup for calculation: Michelson interferometer
with perfect beamsplitter and perfect mirror in one arm,
and 'imperfect mirror' (phase map) in the other arm

Figure 3

z02_0_0_2.flt : Wed May 13 22:19:44 1992



Dark fringe for beam at x=0,y=0,w=2 cm

Figure 4