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Quantum-non-demolition (QND) measurements of the energy of photons in resonators and waveguides are discussed. Feasible methods to realize the QND energy measurements may be based on the use of nonlinear whispering-gallery modes for standing photons, and the anomalous Compton effect for flying photons.

§1. Short Historical Introduction

The first note on the problem of quantum-non-demolition measurement (QND) can be found in the article by L. Landau and R. Peierls[1] which was published in 1931. This paper did not trigger an interest at that time. Next publication, in which there was a comment on the possibility of QND measurements, was the book by D.Bohm[2]. It appeared 20 years after the Landau-Peierls article. The problem remained non-elaborated for additional 15 years till the experimentalists who were making gravitational-wave antennae confronted the necessity to take into account the quantum features of a single macroscopic object when only one measurement was performed and $\hbar\omega \gg kT$.[3] The interest to this narrower problem led to the necessity to get answers on the following questions:

1) what quantum measurements with a single quantum object are possible;

2) what measurements (called the quantum-non-demolition ones) can be repeated without perturbation of the quantum state;

3) how can different types of QND measurements with single object be realized.

Very soon the answers on the questions 1) and 2) for a simple case of macroscopic mechanical oscillator were formulated. But by present moment there is no full set of answers on the question 3). Moreover, there exist no successful realizations of QND measurement with mechanical objects; this task proved to be too complicated.

For those readers who are interested in these problems we can recommend review articles[4-5] (state-of-the-art by the year 1980) and the review article[6] (state-of-the-art with mechanical objects by 1987).

Important contributions to the solution of the questions 1), 2) and partly of 3) were made by Yu. Vorontsov, F. Khalili, K. Thorne, C. Caves and W. Unruh.

Quantum measurements with single object attracted later the attention of physicists who were far from gravitational wave antennae. The possibility of QND measurements in optical region, preparation of the nonclassical states in optical resonators and waveguides were found to be attractive for the creation of new methods of information transfer, new methods of optical measurements etc. In the optical region the solution of the above problems seems to be easier since optical quanta are 'heavy' and $\hbar\omega \gg kT$ at normal laboratory conditions. Thus there is no necessity to reach very low level of dissipation which is the must for a mechanical QND experiment. The first success in optical area belongs to M. Levenson and his colleagues, who have demonstrated a QND measurement of quadrature component of optical wave.[7] Substantial contribution to the elaboration of important details as well as the analysis of QND measurements' application in optical communication was made by Y. Yamamoto.[8] Note, here, that the QND measurement of quadrature components, on our opinion, is an easier problem than the QND measurement of energy, while possible application area of a simple and reliable QND energy measurement procedure would be much wider.

The following short review of QND measurements of energy contains the description of present state-of-the-art and the discussion of possible application.

§2. Standing Photons

QND measurement of energy (or of the number of quanta) in an e.m. resonator ('standing' photons) is one of the simplest cases of QND energy measurement. The principal idea of such experiment (in fact, a thought experiment) was proposed by the authors of the article[9] and independently in the article.[10] The scheme of the experiment is very simple. Let us suppose that the energy to be measured ε the value of which is approximately known, is stored in one of the modes of an e.m. resonator. Let's suppose also for simplicity that the resonator has a rectangular shape, the spacing between two parallel walls being d, and that the structure of the mode corresponds to the ponderomotive force $F=\varepsilon d^{-1}$ acting on the walls. It is evident then that if an experimentalist smoothly liberates one of the two walls making it free mass, he can determine the value of ε by measuring the momentum $P=F\tau=\varepsilon\tau d^{-1}$ spending for this the time τ.

It is also evident that after the wall is liberated (this procedure also needs time which is of the order of τ), the uncertainty of the coordinate Δx of the wall and the uncertainty of its momentum Δp must obey the Heisenberg formula:

$$(\Delta x)^2(\Delta p)^2 \geqslant \hbar^2/4. \tag{1}$$

If the experimentalist measures p using permanent recording of the coordinate x and the averaging time is τ, then the smallest possible errors Δx and Δp are equal to the standard quantum limits for a free mass:[11]

$$\Delta x_{SQL}=\sqrt{\frac{\hbar\tau}{2m}} \quad \text{and} \quad \Delta p_{SQL}=\sqrt{\frac{\hbar m}{2\tau}}, \tag{2}$$

where m is the mass of the wall.

From this formula we see that the error in the measurement of ε is equal to

$$\Delta\varepsilon=\frac{\Delta p\cdot d}{\tau}=\frac{d}{\tau}\sqrt{\frac{\hbar m}{2\tau}}. \tag{3}$$

Thus the experimentalist is free in choosing the values of m and d at fixed τ. But to provide the free wall condition during the time τ it is necessary for the rigidity $k=\varepsilon d^{-1}$ produced by e.m. field to be small enough and

$$\tau \leqslant \sqrt{\frac{m}{k}}=\sqrt{\frac{md^2}{\varepsilon}} \quad \text{or} \quad d \geqslant \tau\sqrt{\frac{\varepsilon}{m}}. \tag{4}$$

Combining the formulae (3) and (4) we obtain

$$\Delta\varepsilon \geqslant \sqrt{\frac{\varepsilon\hbar}{2\tau}}, \tag{5}$$

and if $\varepsilon=(N+1/2)\hbar\omega$, then

$$\Delta\varepsilon \geqslant \hbar\omega\sqrt{N+1/2}\cdot\frac{1}{\sqrt{2\omega\tau}};$$
$$\Delta N \geqslant \sqrt{N+1/2}\cdot\frac{1}{\sqrt{2\omega\tau}}. \tag{6}$$

From the formula (6) we see that if $\omega\tau \gg 1$, we can reach $\Delta\varepsilon \ll \hbar\omega$. It is evident that these calculations are valid if $\Delta x \ll d$ and $\omega\tau \gg 1$. In other words, the measurement is accompanied by an adiabatic 'red shift' of standing photons. It is easy to show that, in this procedure of measurement, *only* from the formula (1) it follows that $\Delta N\cdot\Delta\phi \geqslant 1/2$ (where $\Delta\phi$ is the uncertainty of phase). The wall after measurement can be put back in its initial position. During this return the ensemble of photons would be 'blue shifted' backward.

Let us underline that the formula (6) is not the ultimate limit of achievable accuracy, because the fulfillment of the condition (4) is determined by our a priori knowledge of ε. If, after the first measurement, the value of ε is known with good accuracy $\Delta\varepsilon/\varepsilon=[(N+1/2)2\omega\tau]^{-1/2}$, we can compensate the known part of the rigidity k. If then we make the second measurement using the same method, including compensation, the accuracy would be better: $\Delta\varepsilon/\varepsilon=[(N+1/2)\cdot 2\omega\tau]^{-3/4}$. Repeating the measurement several times each time improving the compensation of k, one can reach the ultimate limit:

$$\frac{\Delta\varepsilon}{\varepsilon}\approx\frac{1}{2\omega\tau(N+1/2)}. \tag{6a}$$

It is important to emphasize that the result in the formulae (6) and (6a) $\Delta\varepsilon \ll \hbar\omega$ is the consequence of the use of a quadratic effect, i.e., the ponderomotive force.

Unfortunately trivial estimates show that the discussed procedure is only a thought experiment and not an instruction for experimentalists: the value of p is too small even for optical quanta.

In 1981 in the article[12] a more realistic procedure of QND energy measurement was proposed. Instead of the ponderomotive force it was proposed to employ the cubic nonlinearity $\chi^{(3)}$ of a dielectric.

The idea of this proposal is simple. Suppose that we have to measure the energy ε_1 stored in the mode ω_1 of a resonator. Suppose then that the ω_1 mode field configuration substantially overlaps with that of another mode ω_2. Then the energy $\varepsilon_1 = (N_1 + 1/2)\hbar\omega_1$ in presence of cubic nonlinearity can be measured via the shift of the frequency of the second mode:

$$\frac{\Delta\omega_2}{\omega_2} = \frac{12\pi^2\chi^{(3)}(N_1 + 1/2)\hbar\omega_1}{V}, \qquad (7)$$

where V is the volume occupied by the e.m. field of the modes. From this simple formula one can see that the experimentalist has to find a dielectric with the greatest possible value of $\chi^{(3)}$, and, in addition to that, to have enough time τ to measure $\Delta\omega_2/\omega_2$, this dielectric as well as the resonator as a whole must have the dissipation small enough to fit the following condition:

$$\tau \ll \frac{Q_1}{\omega_1(N+1/2)}, \qquad (8)$$

where Q_1 is the quality-factor of the mode.

Thus the experimentalist has to solve a complicated problem: to design a resonator with sufficiently high nonlinearity and high quality-factor.

Possible reserve in solution of this problem can be the reduction of volume V. One of the ways to employ this reserve, in case of optical quanta, is to use the dielectric resonators with whispering-gallery modes.[3] In these resonators, the value of V can reach $2\cdot10^{-11}$ cm^3, the quality-factor $Q_1 \simeq 10^{10}$ and the strength of electric field $E \simeq 400$ V/cm at $\varepsilon_1 = \hbar\omega \cong 2\cdot10^{-12}$ erg.

In the preliminary experiments with resonators of this type (see details in ref. 14) the quality-factor $Q \simeq 3\cdot10^8$ was obtained with $V \simeq 10^{-9}$ cm^3 and the optical bistability was observed at 10^2 erg/s of pump power. Resonators were made of fused quartz. Even for this 'linear' material these results have to be regarded as preliminary, since there is substantial reserve in further improvement of quality-factor and further reduction of volume. According to the estimates,[14] substitution of fused quartz by the existing high-nonlinearity low-absorption semiconductor-doped glasses would permit to solve the problem of QND energy measurement of optical photons in the dielectric resonators.

§3. Flying Photons

From the point of view of possible applications it is much more attractive to realize the QND measurements of energy of a bunch of flying photons. Let us discuss a QND experiment similar to that described in the previous section. Suppose that we have at our disposal an e.m. resonator in which we have measured the energy by the QND method with the error $\Delta\varepsilon_{res} \ll \hbar\omega$ and thus the number of quanta $N = \varepsilon_{res}(\hbar\omega)^{-1}$ is known exactly. If now we couple the resonator with a waveguide (open the 'gate') we shall have a bunch of flying photons. We can also have a priori information about the length $\Delta x_{bunch} = v\tau_{bunch}$ in which all N photons are located. The value of τ_{bunch} is longer than $\tau^* = Q\omega^{-1}$ (where Q is the loaded quality-factor). The photons are emitted into the waveguide independently, and the relative uncertainty of the energy of each photon is equal to $(\omega\tau^*)^{-1}$. Thus for the whole bunch

$$\frac{\Delta\varepsilon_{bunch}}{\varepsilon_{bunch}} \simeq \frac{1}{\omega\tau^* N^{1/2}}. \qquad (6)$$

It is evident that the increase in the error of the total energy $\Delta\varepsilon_{bunch} > \Delta\varepsilon_{res}$ is due to the interaction between the photons and the gate when it was opening.

Now let us suppose that a part of the waveguide wall with length $l > v\tau_{bunch}$ is used to measure the energy of the bunch via the ponderomotive momentum p. In this scheme it is necessary that the propagation speed of photon to depend on the transverse coordinate of the wall q (in general, q is the generalized coordinate):

$$v = v_0(1 - q/d), \qquad (10)$$

where $1/d$ is the coupling constant; if the wall moves in transverse direction to the waveguide axis, then d is of the order of the waveguide diameter.

The force $F = \varepsilon_{bunch} d^{-1}$, which acts on the wall, produces a momentum $p = \varepsilon_{bunch}\tau_{meas} d^{-1}$, and the error in the measurement of the energy is

$$\Delta\varepsilon_{\text{bunch}} = \frac{d}{\tau_{\text{meas}}} \cdot \Delta p, \qquad (11)$$

where the time of measurement $\tau_{\text{meas}} < l/v$. Due to the uncertainty Δq the value of v will also have uncertainty $\Delta v = v_0 \cdot \Delta q \cdot d^{-1}$. So finally after the measurement of the momentum p the bunch will acquire random displacement along the waveguide

$$\Delta x = \tau_{\text{meas}} \Delta v = \tau_{\text{meas}} v_0 \Delta q \cdot d^{-1}. \qquad (12)$$

This displacement corresponds to an uncertainty of the arrival time of the bunch to a fixed point of the waveguide far from the meter:

$$\Delta\tau = \Delta x \cdot v_0^{-1} = \tau_{\text{meas}} \Delta q \cdot d^{-1}. \qquad (13)$$

As soon as $(\Delta p)^2 (\Delta q)^2 \geqslant \hbar^2/4$, then from the formulae (11) and (13) we obtain:

$$\Delta\varepsilon_{\text{bunch}} \Delta\tau \geqslant \hbar/2, \qquad (14)$$

or

$$\frac{\Delta\varepsilon_{\text{bunch}}}{\varepsilon_{\text{bunch}}} = \frac{\Delta\bar{\omega}}{\omega} = \frac{\hbar}{2\Delta\tau \cdot \varepsilon_{\text{bunch}}} = \frac{1}{2\omega\Delta\tau N}. \qquad (15)$$

Comparing the formulae (9) and (15) we see that the gain due to this measurements corresponds to the factor $\sqrt{N}$ if $\Delta\tau = \tau^*$. It is important, however, to take into account that two conditions have to be fulfilled: $\Delta\tau < l/v_0$ and $\tau_{\text{meas}} < l/v_0$.

It is possible to show, that the frequencies of photons in the bunch become anticorrelated in relation to the measured mean frequency (see details in ref. 15). It can be noted, in addition, that these frequency-anticorrelated quantum states permit, in principle, to realize a QND measurement of momentum of a free particle.

Ponderomotive scheme for the QND energy measurement of a bunch of flying photons is also a thought experiment, as in the case of standing photons in a resonator. Practically, such measurement can be realized by a method similar to that described in the previous section: by inserting two waves (the probe- and the signal one) into the waveguide made of dielectric with high value of $\chi^{(3)}$ and measuring the energy of one wave via the phase shift of the other. In this case, however, an experimentalist is confronted with the necessity to compensate the self-action of each wave. Another way to realize QND energy measurement of a group of flying photons is the use of a quadratic force which repulses an electron flying along the waveguide almost synchronously with photons. The estimates (see details in ref. 16) show that this method can permit to measure the energy of short monophotonic state by registering the repulsion of a few electrons.

§3. Conclusion

There exists an evident line of goals in the problem of QND energy measurement, with increasing complexity of practical realization:

1) QND energy measurement with the error $\Delta\varepsilon < \hbar\omega/\sqrt{N}$ (energy uncertainty in the coherent quantum state) in an optical range;
2) QND energy measurement with the error $\Delta\varepsilon \simeq \hbar\omega$;
3) QND energy measurement with the error $\Delta\varepsilon \simeq \hbar/\tau$.

Ever more complicated would be the achievement of the above three goals in the microwave area and further, in the low-frequency mechanical systems.

It is hard to predict how soon these goals will be implemented. It depends very strongly on how attractive or how fundamental will be physical problems which can be solved with the use of QND measurements of energy. But no doubt the list of these problems should necessarily include the following two: 1) the realization of sensitive gravitational wave antennae and 2) the realization of Landauer-Feynman quantum computer.

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* * *

N. G. van Kampen: You make the impression that photons are little balls that you keep in a cavity and subsequently release by opening a door. My problem is that I have learned that a photon is an elementary excitation of one normal mode of the Maxwell field. How does that agree with your picture?

V. B. Braginsky: All caluculations which I have presented are based on the so-called monophotonic state formalism which is rigorously defined from mathematical point of view. Monophotonic states were realized recently experimentally. In fact all kind of experiments are finite in time, and if radiation process takes place in the experiment, it is more correct to use the term monophotonic state. This means that we have a localized object with not exactly defined energy, but it resembles a photon. Formally photon has infinite dimension and thus it is an idealization.