LIGO-T890023 00-R

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List of 3/1/89

On Local Coincidences, Mid-Stations, and Correlated Noise

1. Summary

The optimum length for mid-length interferometers, and the tolerance thereon, can be determined in principle by finding the extremum of a figure of merit. We examine three figures of merit. Maximizing the precision in the signal ratio argues for a length ratio of unity. Maximizing the discrimination between gravitational waves and spurious influences is best done at a length ratio of 0.453. A figure of merit which includes considerations both of clear discrimination and high event rate argues for a length ratio of 0.69. For each of these figures of merit, the tolerance on the length ratio is never less than about 0.1.

The discriminatory power of mid-length interferometers is examined as a function of the signal-to-noise ratio, using the statistical theory of hypothesis testing. We find, using plausible assumptions, that a mid-length interferometer gives useful local veto information for (full-length) SNRs in excess of 10 sigma. As part of a two-site system, mid-length interferometers give useful discriminants above a threshold around 5 sigma. The limitations of this calculation are discussed.

2. Introduction

This memo grew out of a review of Ron Drever's "Notes on the Concept of a Half-Length/Full-Length Interferometer System." In some ways this work can be considered an extended review, although there is a substantial amount of new quantitative material on two topics. These are the questions of the optimum length ratio and on the statistics of discrimination between true gravitational wave pulses and spurious events.

3. Interferometer Length Ratio: Optimum and Tolerance

It was shown in Ron Drever's memo that coincidences between more than two interferometers will be valuable in rejecting spurious pulses. But there are several aspects of the question of multiple coincidences which I believe deserve quantitative analysis. One of these is, "What is the best length ratio between the full- and mid- length interferometers, and what is the tolerance on that optimum ratio?" It is not immediately obvious that a ratio of one half is the best. In the past some sentiment favored multiple interferometers of equal length as the best way to generate multiple coincidences. It turns out that there is a mathematical way to find at least a partial answer to these questions.

The competing claims of length ratios of unity versus one half have to do with the competition between signal-to-noise ratio and interference rejection. As Ron points out in the last paragraph of his memo, marginal signals may be statistically significant in a full-length interferometer and insignificant in a half-length device. (Indeed, this argument is at the heart of our desire to build a 4 km LIGO instead of a 2 km version.) If, as is quite possible, the first signals that we detect are just at the limits of our sensitivity, then coincidences between full- and half-length interferometers may fail us just when we need them the most. There may as few as one-eighth as many significant signals in the half-length interferometer as in the full-length one (in the case where we are detecting homogeneously distributed sources, a likely though not certain case.) In this case, it would be coincidences between identical interferometers that would be of the greatest value.

Beyond the demand that real gravitational wave pulses give temporally coincident signals from all interferometers, we can further distinguish real gravitational waves from various forms of interference by requiring that all the signals have the right amplitude. Because there is an angular dependence to the interferometer sensitivity which will not be easy to account for unless there are three or more sites, the only signal ratios which are simple to use as checks are the ratios from interferometers at the same site.

We require that a candidate gravitational wave give signals proportional to the length ratio of the two colocated interferometers. Then one figure of merit is the fractional precision with which the ratio can be determined. We can write

$$\frac{\sigma_z}{r} = \sqrt{\frac{{\sigma_s}^2}{s^2} + \frac{{\sigma_l}^2}{l^2}}. \tag{1}$$

Here s is the length of the short interferometer, l is the length of the long interferometer, and $r = s/l$ is the ratio of the lengths. The letter z represents the measured (dimensionless) ratio between the signals in the two interferometers, which will have the expectation value r. σ_s and σ_l represent the noise (in units of length) in the signals from each interferometer. σ_z is then the dimensionless error in our determination of the signal ratio z. (Here we have made the usual assumption that the signal-to-noise ratio is proportional to the length of otherwise matched interferometers. This will usually, but not always, be the case.)

We can simplify this expression to read

$$\frac{\sigma_z}{r} = \frac{\sigma_l}{l}\sqrt{1 + 1/r^2}. \tag{2}$$

Clearly, we minimize the fractional uncertainty by maximizing r. Since $r \leq 1$, then the optimum has $r = 1$. Thus if our requirement is to make the most precise determination of the signal ratio in the two interferometers, then we want the two to be the same length.

The virtue of half-length interferometers, as proposed by Ron, is supposed to be that they will demonstrate that a purported gravitational wave possesses the distinctive signature of signal strength proportional to length. It is hard to imagine any spurious signals which will mimic this property. The optimum we derived in the previous paragraph took no account of this. Using only signal-to-noise ratio as a criterion, that argument favored a signal ratio of unity, which could be mimicked by other effects. It would be nice to find an optimum sensitivity to a ratio other than unity. This would represent the most precise way for us to pick out the characteristic tidal signature of a gravitational wave. Let us consider minimizing the figure of merit

$$\frac{\sigma_z}{r} \times \frac{1}{1-r} = \frac{\sigma_l}{l} \frac{\sqrt{1+\frac{1}{r^2}}}{1-r}, \tag{3}$$

which weights the precision by how far the ratio is from unity.

To find the minimum of this figure of merit, we can follow the standard procedure of finding where the derivative of the expression (with respect to r) is equal to zero. Some involved but boring algebra shows that this requirement is equivalent to the equation

$$r^3 + 2r - 1 = 0. \tag{4}$$

This cubic equation has its sole real root at $r = 0.453$. This is close to $r = 0.5$. In fact, direct evaluation of the expression for the figure of merit shows that the half-length case differs from the optimum by about one percent.

The preceding argument shows that half length is just about optimum for the purpose for which the mid-length interferometers were proposed, to recognize the signature of a gravitational wave and distinguish it from the clutter of local non-gravitational non-statistical effects.

Thus it appears clear that there is value in multiple coincidences both from colocated interferometers of equal length and from sets of half- and full-length interferometers. The advantages of equal length interferometers are optimum precision and equal sensitivity even in the case when signals are of barely significant size. The advantage of half-length interferometers is optimum ability to demonstrate the characteristic gravitational tidal signature.

It is interesting to try to extend the preceding argument to determine the precision to which we ought to set the length ratio. This involves a certain amount of judgement. For one thing, the weighted figure of merit is not God-given, but was proposed as a simple expression which somehow embodies the virtue of the mid-station concept. It is also not always agreed exactly what constitutes an unacceptable degradation of a signal-to-noise ratio. (The site committee spent several hours of conference calls, and more time besides, debating just such questions.) Thus, with all due humility, I propose that we consider both the unweighted figure of merit (expression 2) and the weighted figure of merit (expression 3), at two benchmark levels: at 1.1 times the optimum level, where I claim we would just begin to notice the difference, and at $\sqrt{2}$ times the optimum, at which point the difference is worth attending to very seriously. Graphs of these figures of merit are presented at the end of the text.

Consider the unweighted figure of merit first. Its optimum is $\sqrt{2} = 1.41$, at the value $r = 1$. Ten percent worse is 1.56, at $r = 0.83$. Finally, $\sqrt{2}$ worse is at $r = 0.57$.

The weighted figure of merit has its optimum, 4.43, at $r = 0.453$. It is ten percent greater at $r = 0.31$ and $r = 0.60$. It is $\sqrt{2}$ times the optimum at $r = 0.20$ and $r = 0.73$.

I don't think there is any single obvious interpretation of these numbers. On the face of it, it seems we could say that the mid-stations could be any place between $r = 0.31$ and $r = 0.60$. Whether we ought to be more subtle is the harder question. For example, we might be tempted to say that $r = 0.60$ was preferable to $r = 0.31$ since the signal-to-noise ratio would be a factor of two greater, and the event rate eight times higher (in the cases of the simple models for noise and source distribution discussed above.) But if we really take this line of reasoning seriously, then perhaps we ought instead to be optimizing another figure of merit. For example, we could consider optimizing

$$\frac{\sigma_r}{r} \times \frac{1}{r^{3/2}(1-r)} = \frac{\sigma_l}{l} \frac{\sqrt{1+\frac{1}{r^2}}}{r^{3/2}(1-r)},$$

which multiplies our weighted figure of merit with a factor inversely proportional to the square root of the event ratio in the shorter detector.

This event-weighted figure of merit has its optimum value at $r = 0.69$. It is ten percent larger at $r = 0.57$ and $r = 0.79$. It is $\sqrt{2}$ times poorer than the optimum at $r = 0.46$ and $r = 0.87$.

Now, if we were sure that this figure of merit represented what we want to get out of mid-stations, then we could say that they should be located somewhere between $r = 0.57$ and $r = 0.79$. But I would be surprised if there were immediate agreement to make this the specification. It is a methodological weakness of most optimization schemes that the hardest thing is to come up with the right figure of merit to optimize. As we have seen, different choices can move the optimum location around by a few tenths, comparable to the tolerance determined by any single figure of merit.

4. Utility of Mid-Length Interferometers as a Function of Signal-to-Noise Ratio

Ron Drever's memo lists five "advantages" of mid-length interferometers. In this section I would like to discuss each of these in turn (although out of Ron's original order). My goal is to distinguish between the situations where mid-length interferometers confer a genuine advantage and those where their utility is questionable. Often, the distinction is related to the question of correlated "noise" (or, more accurately, interference or pick-up).

Ron's advantage number 3 is the ability of mid-length interferometers to show that a purported gravitational wave indeed exhibits its character as a strain, or a tidal effect as we called it above. This is, in my opinion, the key feature. Ron's comments also say that this was the original advantage that he had in mind when he first thought of the idea. It is this feature that was the basis of the search for the optimum length which we carried out in the previous section.

Closely related is the feature Ron lists as number 1: since it is unlikely that interference gives the "correct" signal ratio in the two different interferometers, use of the mid-length interferometer gives a powerful and rather unambiguous discriminant against many forms of correlated noise. I think this argument is also basically correct. The only qualification I would apply is that in this function there are some other tools which can do the same job, at least partially. I am referring to our ability to monitor external disturbances which can cause spurious pulses. As a matter of good experimental practice, I am sure that we will want to install monitors of whatever disurbances we imagine can cause spurious events. Knowing the sources of these events would not only help us to characterize our instrument better, but should suggest modifications to make the interferometers more nearly immune to these influences.

Even so, mid-length interferometers still fulfill a useful role. One is that we still get a discriminant, even from influences we haven't thought of in advance. Secondly, we get a discriminant against events which aren't due to external forces, but which are instead caused by things internal to a single interferometer. (The archetype of this class is a sudden stress relaxation in a suspension element.) But note that this last function would be served as well, if not better, by a second interferometer of full length.

The point on which there has been the most confusion is the one Ron lists as number 2. This is the argument that use of mid-length interferometers allows a lower detection threshold for pulses, because the accidental coincidence rate is lower. In support of this claim, Ron presents calculations which compare the accidentals rate of a set of two interferometers at different sites with that obtainable with a set of four interferometers, two at each site. He shows a dramatic difference.

I think there is a benefit here as Ron claims, but not for the reason he gives. His calculations assume that the pulses in each of the four interferometers are uncorrelated. If that were the case, then the benefit of four interferometers over two would be substantial, and indeed three interferometers might even be sufficient (depending on the accidentals rate). But, if the pulses were truly uncorrelated, then we would be better off still if our extra interferometers were full-length ones rather than mid-length. So this calculation is not an argument for mid-length interferometers. It instead says that, for the same reason two interferometers are good, more are better.

In practice, we won't be able to assume that our accidental pulses are uncorrelated, at least until we know a lot more about our interferometers. If there are substantial correlations, then we will in fact be somewhat better off if extra interferometers have different length, since then we will be able to reject many coincidences for failing to have the correct amplitude ratio. But the calculation of how great the reduction in the accidentals rate will be can't be done the way it was in Ron's memo. Instead, the improvement depends on the signal-to-noise in the amplitude ratio of the spurious pulses.

We can make some progress toward a quantitative understanding of the benefits of mid-length interferometers by analyzing a simplified model. Let's set the length ratio at the canonical value of one half, for definiteness. Thus we expect that real gravitational signals give one half as large a signal in the shorter interferometers as in the longer ones. Next we need to model the spurious events. The only simple way to do this is the way Ron proposed, namely that spurious events are likely to be either the same size in both interferometers (assuming they are suitably well-matched) or completely absent in one while present in the other. In particular, uncorrelated internal events are of the second category, correlated pick-up in the electronics "down-stream" from the photodetectors is in the first category, while external effects on the interferometer itself (seismic noise or gas bursts, for example) should give a roughly equal mix of each kind.

(We should pause to note that, in the case of some of these kinds of spurious events, the assumption of a clean bimodal distribution represents a drastic simplification. To match electronic pick-up to better than a factor of two may not be so easy to achieve.

Similarly, seismic or acoustic coupling can easily vary at the factor of two level, because of the structure of standing waves in the environment of the interferometers. We should return at the end of the calculation to consider how much our results could be affected if this model breaks down.)

With the model in hand, we can turn to the standard techniques for the detection of signals in the presence of noise. (I found Anthony D. Whalen's book *Detection of Signals in Noise* very useful, especially chapter 5.) Whenever we get signals, we need to distinguish between two hypotheses. The first is that the signal is due to a gravitational wave. The second is that it is due to some spurious influence. The most powerful statistical technique for deciding between hypotheses is based on examining likelihood functions, which are conditional probability distributions for the signals, given the assumed truth of each of the competing hypotheses.

The measurable quantity we are concerned with is the ratio, z, in the signals from the two colocated interferometers. If we assume for the moment that a gravitational wave arrived with a certain signal-to-noise ratio, then we could derive the expected probability density for the measured z. Call this the likelihood function $p_1(z)$. If we assume instead that the signal is spurious, then we can write an alternative probability density (either centered on $z = 0$, $z = 1$, or a superposition of the two.) Call the second probability density $p_0(z)$. We can then construct the so-called likelihood ratio

$$\lambda(z) \equiv \frac{p_1(z)}{p_0(z)}.$$

The essence of the test is to identify those values of $\lambda(z)$, or equivalently values of z itself, for which it is sufficiently likely that the gravitational wave hypothesis is true.

Much of the statistical literature revolves around different ways of specifying how "sufficiently likely" the hypothesis is. Most methods involve either knowledge of the *a priori* probabilities of the different hypotheses, or the quantitative specification of the "costs" of making errors. One method requires neither. It is the Neyman-Pearson criterion, which maximizes the probability of detection for a given probability of false alarm.

To apply the Neyman-Pearson criterion, we first specify the allowed probability of false alarm, that is the probability that we will say the gravitational wave hypothesis is true when in fact the null hypothesis was correct. By examining the likelihood function $p_0(z)$ (or more specifically an integral of $p_0(z)$), we can find those values of the measured z for which the probability of false alarm is higher than our specification. The values of z not so excluded constitute the window in which we will claim that a signal ratio corresponds to a gravitational wave. The integral of $p_1(z)$ inside this window gives the probability of detection, that is the fraction of gravitational waves which will pass our test. If there exists a window satisfying our false alarm criterion for which the probability of detection is close to unity, then we have a successful hypothesis testing scheme. If, on the other hand, the probability of detection is zero or much smaller than unity, the information at our disposal is not very useful.

First we need to find the likelihood functions for our problem. It would be best to derive analytic expressions, which is possible in simple cases. But for the case of a quotient of two normally-distributed variables, there exists no closed-form expression for the probability density in the general case where the mean may be different than zero. (See J. H. Curtiss, *Annals of Mathematical Statistics*, vol. 12, pp. 409 ff, (1941), and references therein.) Besides, we will want to study the effects of thresholds on the statistics, which complicates matters further. Thus, we have no choice but to perform a Monte Carlo calculation. I have calculated the relevant likelihood functions for signal ratios of zero, one half, and unity, for signal-to-noise ratios in the full-length interferometer of 4,6,8, and 10 sigma. I required that the "measured" signal in the full-length interferometer be greater than 3 sigma. Histograms of these functions are presented in the appendix.

Next it is necessary to specify the false alarm probability that we can tolerate. This depends on the rate at which large events occur in the full-length interferometers, through the relation

$$\alpha = N_{accidentals} P_{fa}. \tag{5}$$

Here α is the false alarm rate, $N_{accidentals}$ is the rate of threshold-crossing events, and P_{fa} is the false alarm probability, that is the probability that we claim an event matches the gravitational wave signature when it is in fact due to a spurious cause. $N_{accidentals}$ is determined by the details of the interferometer, which we won't know until it is built. For the sake of definiteness, let us assume it takes the value of $20hr^{-1}$ for a single interferometer, the value Ron took on the basis of our experience to date in the prototypes. Ron went on to show that this corresponds to a chance coincidence rate of $27yr^{-1}$ for the case of two spatially separated (and thus uncorrelated) interferometers. The singles rate corresponds to a threshold of around 5 sigma, assuming gaussian statistics and a template "spacing" of 3 msec.

I assume that the net false alarm rate that we can tolerate is of the order of $0.1yr^{-1}$.

Translation of this into the relevant P_{fa} requires specifying $N_{accidentals}$, which in turn depends on how we imagine we will use the mid-length interferometers. I consider three cases. We place the most extreme demand if we imagine coincidences only between one full-length interferometer and one co-located half-length interferometer. Then, the relevant $N_{accidentals}$ is the singles rate $20hr^{-1}$. This yields the demand that P_{fa} be less than 6×10^{-7}. On the other hand, we actually plan to use mid-length interferometers to supplement two full-length interferometers. The accidentals rate from the two in coincidence is only $27yr^{-1}$, so $P_{fa} = 4 \times 10^{-3}$. This is the false alarm probability we require from mid-length information. If we have only one mid-length interferometer, this is the false alarm probability we require from it. If we have two independent ones, then the allowed P_{fa} becomes the square root of the above number, or 0.06.

Now we need to examine our model likelihood functions in the context of these benchmark P_{fa}s. The window we expect in the variable z will have its lower bound set by the high-z tail from events with expectation value zero, and its upper bound determined by the low-z tail from events with expectation value unity. For this reason, the most convenient way to present $p_0(z)$ is in the form of its integral, from positive infinity down to z in the case of expectation value zero, and from negative infinity up to z for the case of expectation value unity. Graphs of these integrals are presented in the appendix. We also show plots for $p_1(z)$, which give the integral from minus infinity to z for $z < 0.5$ and the integral from plus infinity down to z for $z > 0.5$.

The results of this analysis are summarized in Table 1. For each of the three thresholds, the table shows the allowed range of z. Also given is the integral of $p_1(z)$ within the allowed window, the so-called "probability of detection". Note that, for computational speed, I restricted the Monte Carlo calculations to 10^5 trials each, so there is no information presented for probabilities below 10^{-5}. This is not a serious problem for the cases considered, since for all except the 10 sigma case there is no allowed window at all even at the 10^{-5} level. For 10 sigma, the integrals are so steep at 10^{-5} that extrapolating can be done without introducing serious error, I hope.

As the table shows, only at full-length signal levels of around 10 sigma do we start to obtain reliable discrimination ($\leq 0.1yr^{-1}$) by combining a single mid-length interferometer with a single full-length device. It appears that this elaborate statistical procedure is confirming the straightforward conclusion that a half-length interferometer is half as sensitive as a full-length one, which we assumed was operating at a five sigma threshold.

The other false alarm thresholds are more interesting. Useful information from mid-length interferometers in conjunction with two full-length interferometers is available at signal levels in the 4 to 6 sigma range, roughly matching our assumption about the singles rates in the full-length interferometers. Two mid-length interferometers clearly give reliable discrimination at lower levels than just one.

This calculation shows quantitatively how mid-length interferometers can provide useful information in two different regimes. For pulses with very high (in excess of 10 sigma) SNR, mid-length interferometers can provide an important local veto. For the lower SNR levels at which a two-site search is likely to operate, mid-length interferometers can provide supplemental information to reduce the accidental coincidence rate.

We should consider how this judgement of the utility of mid-length interferometers depends on the estimated accidentals rate. If the rate were higher, we would need larger SNR to compensate for the required lower P_{fa}. If the rate were substantially lower (a little more than a factor of ten, say), then just the two-site coincidence between the full-length interferometers would be sufficient to meet our false alarm criterion. How likely is this latter case is a matter of judgement at this point. In any case, the mid-length interferometer would still provide a qualitatively unique piece of information, the tidal signature.

Remember that this calculation rests on the assumption that correlated noise produces signals of equal size in the two interferometers. Breakdown of this assumption could be determined, after the fact, by examination of the actual histogram $p(z)$ of the signal ratios. If our model were in fact substantially violated, then the advantages of mid-length interferometers are reduced below the levels presented in Table 1. There would still be statistical information available in the histogram itself, as it is unlikely that the distribution would peak at $z = 0.5$.

Thus, I conclude from this calculation that although mid-length interferometers provide valuable information, the improvement in pulse sensitivity is not nearly as great as that claimed in Ron's memo.

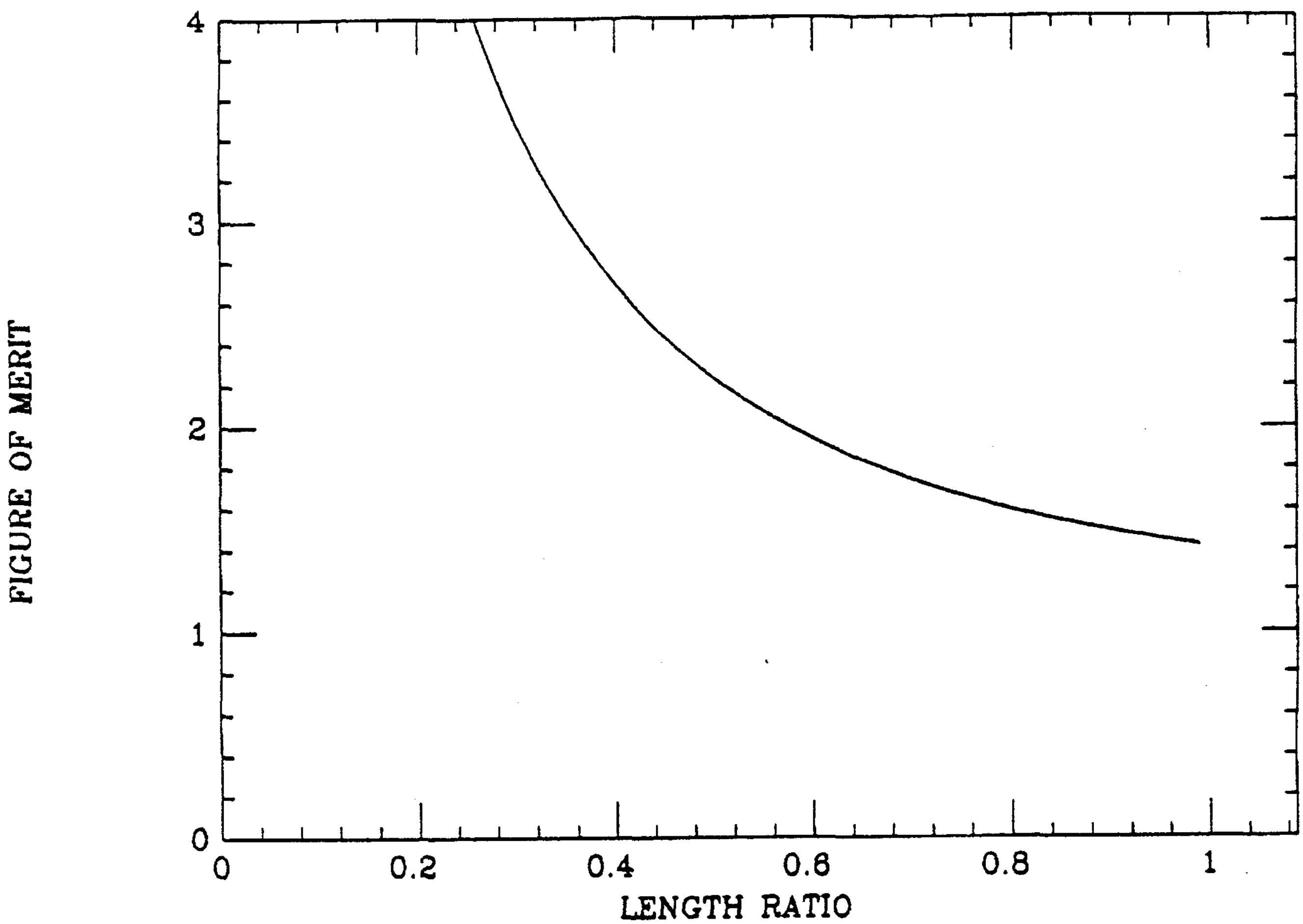
Finally, I would like to comment briefly on the utility of mid-length interferometers for periodic and stochastic searches, Ron's advantages 4 and 5. In the case of periodic searches there is already another discriminant between gravitational waves and false signals, in addition to requiring a match between full-length interferometers at two sites. True periodic gravitation waves will show a Doppler shift due to the earth's motion. Finally, stochastic noise upper limits could be more sensitive from co-located interferometers, since they respond in phase to the whole sky instead of the small part of the sky which separated interferometers share. But this advantage can only be exploited if the amount of correlated noise in the co-located interferometers is small. If this were the case, a pair of full-length interferometers might give a better upper limit still.

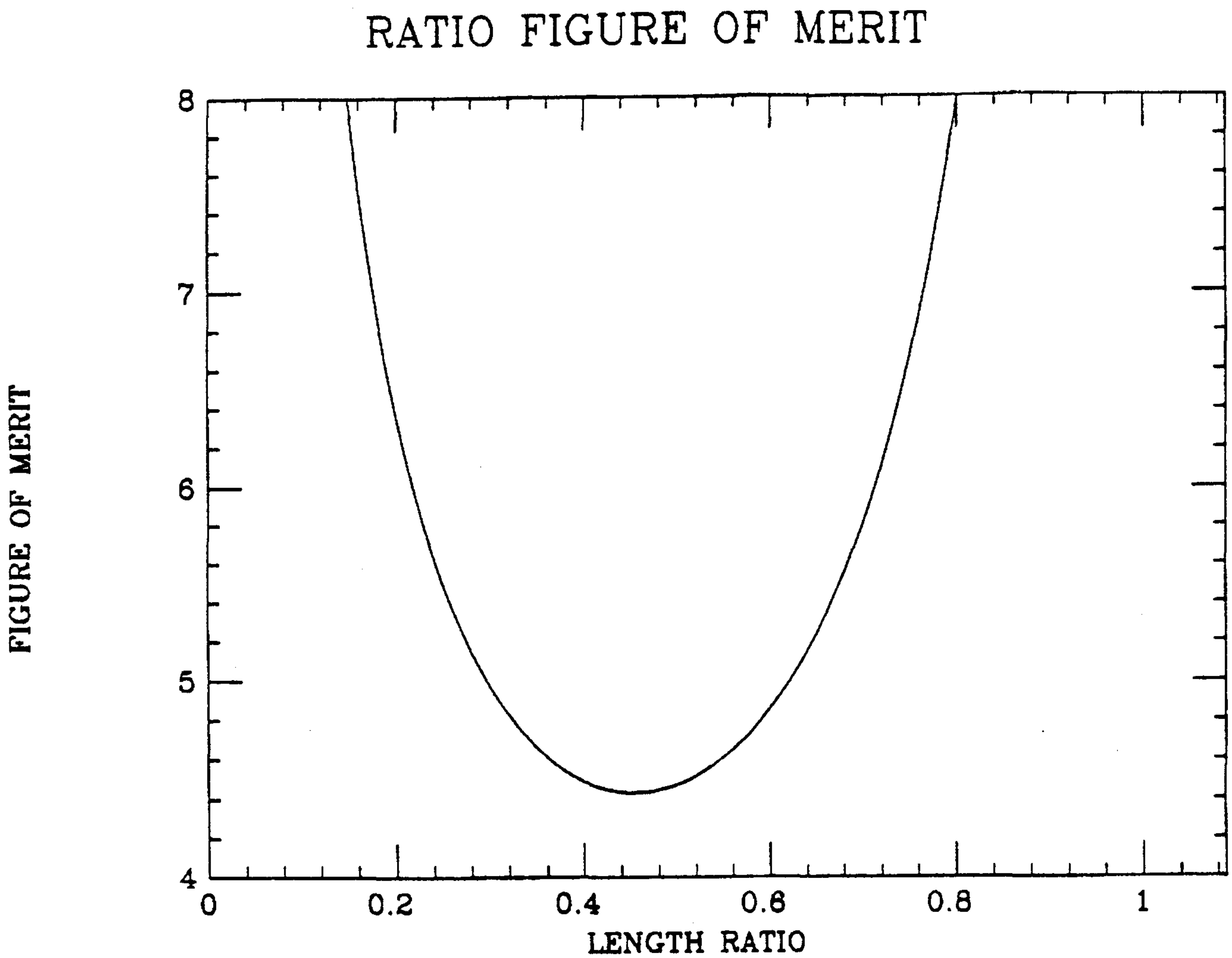
5. Discussion

It seems likely that, in the early phase of receiver operation, tracking down the sources of spurious pulses will be of paramount importance. For that purpose auxiliary sensors will have an important role, but clearly mid-length interferometers give a characteristic discriminant which will be very valuable. In the mature phase of the operation of a receiver system, one can hope that the number of correlated spurious pulses has been reduced substantially, or at least that the appropriate veto sensors have been found. In that case, we could consider reducing our reliance on the mid-length interferometers. If we think this is a realistic expectation, then we might plan to provide fewer spaces for mid-length interferometers than we might otherwise have done. A radical plan would envision all receiver shake-down at one prime site. Then, it might even be sufficient to build no mid-length interferometers at all at the subsidiary site, if we thought the statistics looked promising for two full- plus one mid-length interferometer.

Peter R. Saulson
February 21, 1989

UNWEIGHTED FIGURE OF MERIT




RATIO FIGURE OF MERIT
FIGURE OF MERIT
8
7
6
5
4
0
0.2
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0.8
1
LENGTH RATIO

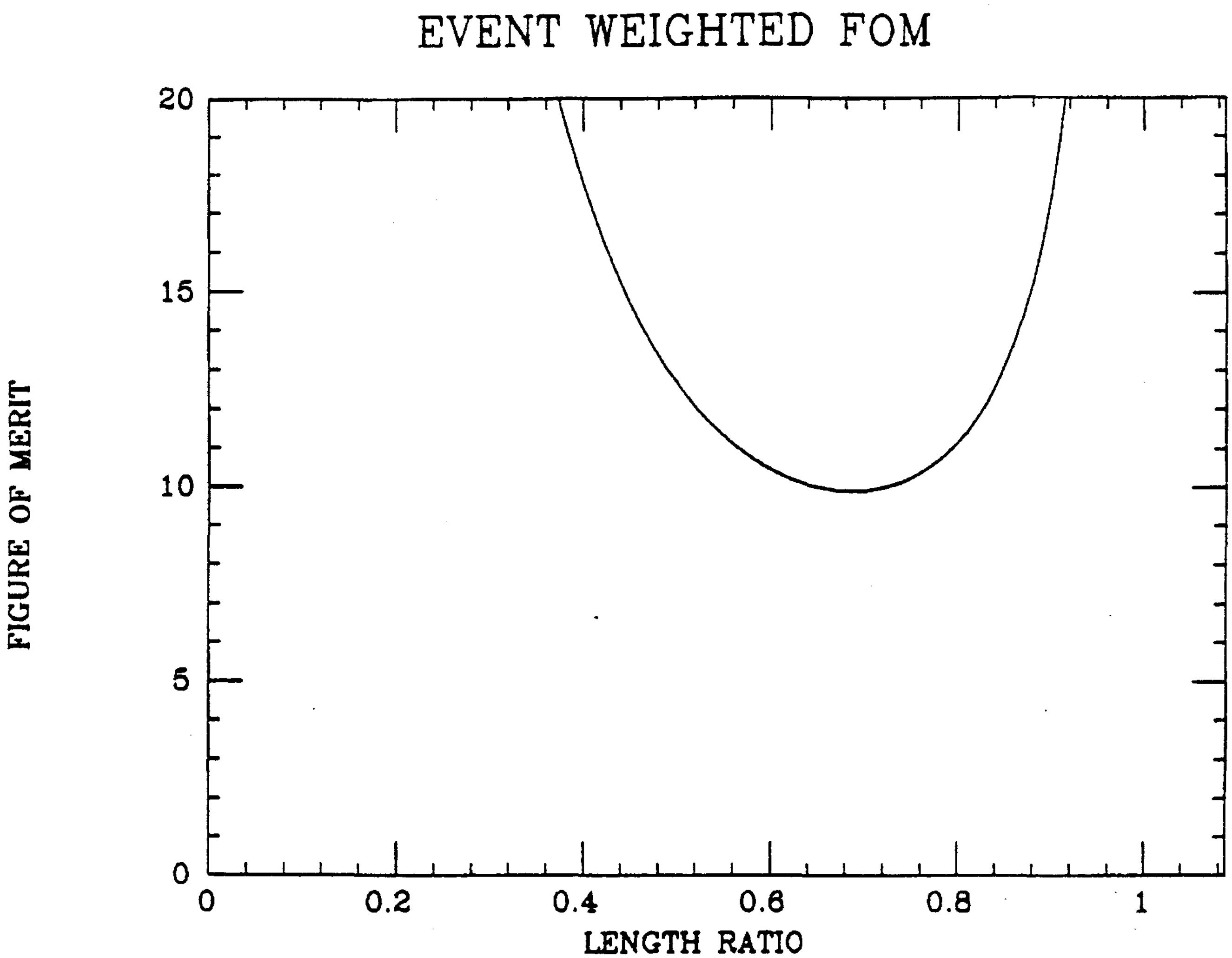
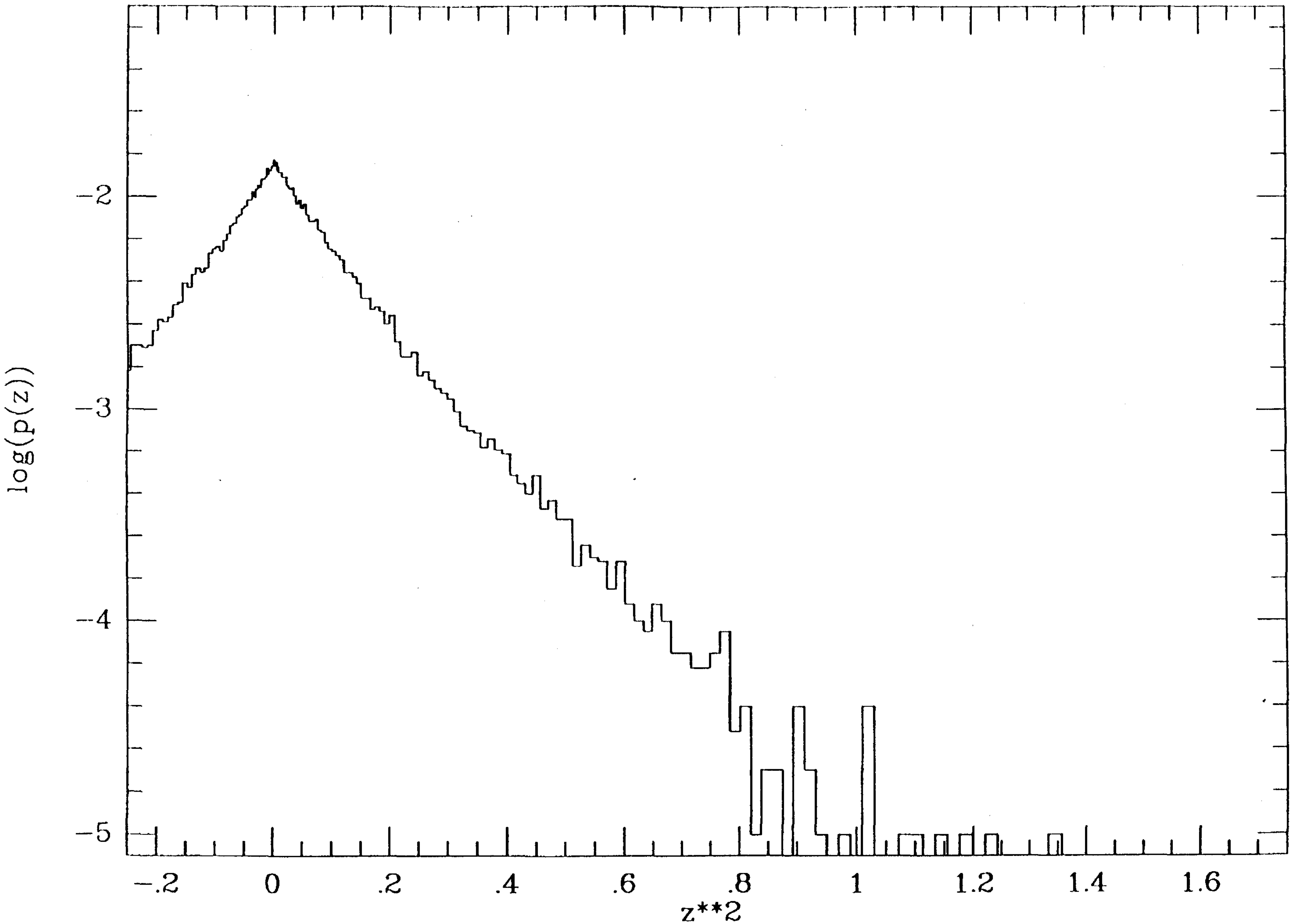

EVENT WEIGHTED FOM
FIGURE OF MERIT
LENGTH RATIO
0
5
10
15
20
0
0.2
0.4
0.6
0.8
1

TABLE 1

SNR, Threshold		Signal Ratio Window	Probability of Detection
4 sigma			
	log P_fa < -5	1 < z < 0 (!)	0
	< -2.4	0.67 < z < 0.29 (!)	0
	< -1.2	0.36 < z < 0.54	0.43
6 sigma			
	log P_fa < -5	0.84 < z < 0.29 (!)	0
	< -2.4	0.49 < z < 0.50	small
	< -1.2	0.29 < z < 0.68	0.72
8 sigma			
	log P_fa < -5	0.59 < z < 0.43 (!)	0
	< -2.4	0.36 < z < 0.61	0.62
	< -1.2	0.20 < z < 0.75	0.90
10 sigma			
	log P_fa < -5	0.45 < z < 0.53 (est.)	0.2
	< -2.4	0.27 < z < 0.67	0.90
	< -1.2	0.16 < z < 0.80	0.99

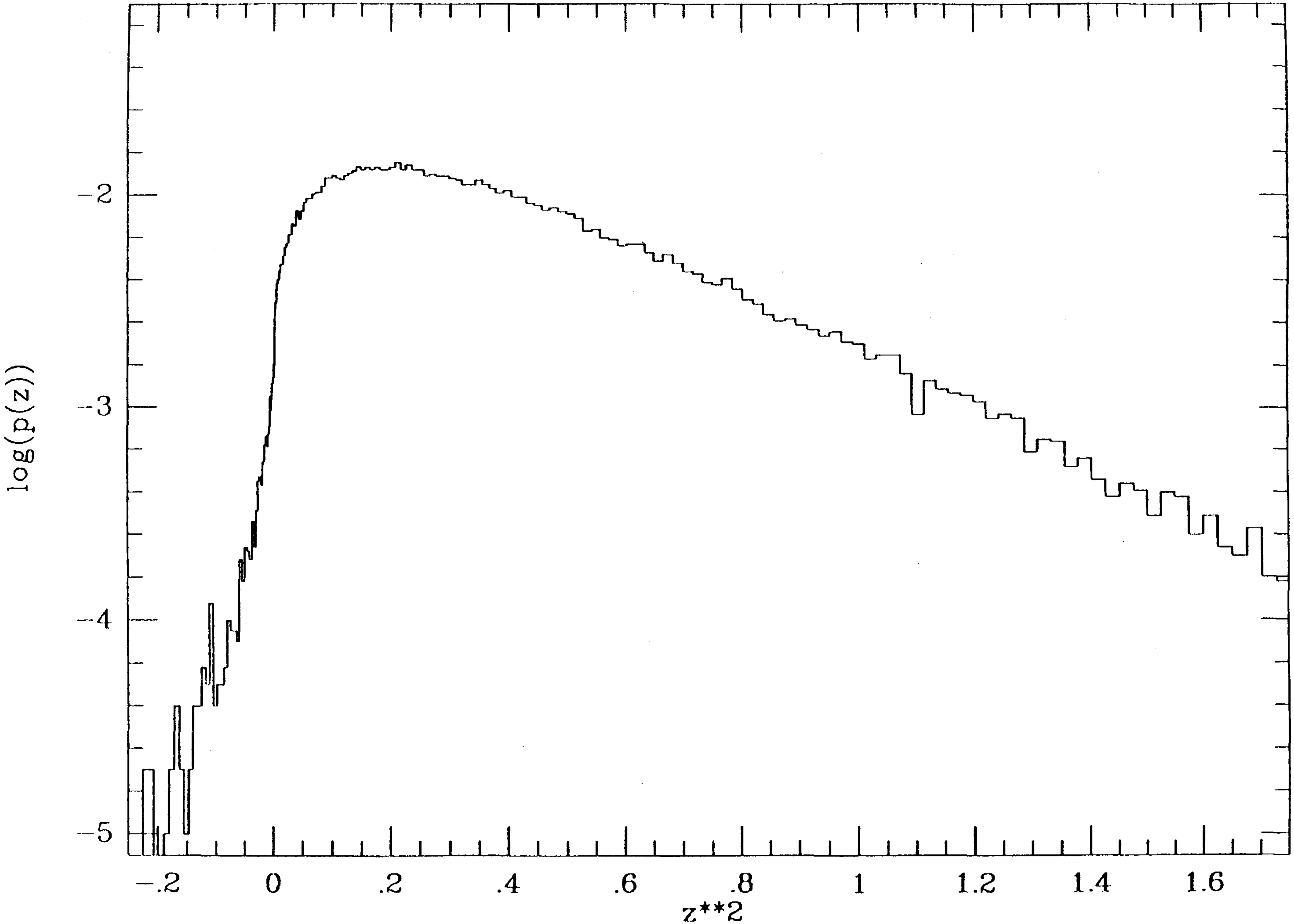
APPENDIX

Contains graphs of likelihood functions, and of integrals of likelihood functions, for signal ratios between half- and full-length interferometers. At each of 4 signal-to-noise ratios in the full-length interferometer, we present graphs for the cases of expected ratios of 0, one half, and 1. These are used to derive the results summarized in Table 1.

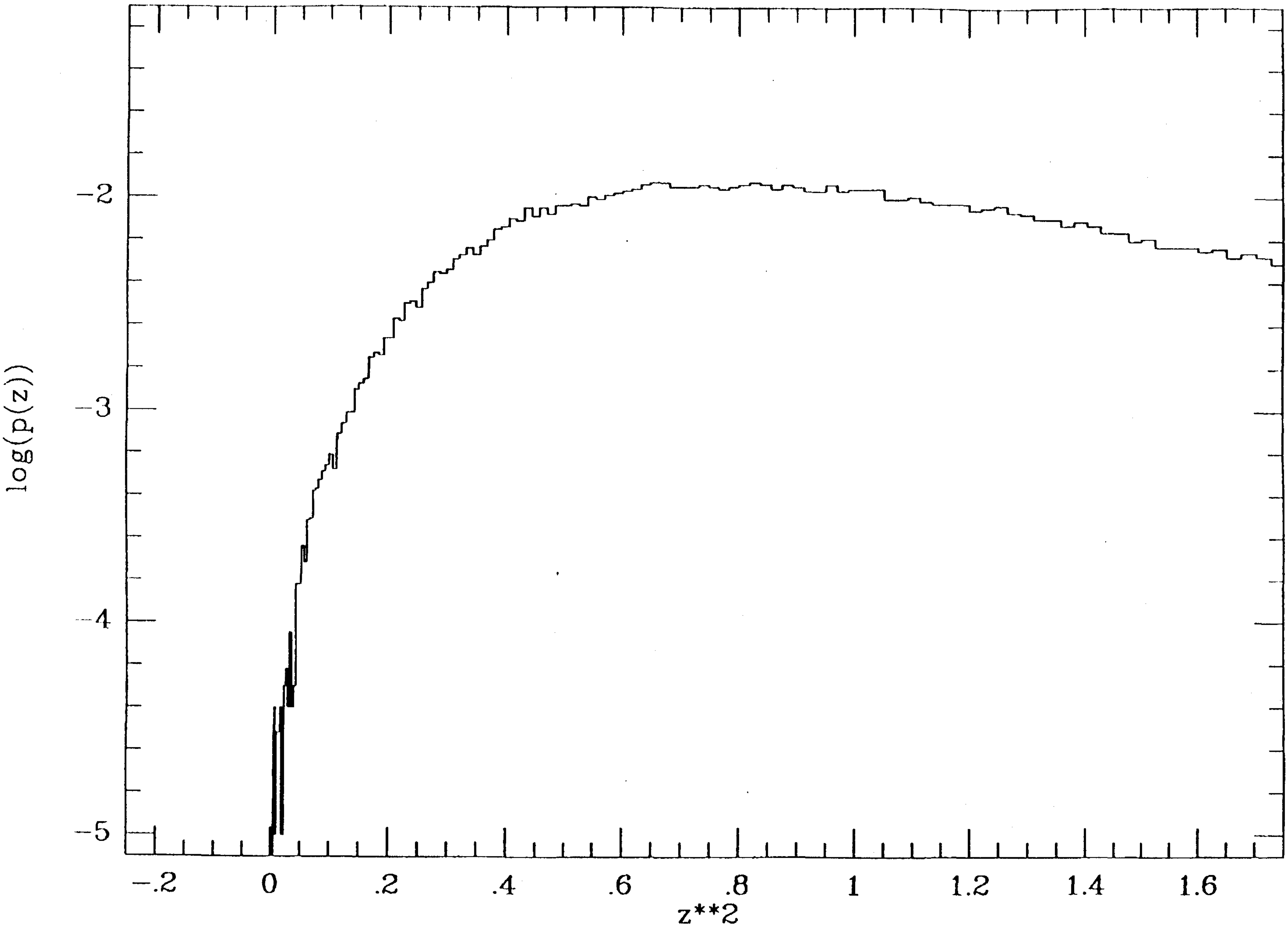


ratiocut.dat

ratio: 0 4σ



ratiocut.
cpl
4σ



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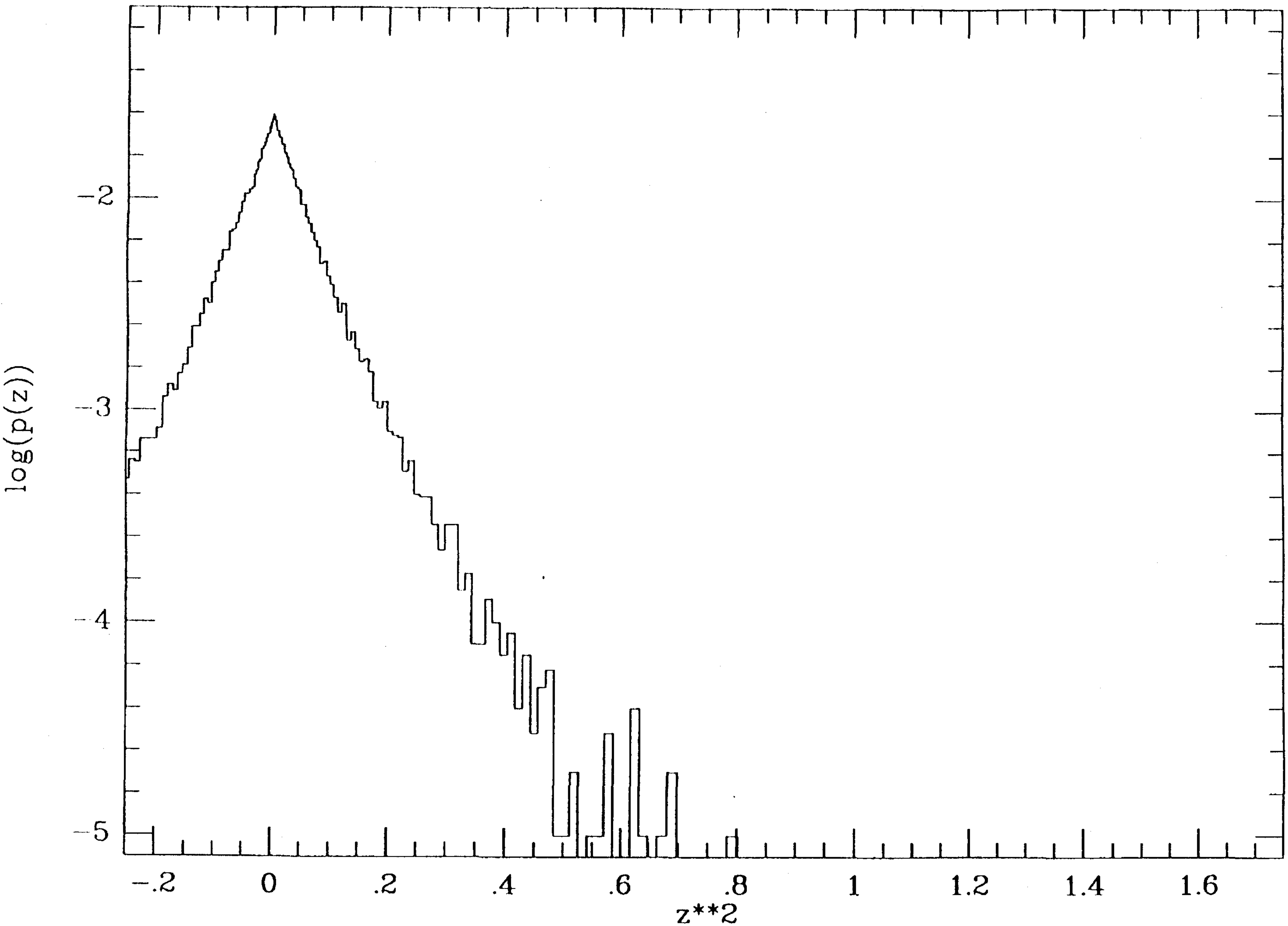
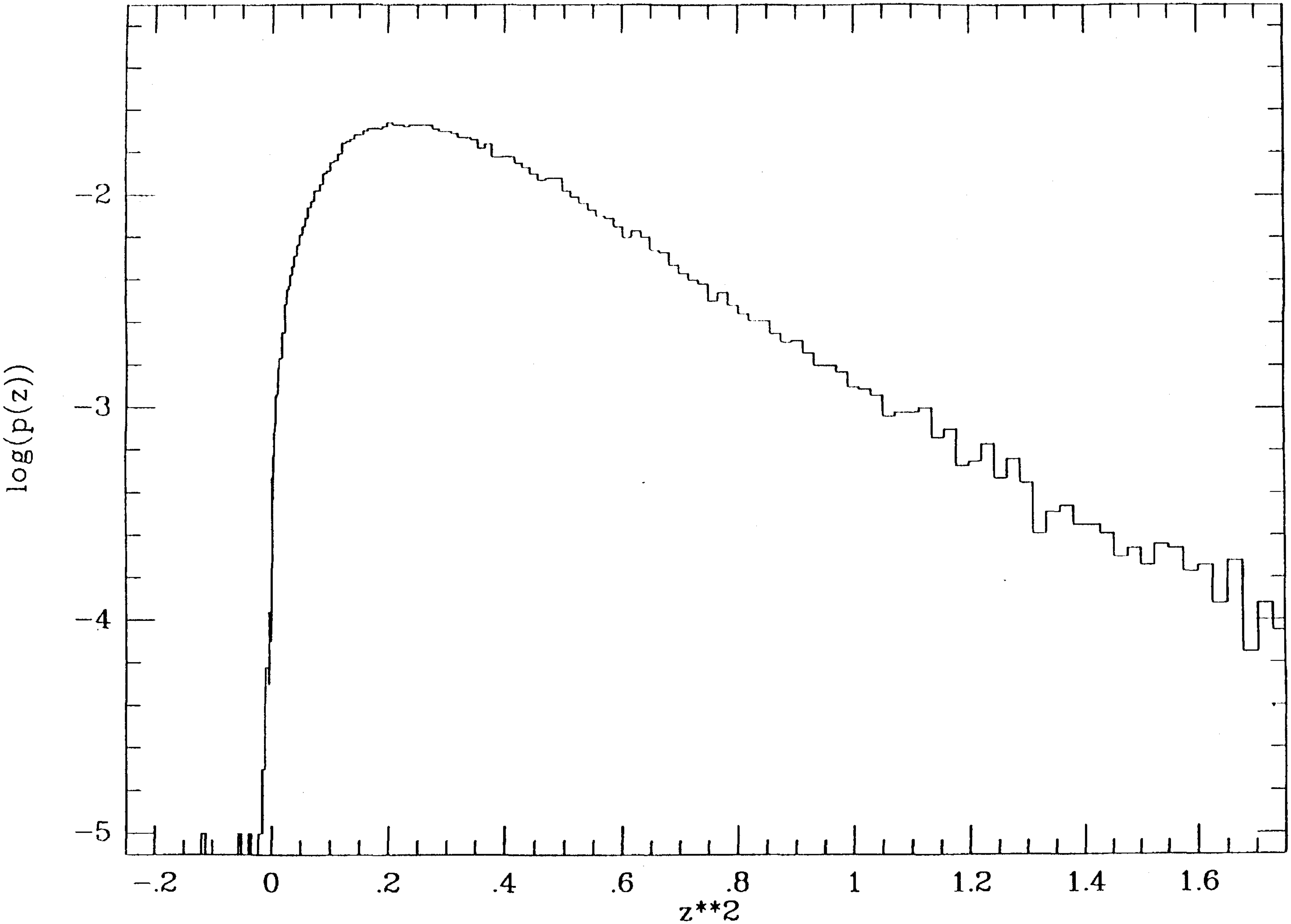
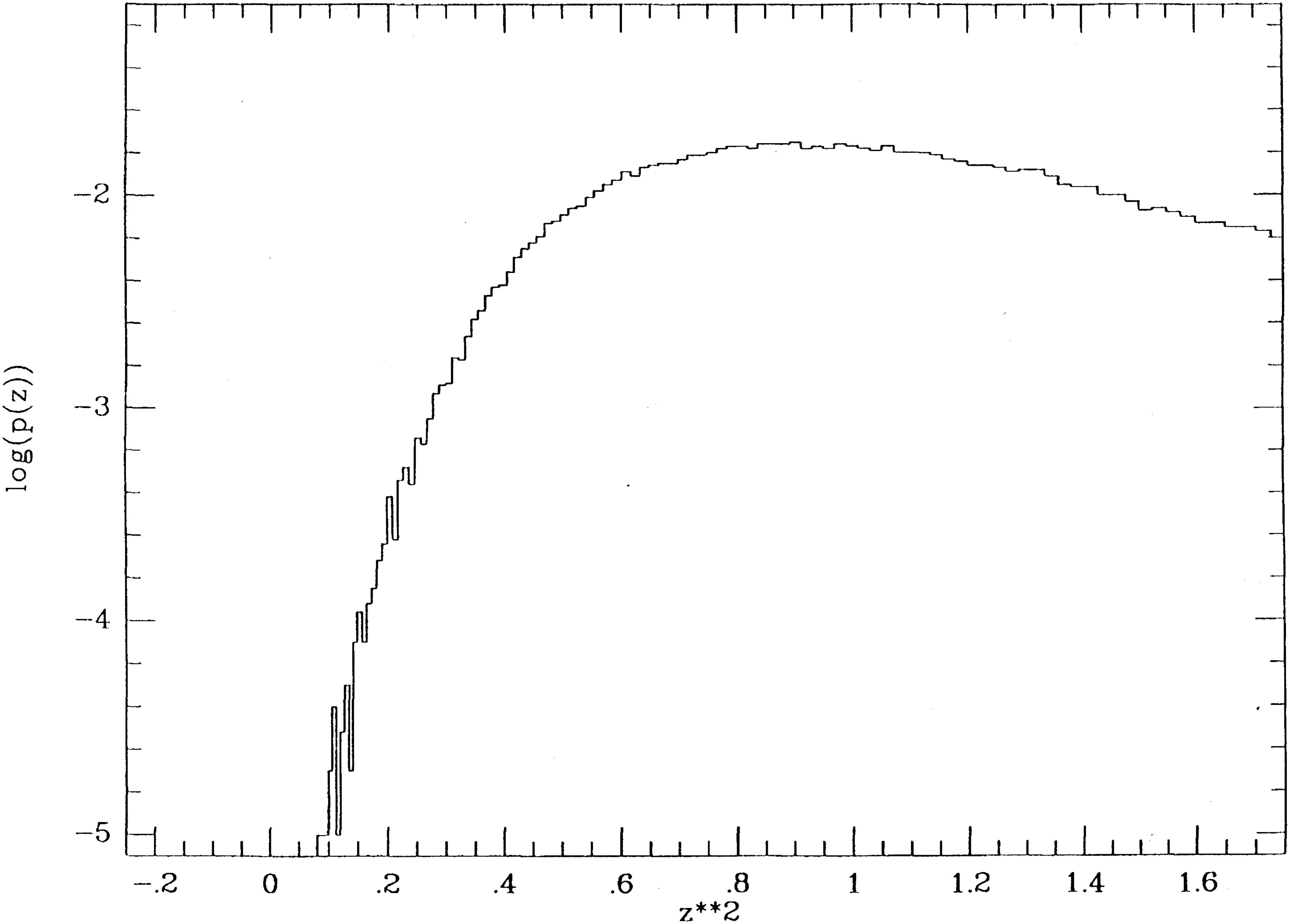


ratiocu

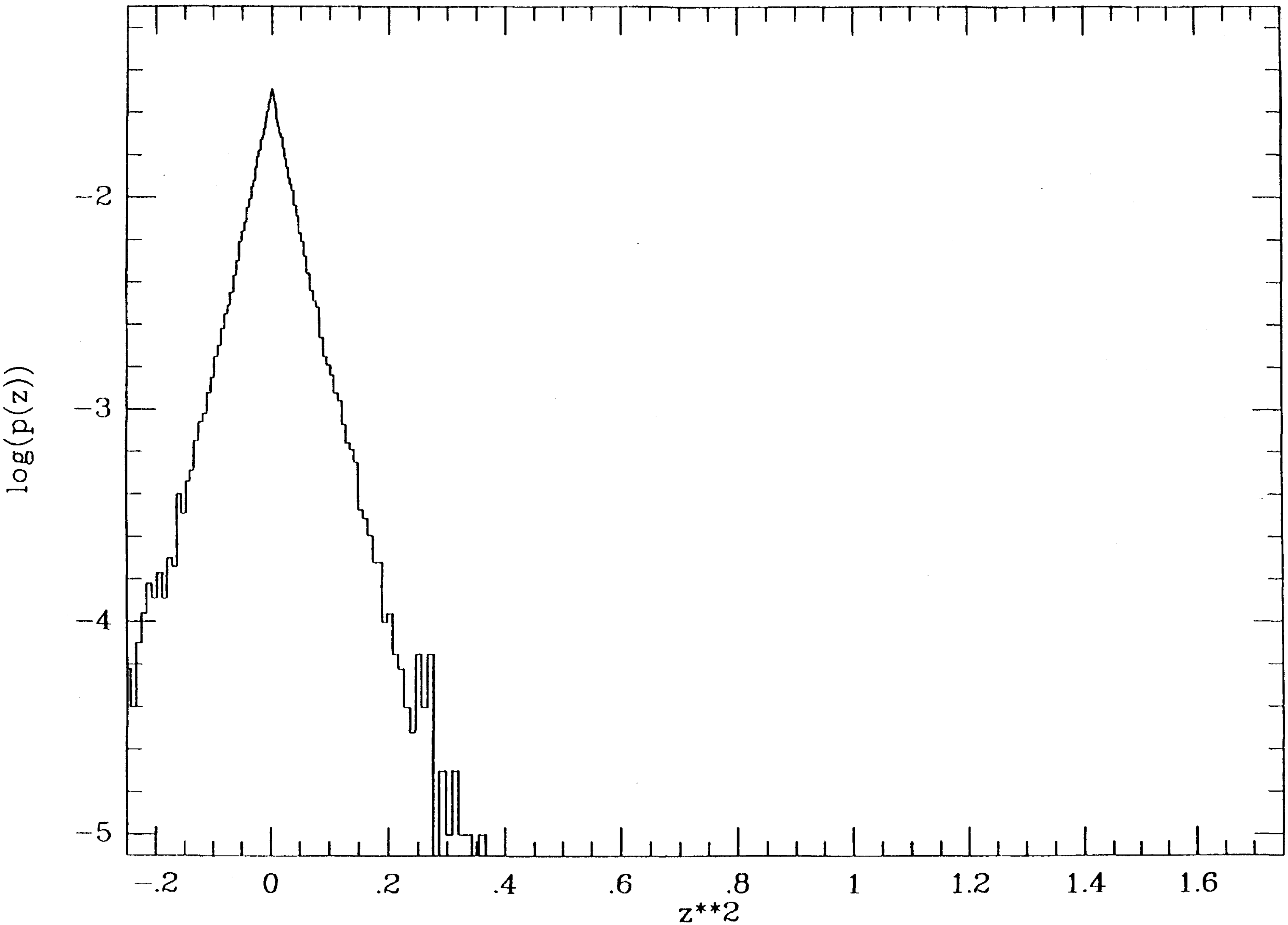
ratio=0, 6σ



ratio = 1/ 6σ



log(p(z))
-2
-3
-4
-5
-.2
0
.2
.4
.6
.8
1
1.2
1.4
1.6
z**2

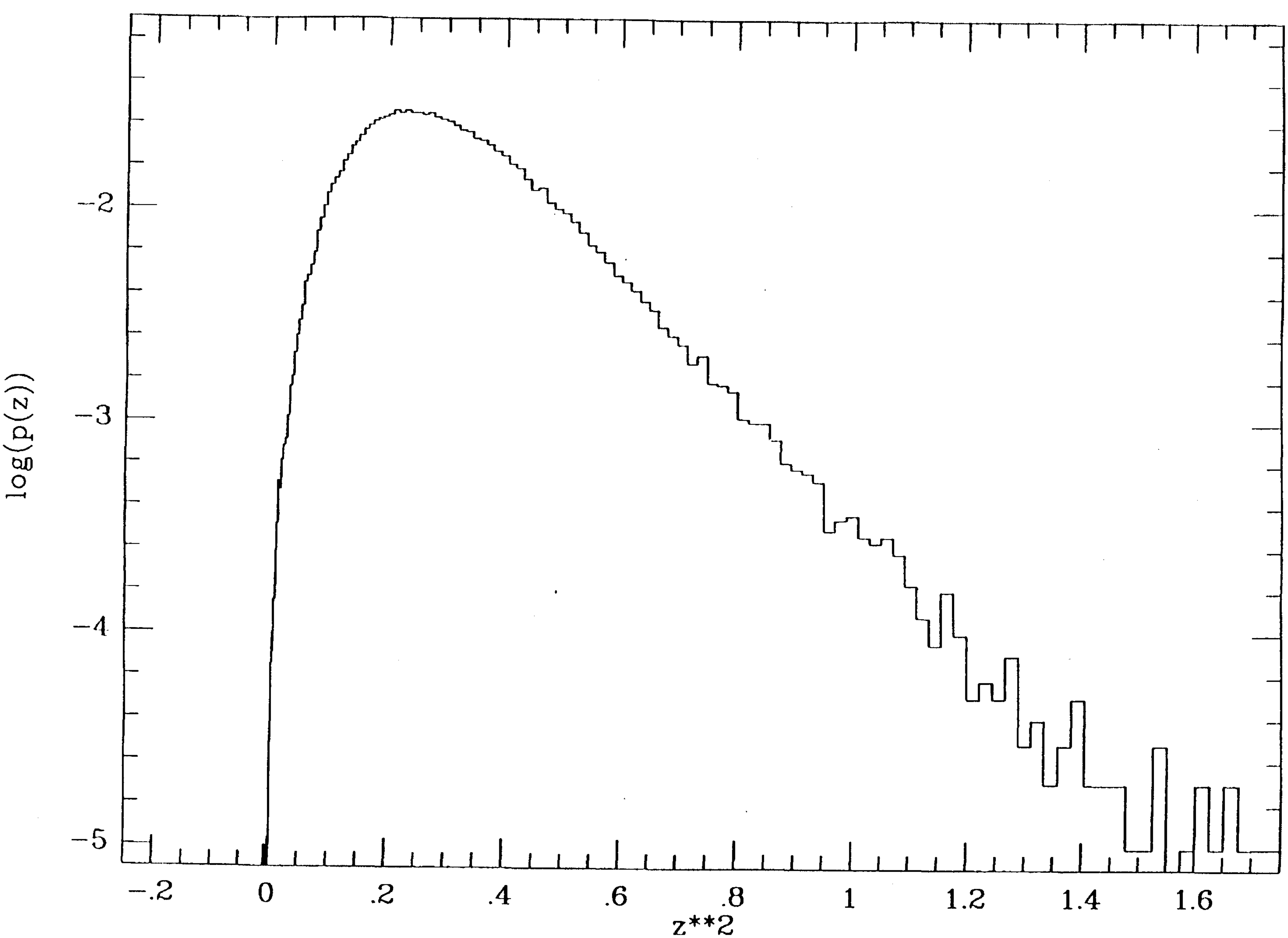


ratiocut.

ratio= 0 , 8σ

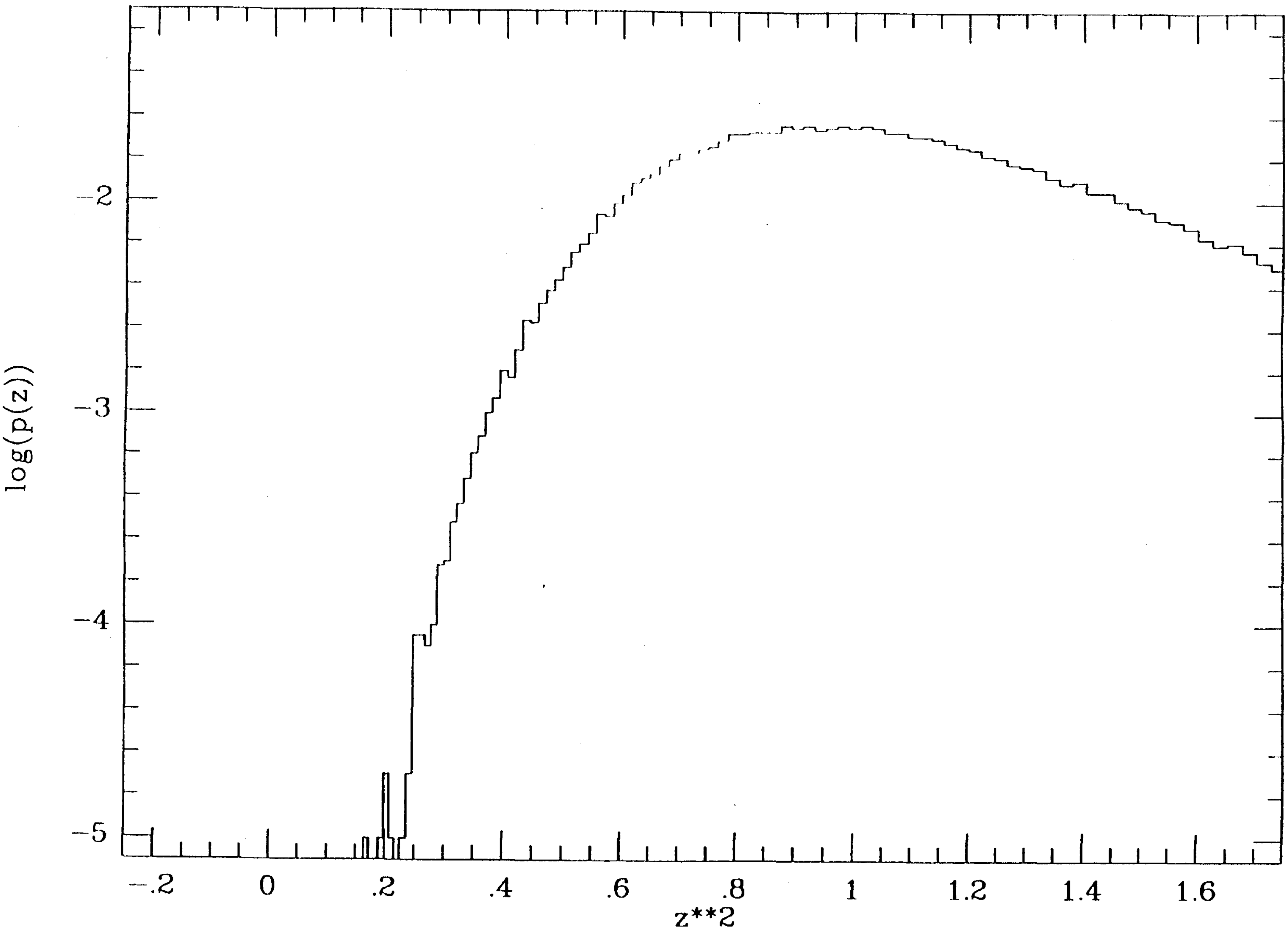
co

02/27/89 13:55 ☎617 253 7014 MIT GRAVITY GRP. → LIGO ☎025



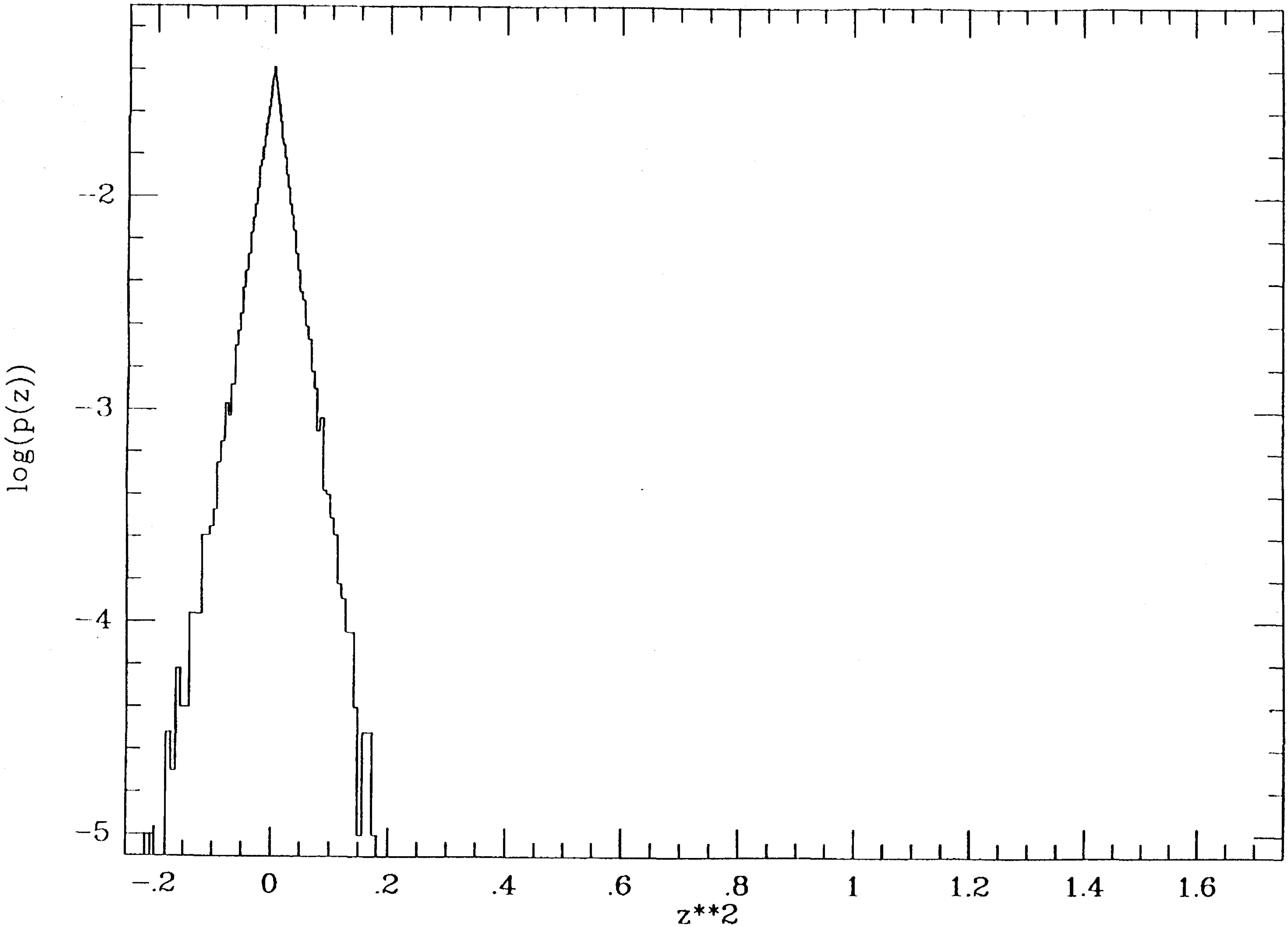
ratiocut.

ratio = $\frac{1}{2}$, 8σ



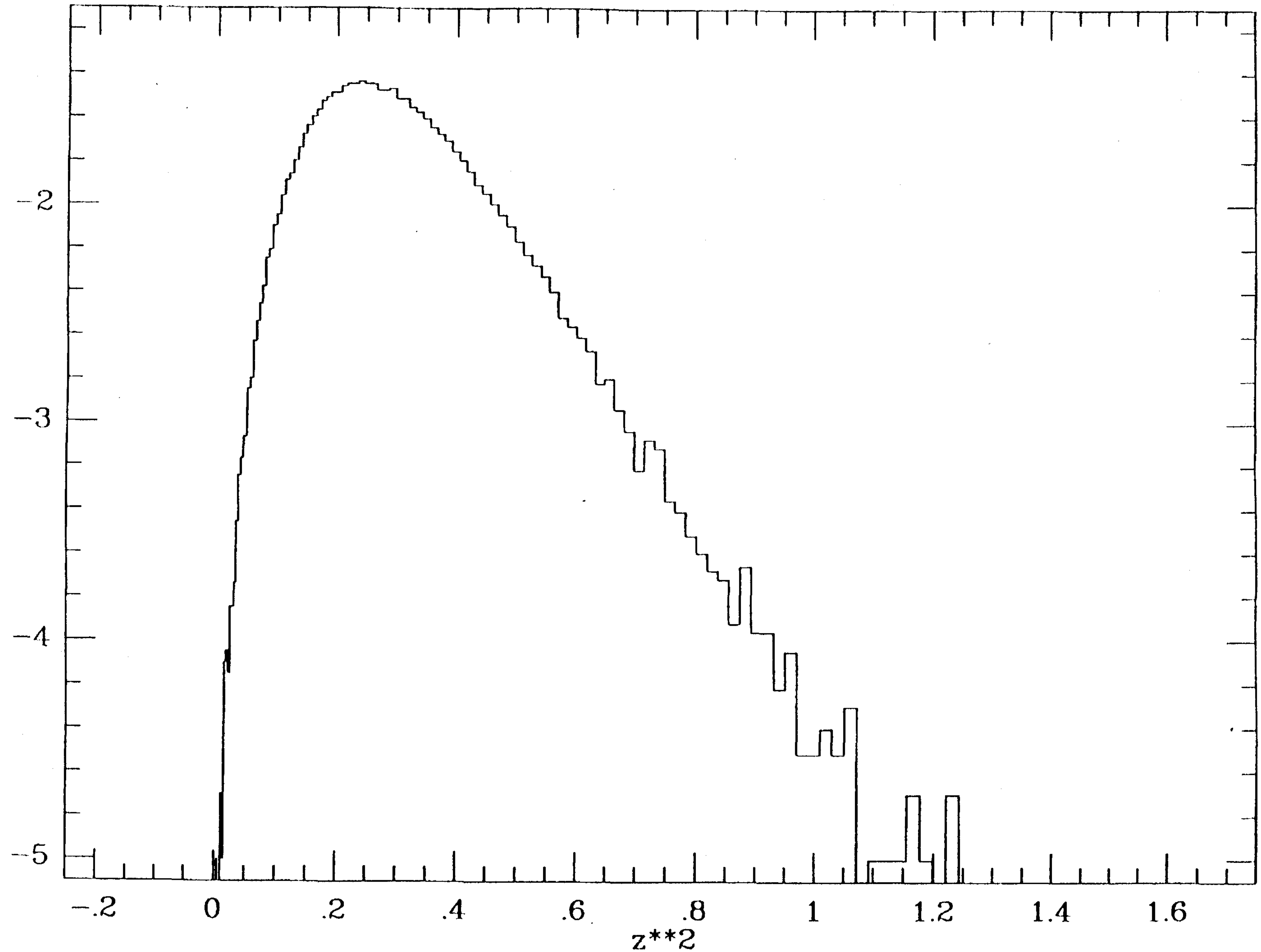
ratio
ratio= 1 . 8σ

02/27/89 13:57 ☎617 253 7014 MIT GRAVITY GRP. →→→ LIGO ☑027



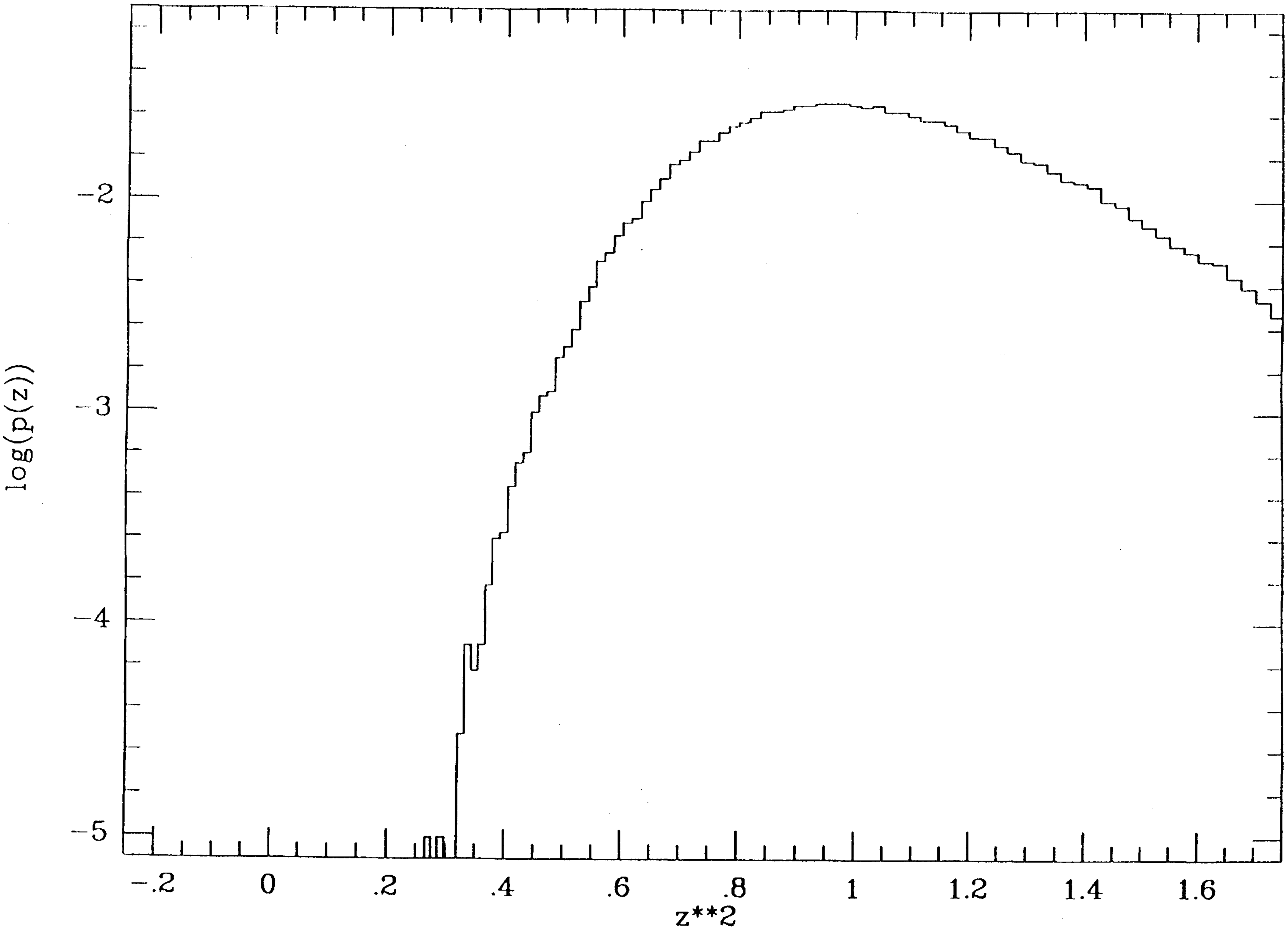
ratio=0,105

ratiou



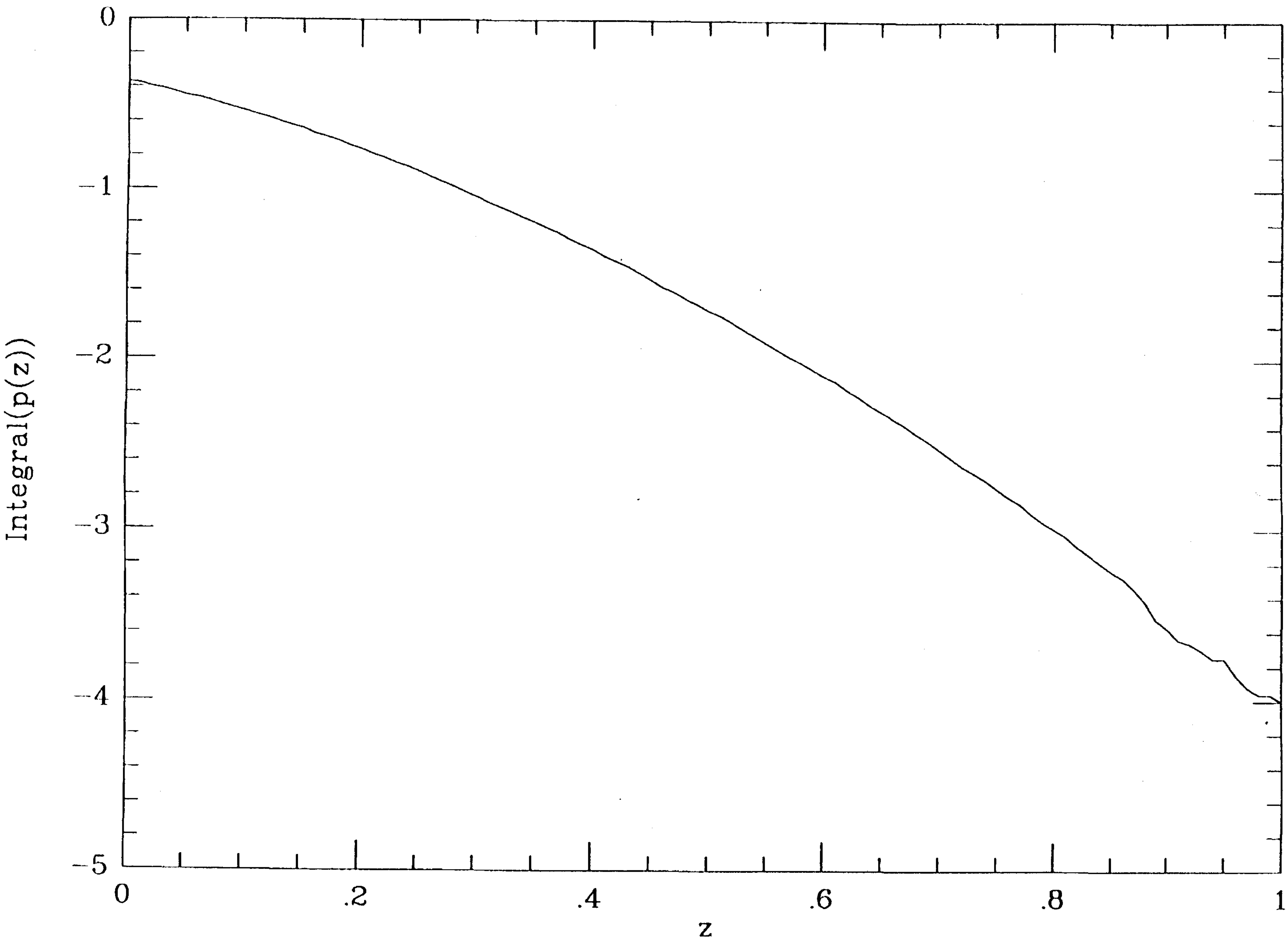
ratio = 1/2, 100

ratiout d
col



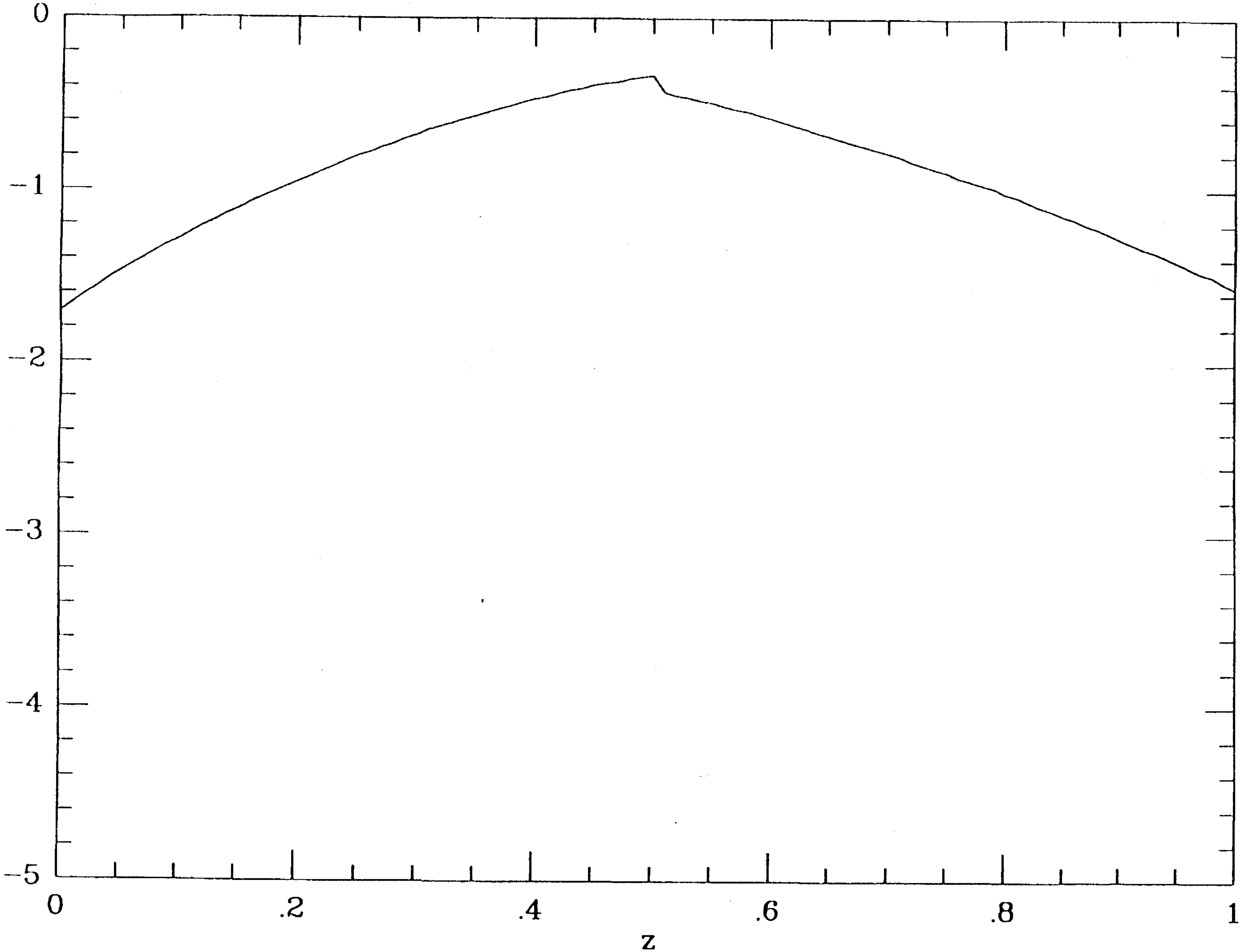
ratiocut.
ratio=1.
col

02/27/89 13:59 ☎617 253 7014 MIT GRAVITY GRP. →→→ LIGO ☑030



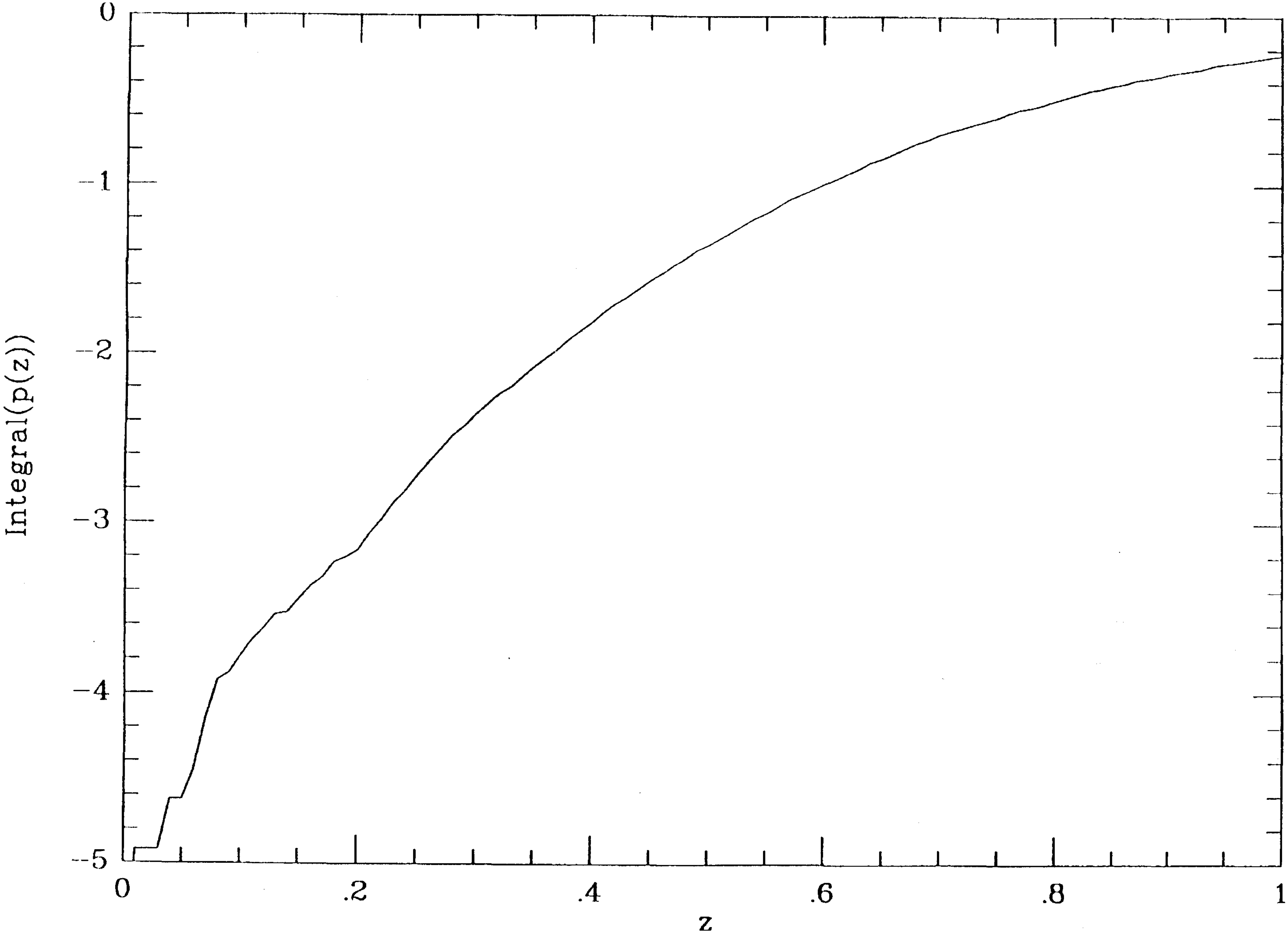
ratio=0, 4σ

ratio=cut
tail



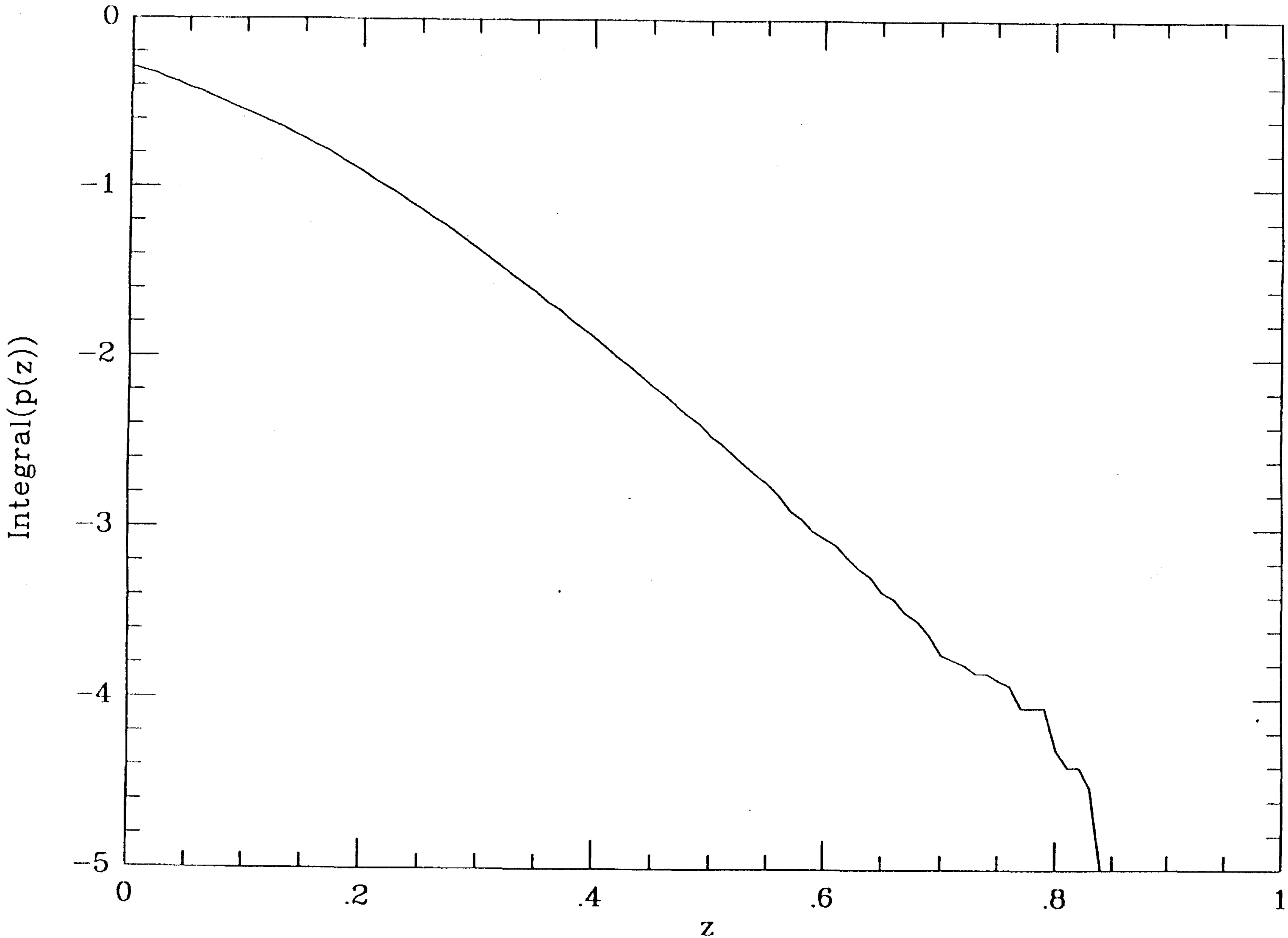
ratio $=\frac{1}{2}$, 4σ

ratio



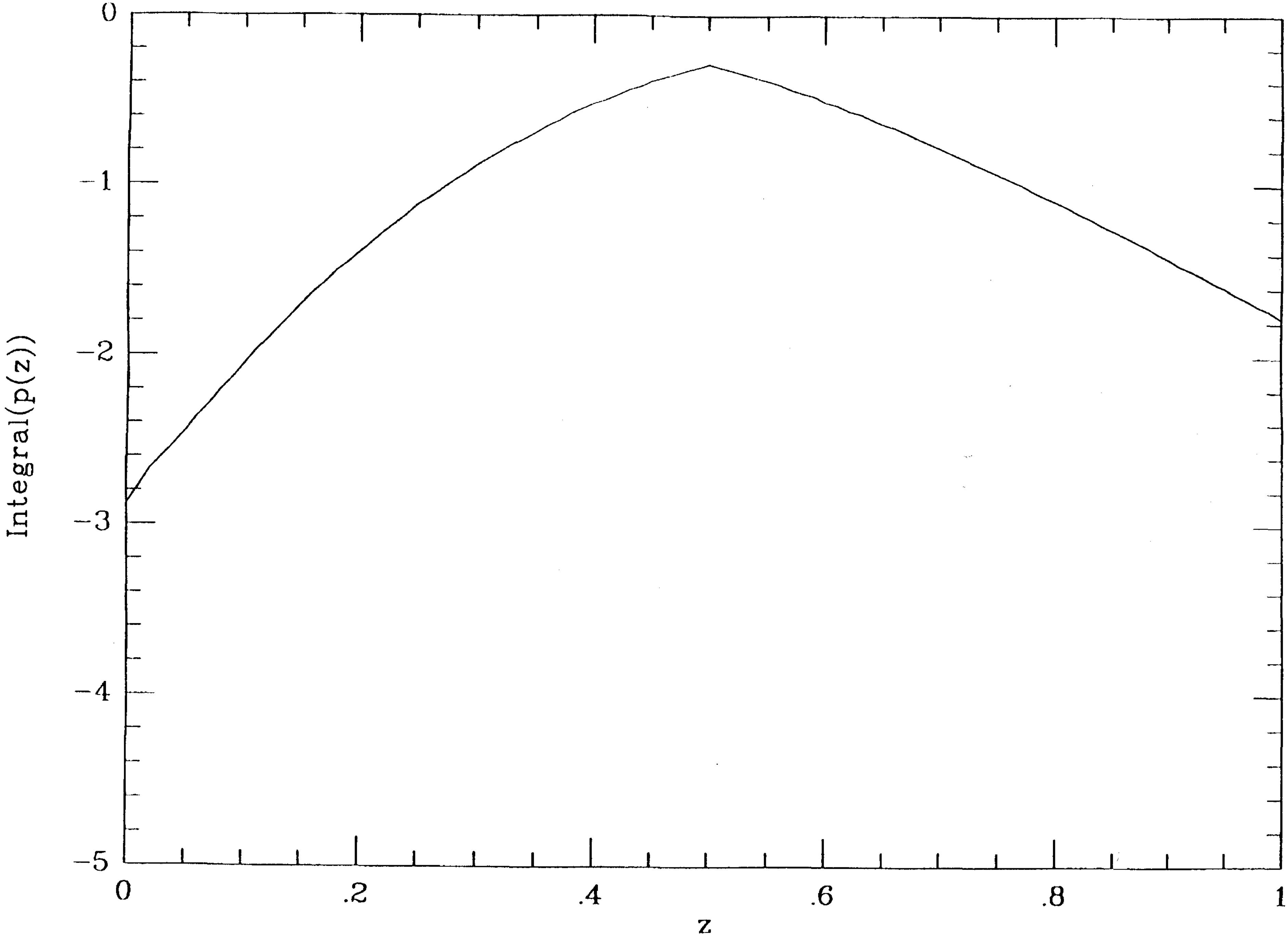
ratio=1, 4σ

ratio
wl



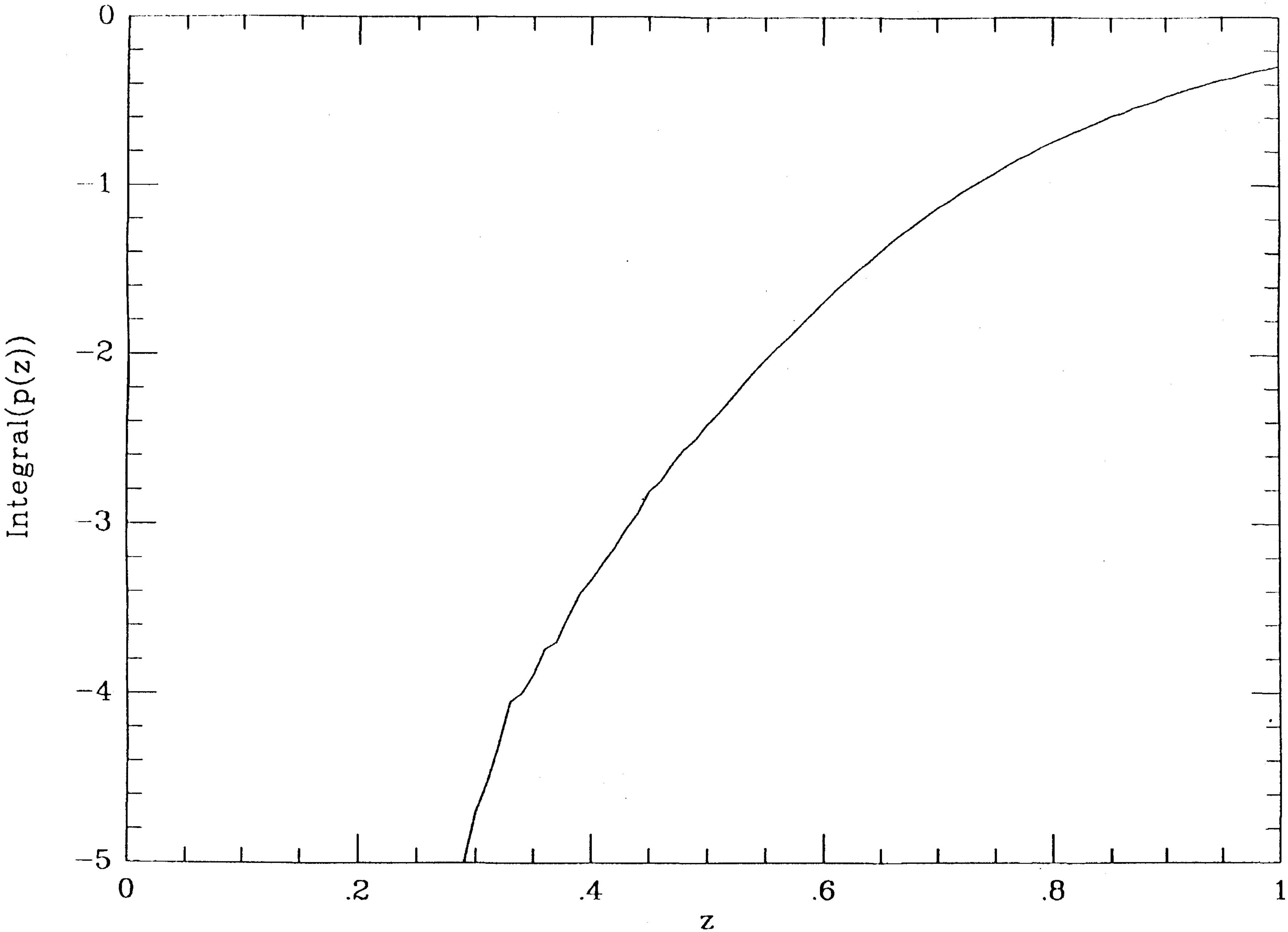
ratio = 0, 6σ

ratio=
sol



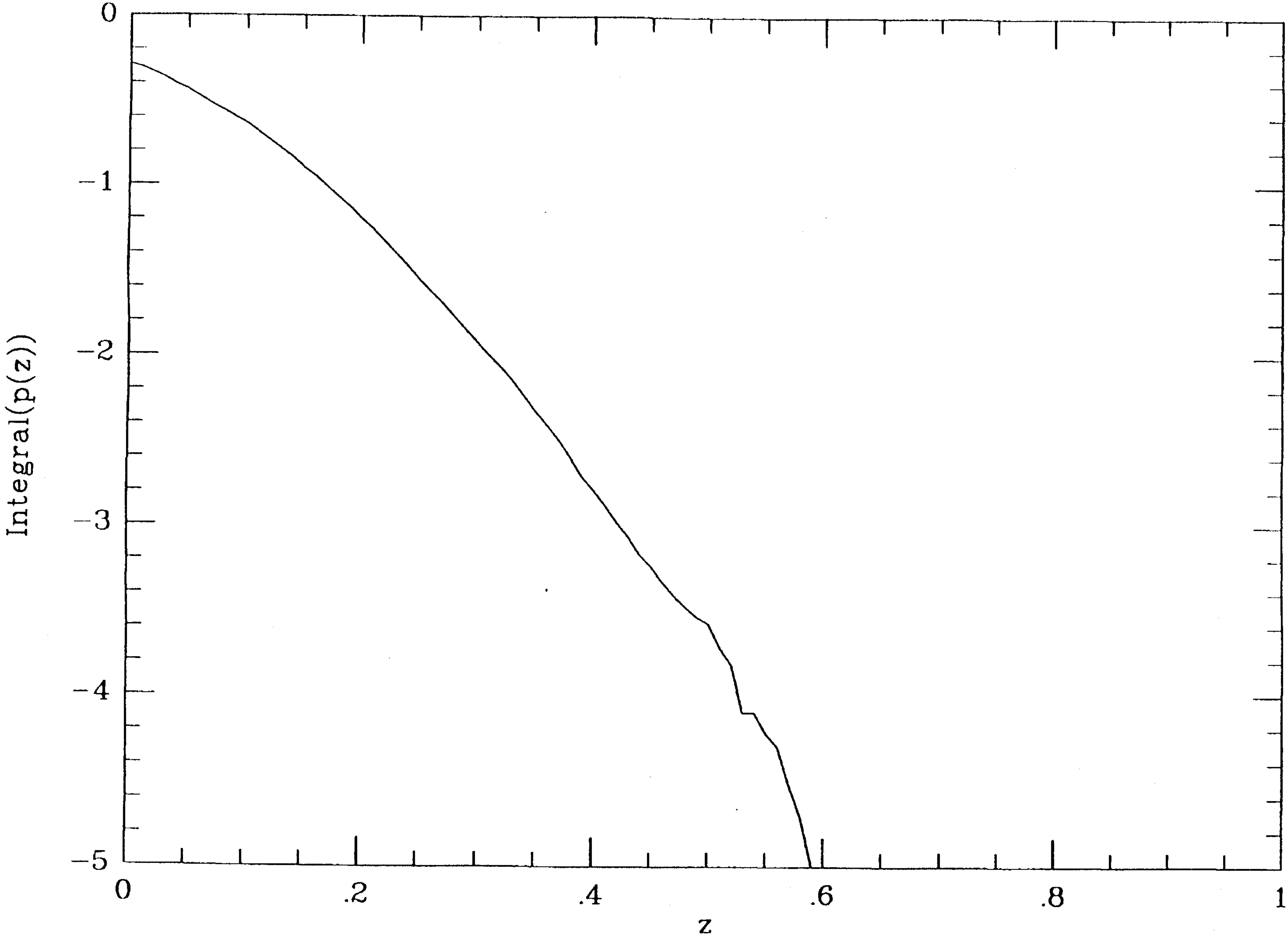
ratio = $\frac{1}{2}$, 6σ

ratio
at (



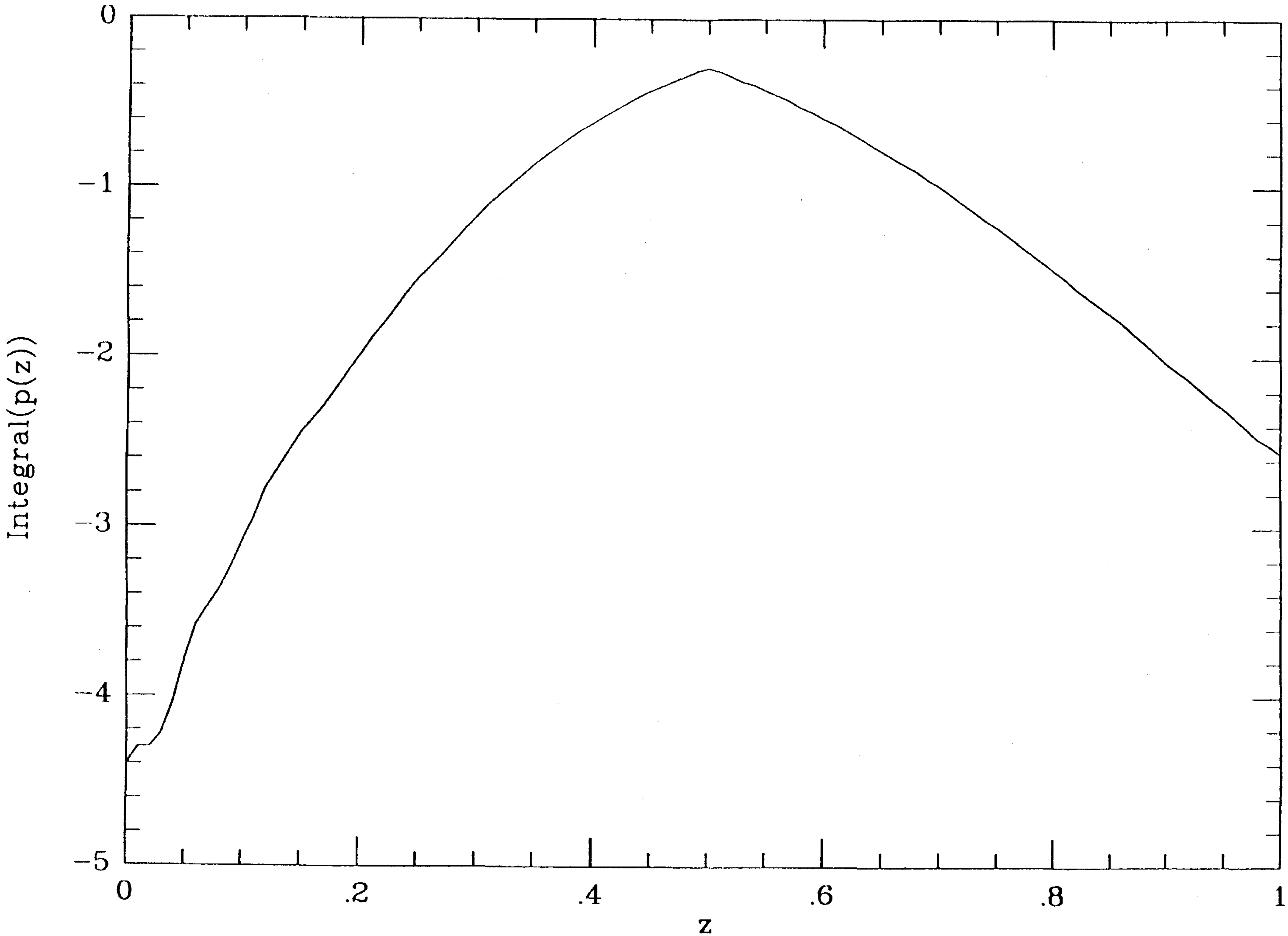
ratio=1, 6σ

ratio

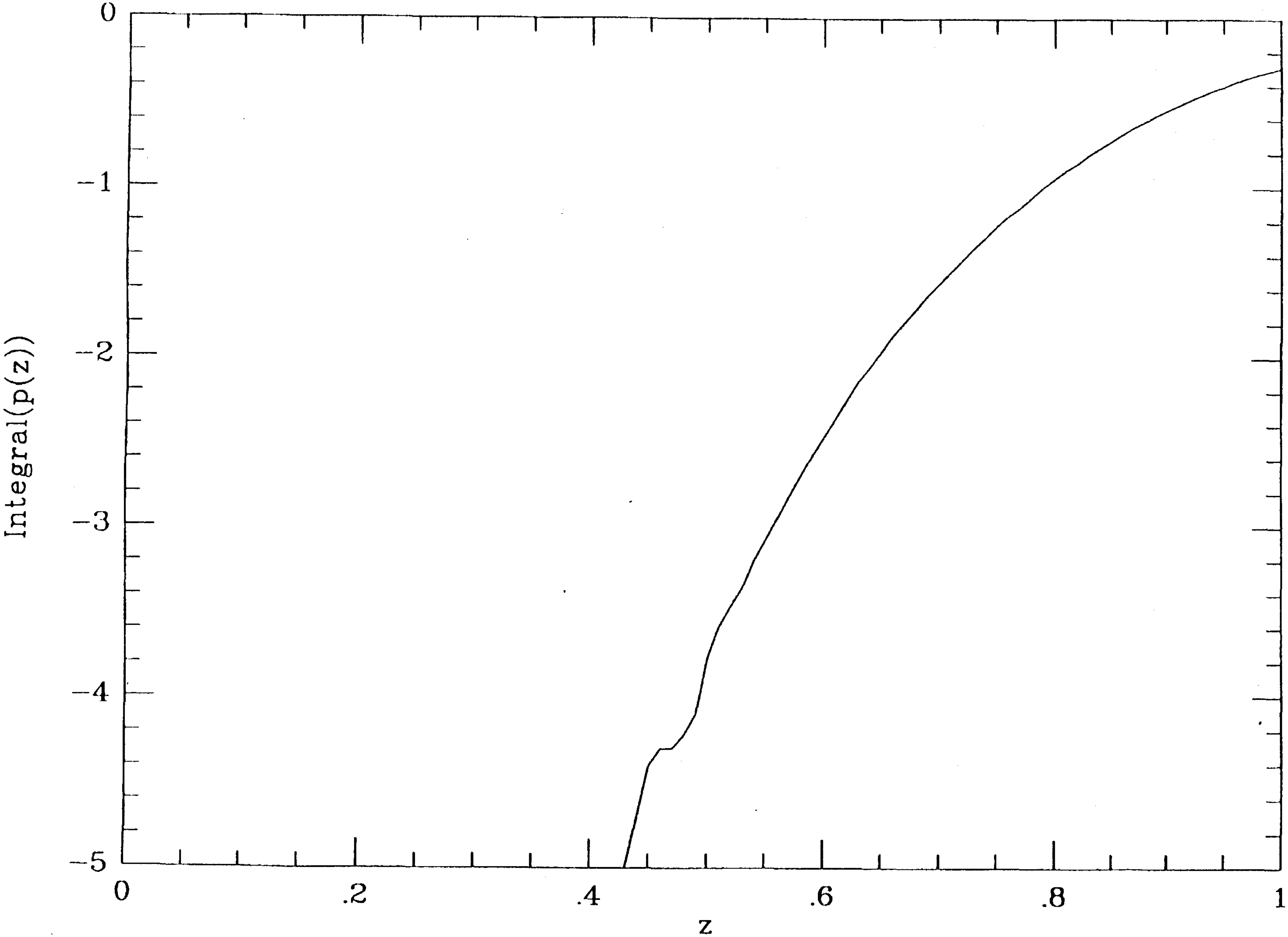


ratio=0, 8σ

ratio
col

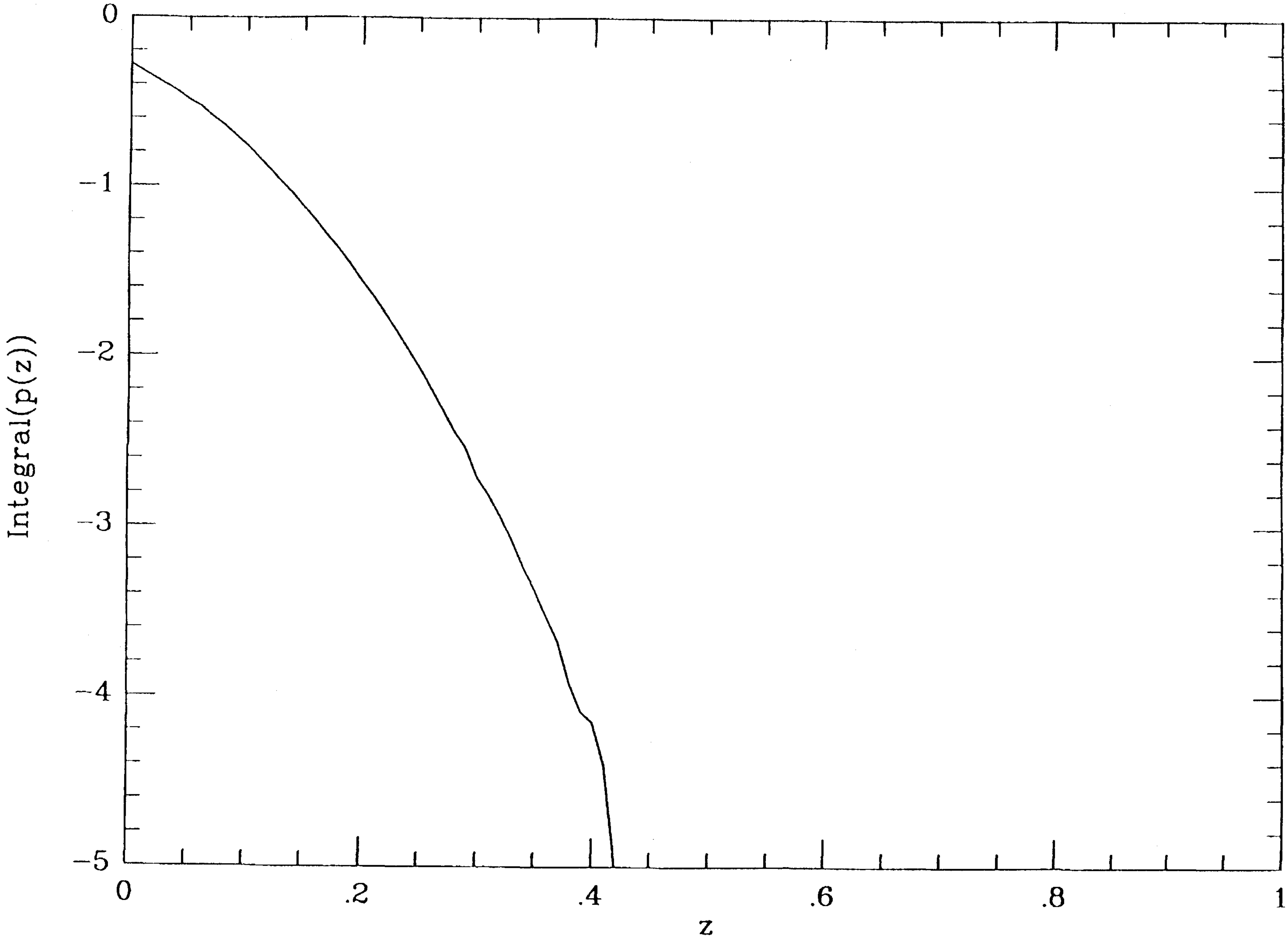


ratio = 1/2, 8σ



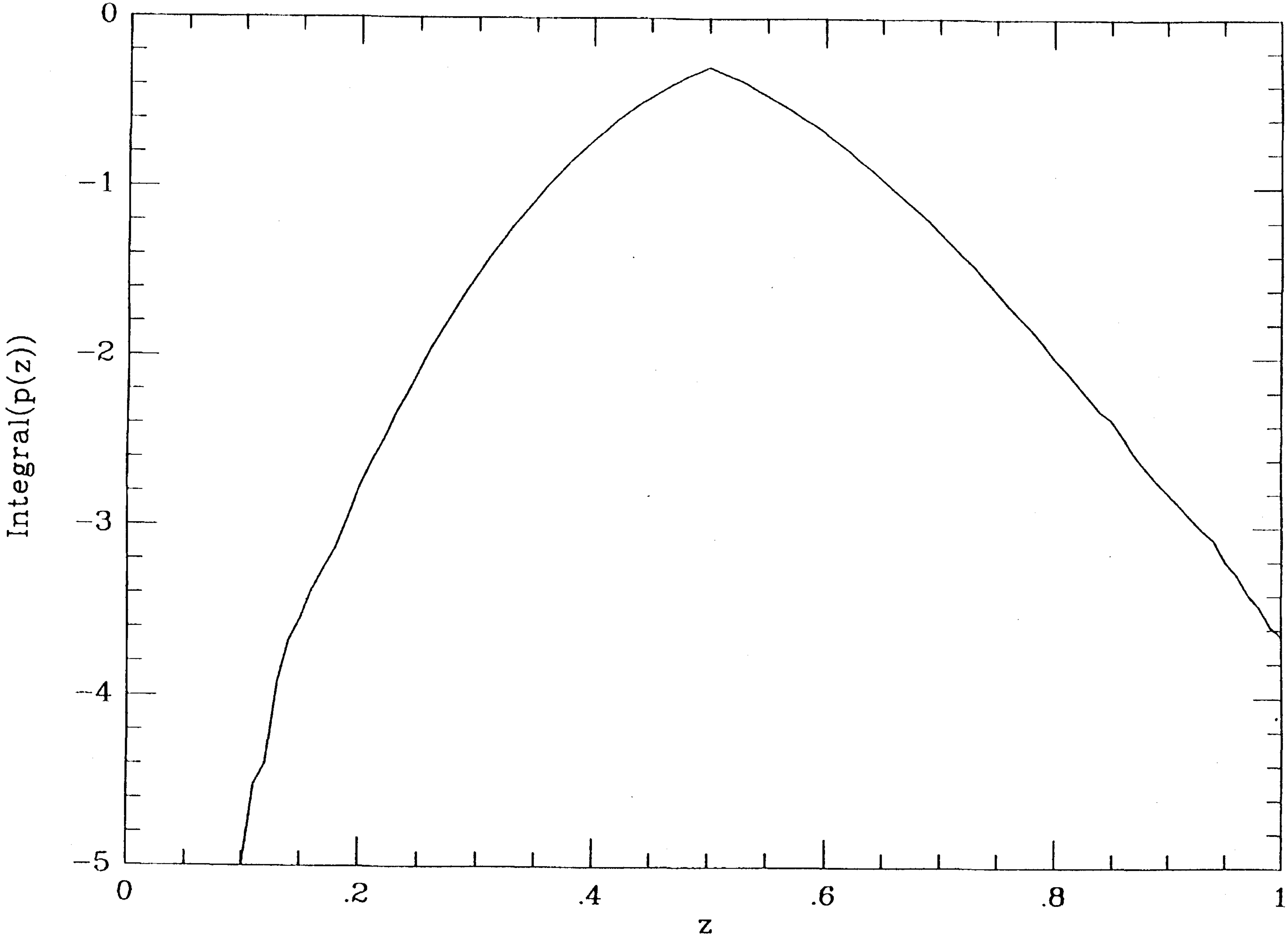
ratio=1 , 8σ

02/27/89 14:07 ☎617 253 7014 MIT GRAVITY GRP. → LIGO Ø039

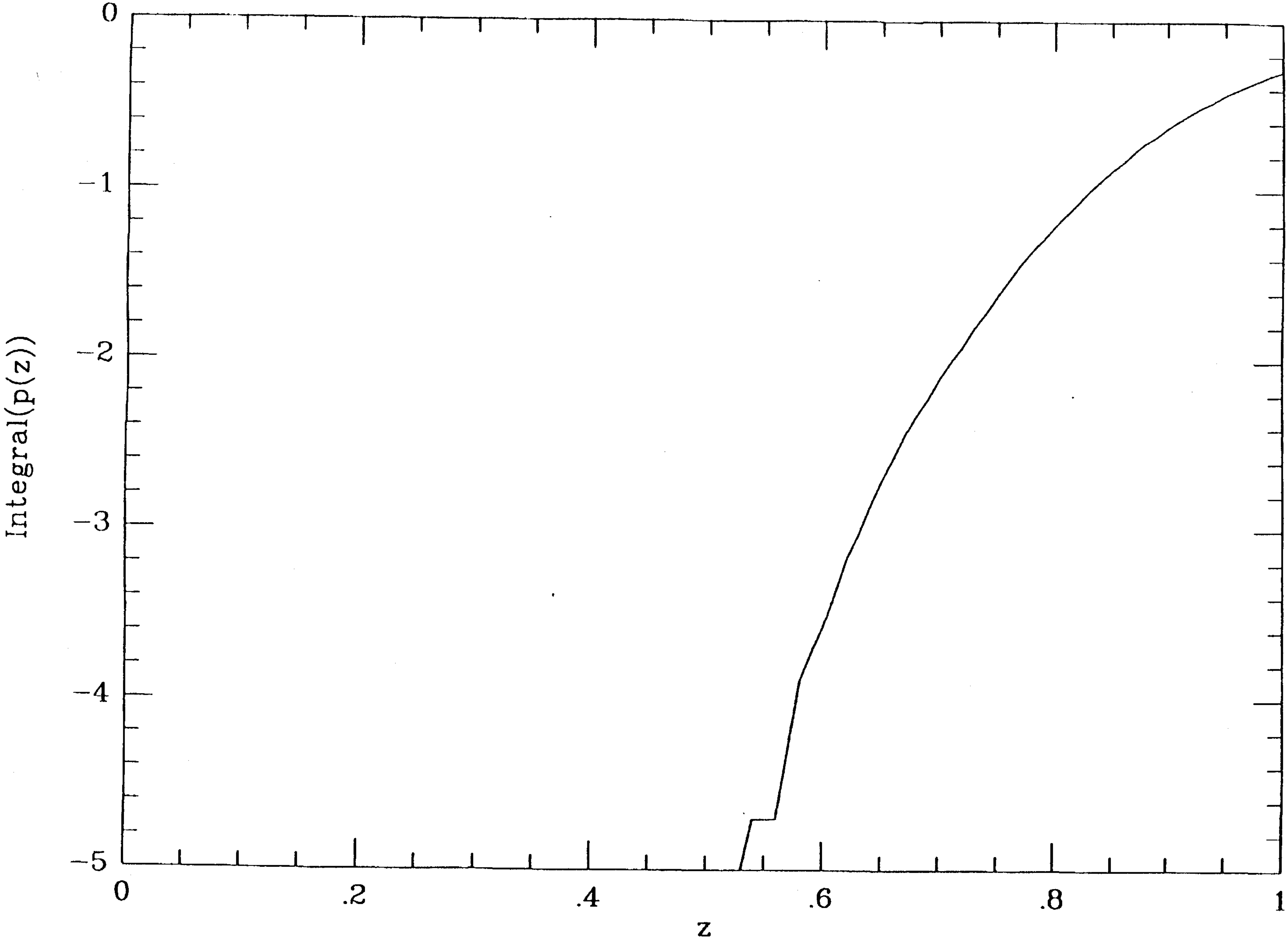


ratio=0 , 10σ

ratio ...
cal ...



ratio: 1/2,



ratio=1, 10σ

ratio…