New Folder Name Intensity Noise

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SUBJECT Intensity Noise

Effect of intensity variations of the incident light in a Fabry-Perot cavity RF reflection locking system was analyzed. The model were verified using the 40m prototype.

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1. Intensity Variations in a Fabry-Perot Cavity

In a Fabry-Perot cavity RF reflection locking system, the transfer function from the mirror displacement or the light frequency to the demodulation signal is proportional to intensity of the incident light. Therefore the intensity of the light can be treated as a multiplier in the process of the demodulation.

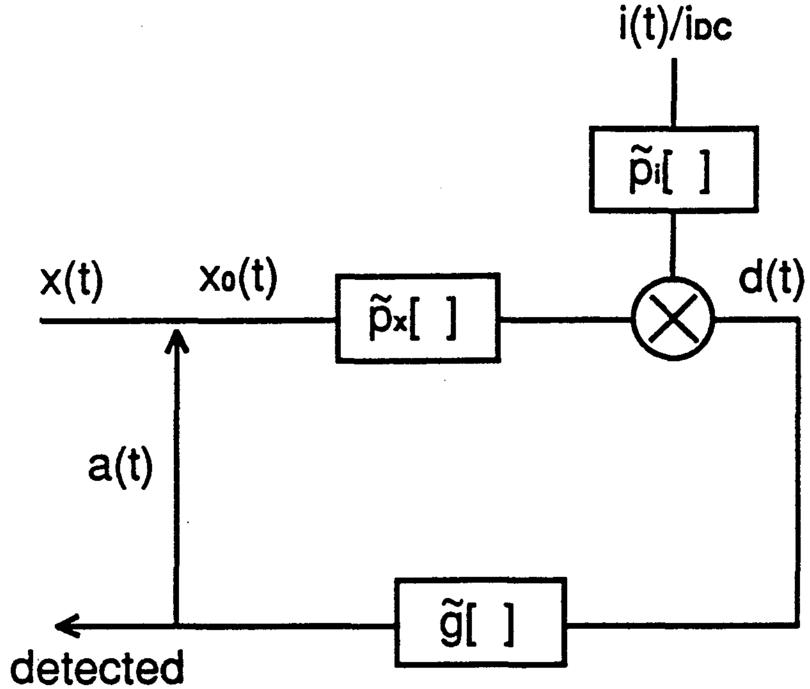


Fig.1 Simplified servo diagram for the FP cavity locking system. Intensity of the light is treated as a multiplier in a time domain.

Figure 1 shows the simplified[1] servo diagram for the FP cavity locking system with variable intensity of the light.[2] The original displacement of the cavity mirrors x(t) is suppressed by the feedback displacement a(t) to the residual displacement $x_0(t)$, which is then detected by the RF modulation scheme as the demodulation displacement d(t) through the cavity pole operator $\tilde{p}_x[\ ]$ (unity at DC). In the process the ratio of instantaneous laser intensity to DC intensity $i(t)/i_{DC}$, through the intensity cavity pole operator $\tilde{p}_i[\ ]$ (unity at DC), is multiplied by. The demodulation displacement d(t) is then, through the filter-amplifier operator $\tilde{g}[\ ]$, fed back as a(t), which is detected.

[1] All quantities in the main loop are defined in terms of mirror displacements.

[2] It should be emphasized that the model can be a good framework for analyzing certain types of noise which change the servo gain in some ways, such as amplitude variations of the RF modulation.

The feedback displacement a(t) is expressed in a time domain by

$$a(t) = \tilde{g}\,[\,\tilde{p}_i\,[i(t)/i_0] \times \tilde{p}_x[x(t) - a(t)]\,] \tag{1}$$

The Fourier coefficient of the feedback displacement An can be derived by Fourier transforming Eq. 1:

$$\begin{aligned} A_n &= A_n^{dis} + A_n^{int} \\ A_n^{dis} &\equiv \frac{\tilde{G}_n \tilde{P}_n^x}{1 + \tilde{G}_n \tilde{P}_n^x}\, X_n \\ A_n^{int} &\equiv \frac{\tilde{G}_n \tilde{P}_n^i}{1 + \tilde{G}_n \tilde{P}_n^x}\, \frac{I_n}{I_0}\, X_0^0 \\ &+ \frac{\tilde{G}_n}{1 + \tilde{G}_n \tilde{P}_n^x} \sum_{n':n' \neq 0,n} \tilde{P}_{n-n'}^i\, \frac{I_{n-n'}}{I_0}\, \tilde{P}_{n'}^x\, X_{n'}^0 \end{aligned} \tag{2}$$

where the large letter symbols represent the Fourier coefficients of the corresponding quantities or operators. A_n^{dis} is the feedback displacement purely due to original displacement; the feedback displacement with constant intensity. A_n^{int} is the feedback displacement due to intensity variations.

Now we wish to obtain the mimic displacement X_n^{int} caused by intensity variations. It can be derived by attributing intensity-caused feedback displacement $A_n^{int}$to displacement-caused feedback displacement A_n^{dis}. When there is no intensity variations, the original displacement X_n^{dis} is derived from A_n^{int} of Equation 2:

$$X_n^{dis} = \frac{1 + \tilde{G}_n \tilde{P}_n^x}{\tilde{G}_n \tilde{P}_n^x}\, A_n^{dis} \tag{3}$$

Equation 3 shows how we obtain the original displacement from the measured signal. And we use the same Equation 3 to obtain the mimic original displacement X_n^{int} even for intensity-caused feedback displacement A_n^{int}:

$$X_n^{int} = \frac{1 + \tilde{G}_n \tilde{P}_n^x}{\tilde{G}_n \tilde{P}_n^x}\, A_n^{int} \tag{4}$$

Therefore from Equation 2, we get

$$X_n^{int} = \frac{\tilde{P}_n^{\,i}}{\tilde{P}_n^{\,x}} \frac{I_n}{I_0} X_0^0 + \frac{1}{\tilde{P}_n^{\,x}} \sum_{n':n' \neq 0,n} \tilde{P}_{n-n'}^{\,i} \frac{I_{n-n'}}{I_0} \tilde{P}_{n'}^{\,x} X_{n'}^0 \tag{5}$$

Equation 5 contains a linear term (first term) and a convolution term (second term). The linear effect is proportional to the DC residual displacement, and intensity variations at a particular frequency induces a Fourier component of mimic displacement X_n^{int} at the same frequency.

More generally the mimic displacement component at a given frequency is given by the convolution of the spectrum of fluctuating residual displacements with the spectrum of intensity variations.

At low frequencies, where $\tilde{P}_n^{\,x} = 1, \quad \tilde{P}_n^{\,i} = 1$ Eq. 5 becomes much simpler:

$$X_n^{int} = \frac{I_n}{I_0} X_0^0 + \sum_{n':n' \neq 0,n} \frac{I_{n-n'}}{I_0} X_{n'}^0 \tag{6}$$

Now let's consider $\tilde{P}_n^{\,x}$ and $\tilde{P}_n^{\,i}$. Obviously the detection system experiences a pole at a frequency determined by the cavity storage time. Therefore[3]

$$\tilde{P}^{\,x}(s) = \frac{1}{1 + \frac{s}{c}} \tag{7}$$

where c is the cavity pole.

As for $\tilde{P}_n^{\,i}$, considering that only the light coming out of the cavity experiences the cavity pole and that RF phase modulation is used as a detection system, after some calculation (Appendix A), we get

$$\tilde{P}^{\,i}(s) = \frac{1 + \frac{s}{2c}}{1 + \frac{s}{c}} \tag{8}$$

Therefore in the linear term in Equation 5,

$$\frac{\tilde{P}^{\,i}(s)}{\tilde{P}^{\,x}(s)} = 1 + \frac{s}{2c} \tag{9}$$

[3] In this paragraph and the next, Laplace transformation is used for convenience.

which indicates that frequency dependence of the linear coupling from intensity to mimic displacement is described by one zero at twice the cavity pole.

Incidentally it can be shown that the relative importance of the linear and convolution terms in determining the overall spectrum is characterized by the ratio between the DC residual displacement $x_{0,DC}$ and twice the rms residual displacement $x_{0,rms}$.

2. Experimental Test

Equation 5 and its predictions were tested on the 40m prototype. It should be noted that the mimic displacement obtained using the 40m prototype is a combination between the mimic displacement in the primary cavity system and that in the secondary cavity system: the mimic displacement produced in the primary cavity system is imposed on the frequency of the incident light to the secondary cavity. Then using this light, the secondary cavity length is measured, where another mimic displacement is produced.

Linear coupling A intensity variation ($\delta I/I_0=7.1\times10^{-3}{}_{rms}$) was applied at 400Hz using an A/O modulator. The displacement signal from the interferometer was monitored at the same frequency as a function of the DC residual displacement in the primary cavity system.[4] The result is shown in Fig. 2 along with the predicted behavior.

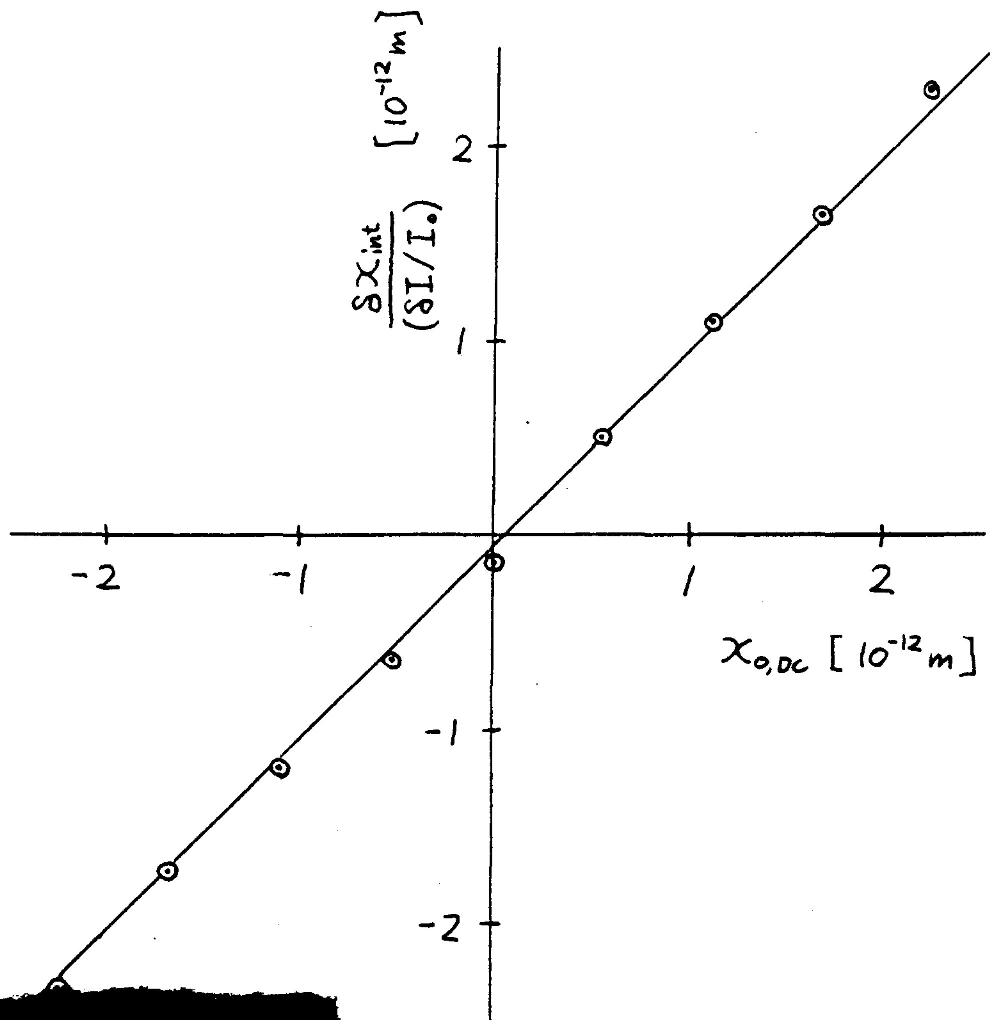
The frequency response of the coupling between 100Hz and 10kHz was also measured with a DC residual displacement of 8.3×10^{-13}m. As shown in Fig. 3, the response has a zero around 1kHz, which is approximately double the cavity pole frequency (500Hz).

Sideband Measurement To examine the convolution of the intensity and residual displacement spectra, the spectral neighborhood of the 400Hz displacement signal caused by the intensity variations was examined (Fig.4). The natural sidebands, symmetric around 400Hz, have a detailed spectral shape identical to low-frequency spectrum of demodulation signals in the secondary cavity system, verifying that the monochromatic 400Hz test signal has "picked out" of the convolution sum a replica of the residual displacement.

Broadband Spectral Convolution For more realistic situations a random intensity noise below 500Hz was generated and the displacement signal at 400Hz was monitored as a function of the DC residual displacement in the primary cavity system with and without bypass servo in the secondary cavity system. The rms residual displacement in the secondary cavity system is 8.0×10^{-14}m with bypass and 1.2×10^{-12}m without the bypass. As shown in Fig. 5, the effective transfer function for the random test signal is approximately proportional to the DC residual

[4] The residual displacement was obtained by calibrating the demodulation signal with known displacement signals and measuring the open loop gain of the primary cavity servo.

[redacted]



[redacted]

[redacted]

Fig. [redacted] of interferometer responce to intensity at 400 Hz.

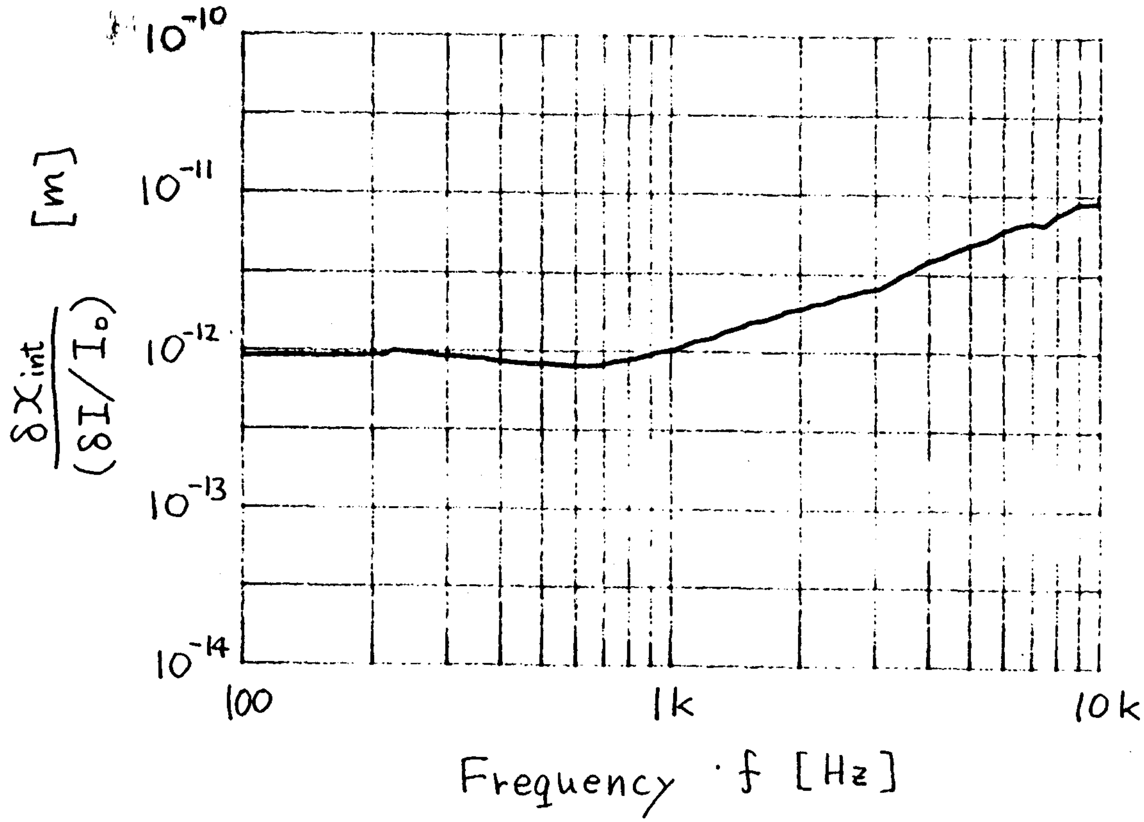


Fig. 3 Frequency dependence of the linear coupling of intensity variations into mimic displacement. The DC residual displacement is $x_{0,DC} = 8.3 \times 10^{-13}$ m.

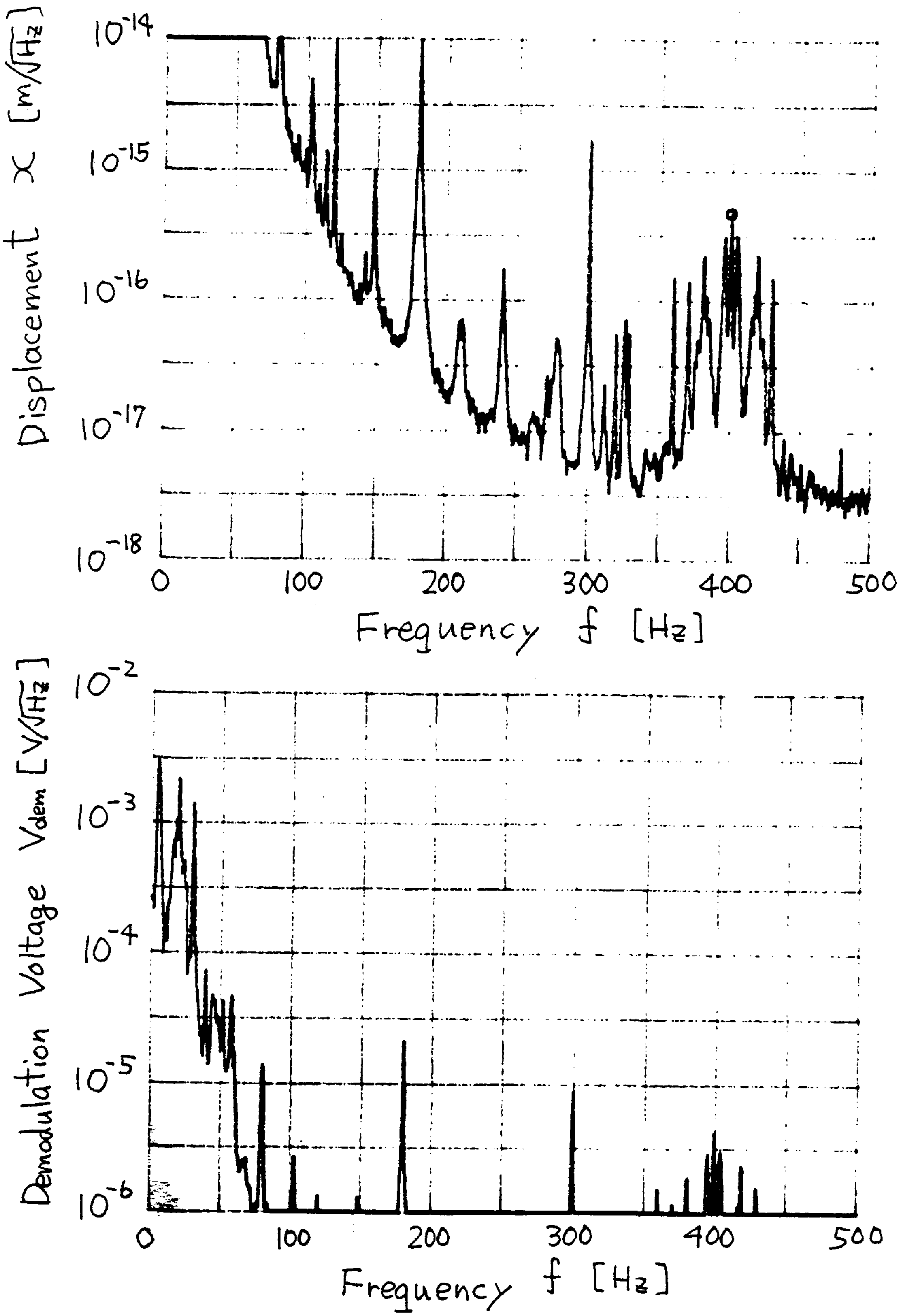
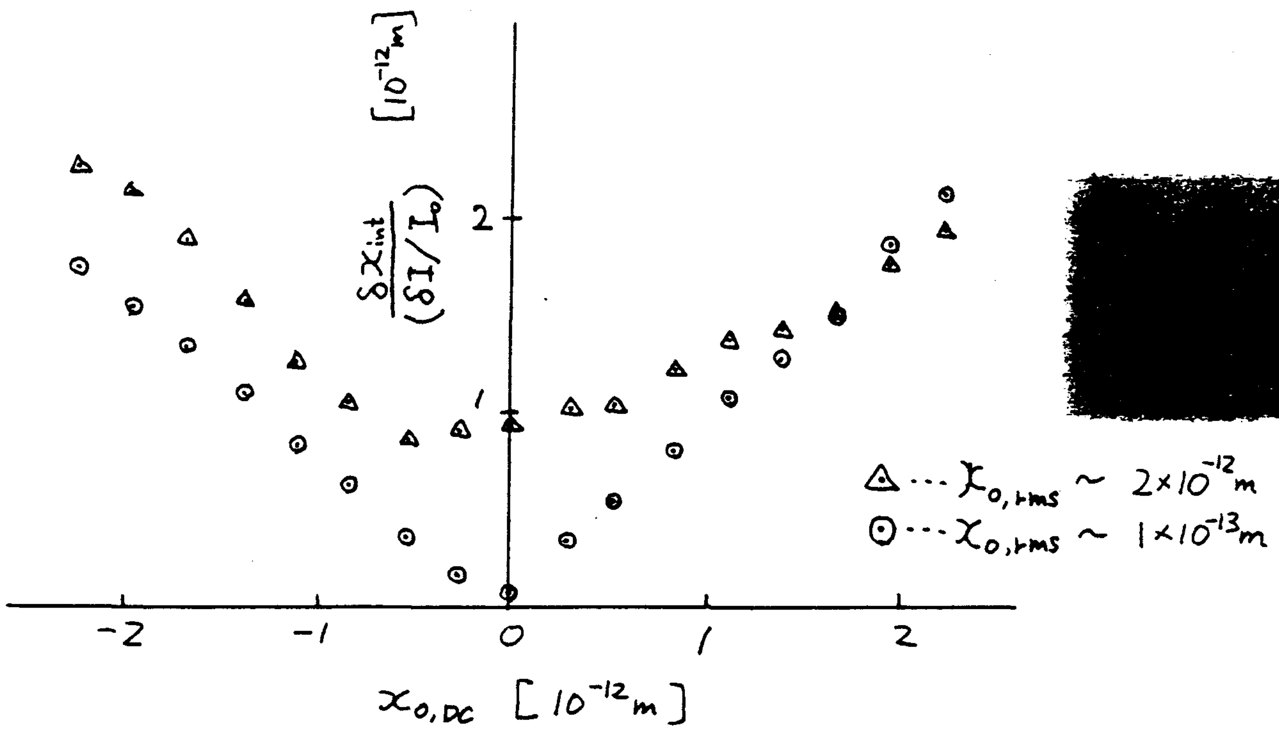


Fig. 4 Displacement spectrum when intensity noise was applied at 400 Hz (top) and the demodulation voltages in the secondary cavity system (bottom).

[redacted]



[redacted]

Fig. 5 Convolution effect of interferometer response t[redacted]00 Hz .

displacement when it is large compared to the rms residual displacement, but fails to track with the DC residual displacement as it decreases to the rms one.

Down-conversion Intensity noise at very high frequencies can be down-converted by coupling with the frequency noise (displacement noise) at comparably high frequencies. It is especially important because high-frequency noise of the gas laser around 100kHz cannot be stabilized very much because of lack of high gain. (Experiment is not completed.)

3. Conclusion

The model of the coupling mechanism from intensity to mimic displacement in a Fabry-Perot cavity RF reflection locking system has been verified using the 40m prototype.

Appendix A

For a given deviation of the cavity from the resonance, the demodulation voltages are proportional to the electric field of the directly-reflected light e(t) and that of the out-of-cavity light αe'(t), where α is the relative amplitude of the out-of-cavity light to the directly reflected light at DC.

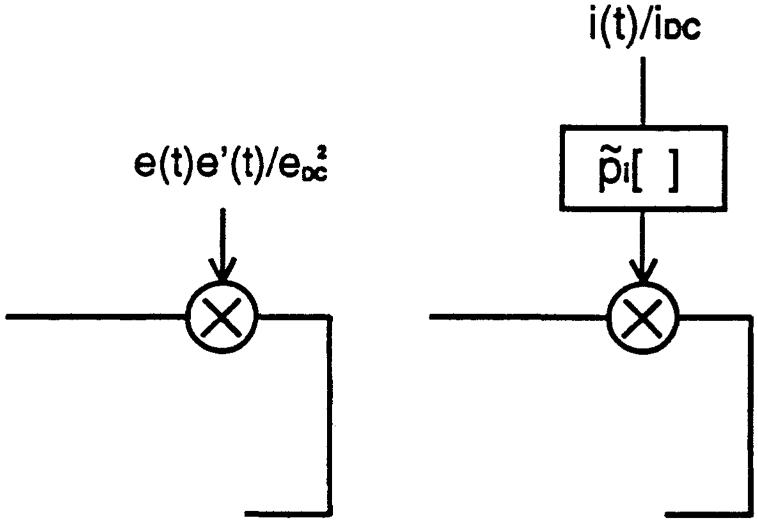
Although e(t) and e'(t) are identical at low frequencies, e'(t) cannot follow e(t) above the cavity pole frequency:

$$e'(t) = \tilde{p}_e[e(t)] \tag{10}$$

Laplace transforming it, we get

$$\begin{aligned} E'(s) &= \tilde{P}^e(s)E(s) \\ &= \frac{1}{1+\frac{s}{c}}E(s) \end{aligned} \tag{11}$$

In the fig. 1, we can re-write the effect of intensity in the following figure (left):



Laplace transforming $e(t)e'(t)/e_{DC}^2$, using Equation 11, we get

$$\begin{aligned}\mathcal{L}\left[e(t)\tilde{p}_e[e(t)]/e_{DC}^2\right] &= \frac{1}{E(0)^2}\sum_{s'} E(s-s')\tilde{P}^{e}(s')E(s') \\ &= \frac{E(s)}{E(0)}\left\{1+\tilde{P}^{e}(s)\right\} \\ &= \frac{I(s)}{I(0)}\,\frac{1+\tilde{P}^{e}(s)}{2}\end{aligned} \tag{12}$$

Therefore comparing the two diagrams in the Figure, we get

$$\begin{aligned}\tilde{P}^{i}(s) &= \frac{1+\tilde{P}^{e}(s)}{2} \\ &= \frac{1+\frac{s}{2c}}{1+\frac{s}{c}}\end{aligned} \tag{13}$$