HIGH PRECISION MEASUREMENTS OF SINGLE MACROSCOPIC OBJECTS.

I. HISTORICAL INTRODUCTION:

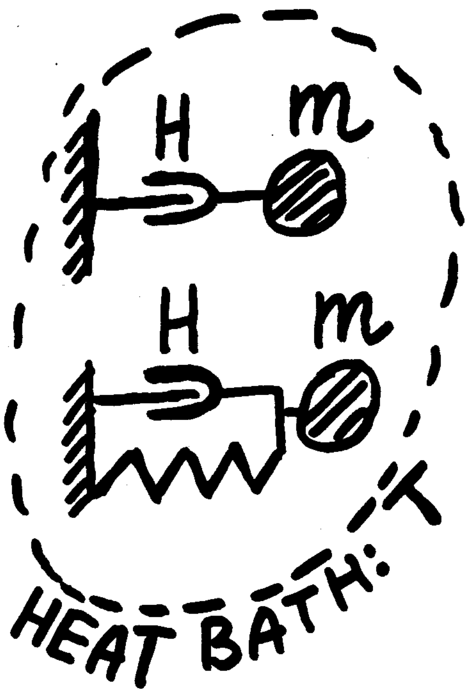
a) WHEN MACROSCOPIC OBJECT HAS TO BE REGARDED AS A QUANTUM ONE?

b) QND MEASUREMENTS: GENERAL PRINCIPLES

II QND MEASUREMENTS OF THE ENERGY OF STANDING AND FLYING PHOTONS: PROBLEMS AND PROSPECTS.

III QND SPEED METER FOR THE DETECTION OF GRAVITATIONAL WAVES.

LIGO-T880081-00-R

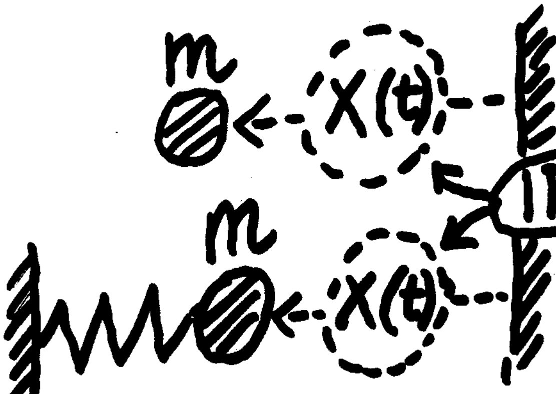


$$\Delta X_T \simeq \sqrt{\frac{k_B T \tau^3}{m \tau^*}}$$

$$\tau \ll \tau^* = \frac{m}{H}$$

AVERAGING TIME

$$\Delta X_T \simeq \sqrt{\frac{k_B T \tau}{m \omega^2 \tau^*}}$$



$$\Rightarrow \Delta X_{SQL} = \sqrt{\frac{\hbar \tau}{2m}}$$

IDEAL COORDIN. METER

$$\Rightarrow \Delta X_{SQL} = \sqrt{\frac{\hbar}{2m\omega}}\,; \quad \tau \lesssim \frac{1}{\omega}$$

$$\Delta X_T \lesssim \Delta X_{SQL}$$

FREE MASS

$$\frac{2 k_B T \tau^2}{\tau^*} \lesssim \hbar$$

$$\frac{2 \cdot 4 \cdot 10^{-14} \cdot (10^{-3})^2}{4 \cdot 10^{7}} \lesssim \hbar$$

OSCILLATOR

$$\frac{2 k_B T \tau}{Q} \lesssim \hbar$$

$$\frac{2 \cdot 3 \cdot 10^{-16}\, 10^{-4}}{5 \cdot 10^{+9}} \lesssim \hbar$$

IF A COORDINATE METER IS USED:

$$(F\tau)_{SQL} = \Delta P_{SQL} = \sqrt{\hbar m \omega}\ ; \quad \Delta \mathcal{E}_{SQL} \simeq \sqrt{\hbar \omega \mathcal{E}}$$

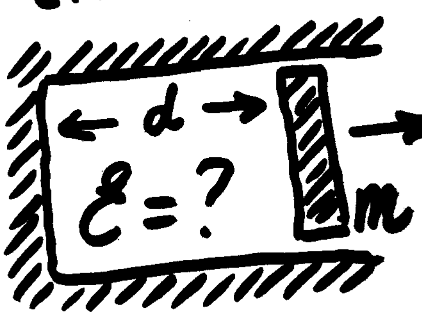
$$\Delta P < \Delta P_{SQL}? \qquad \Delta \mathcal{E} < \Delta \mathcal{E}_{SQL}?$$

QND OBSERVABLES → INTEGRALS OF MOTION

FREE MASS
ENERGY,
SPEED

OSCILLATOR
ENERGY,
QUADRATURE AMPLITUD

E.M. RESONATOR



$$F = \frac{\mathcal{E}}{d}\ ; \quad P = \frac{\mathcal{E}\tau}{d}.$$

$$\Delta \mathcal{E}_{MEAS} = \frac{\Delta P_{MEAS}\, d}{\tau}$$

$$\Delta P \Delta X \geq \frac{\hbar}{2}$$

$$\Delta \varphi_{PERT} = \omega\tau \cdot \frac{\Delta\omega}{\omega} = \omega\tau \frac{\Delta X}{d}$$

$$\Delta \mathcal{E}_{MEAS} \cdot \Delta \varphi_{PERT} \geq \frac{\hbar\omega}{2}$$

$$\Delta \mathcal{E}_{MEAS} \ll \hbar\omega\ ; \quad \Delta\varphi_{PERT} \gg \frac{1}{2} \ \rightarrow\ N \text{ STATE}$$

IF $\Delta P = \Delta P_{SQL}$, THEN $\Delta \mathcal{E}_1 = \frac{d}{\tau}\sqrt{\frac{\hbar m}{2\tau}}$.

IF apriori $\Delta \mathcal{E} \simeq \mathcal{E}$,

THEN $\Delta \mathcal{E}_1 = \sqrt{\frac{\mathcal{E}\hbar}{\tau}} = \frac{\Delta \mathcal{E}_{SQL}}{\sqrt{\omega\tau}}$.

$$\frac{\Delta \mathcal{E}_1}{\mathcal{E}} = \frac{\Delta N_1}{N_1} = \frac{1}{\sqrt{N_1+\frac{1}{2}}\cdot\sqrt{2\omega\tau}}$$

SOV. PHYS. JETP, v. 46, 705 (1977)

2-nd measurement

$$\left(\frac{\Delta \mathcal{E}_2}{\mathcal{E}}\right)_2 = \left(\frac{\Delta N_1}{N_1}\right)_2 = \frac{1}{\left[(N_1+\frac{1}{2})2\omega\tau\right]^{3/4}}$$

PROC. 3-d INT. SYMP. ON FOUND. QUANT. MECH. TOKYO 1989

: : : : : : : : : : : :

$$\left(\frac{\Delta \mathcal{E}_1}{\mathcal{E}}\right)_{lim} = \left(\frac{\Delta N_1}{N_1}\right)_{lim} = \frac{1}{(N_1+\frac{1}{2})\cdot 2\omega\tau}\ ; \quad (\Delta \mathcal{E})_{lim} \simeq \frac{\hbar}{\tau}$$

$\uparrow\uparrow$ THOUGHT EXPERIMENT $\uparrow\uparrow$

$\downarrow\downarrow$ REALISTIC QND$\mathcal{E}$ METER OF STANDING PHOTONS $\downarrow\downarrow$

PROBE ω_2

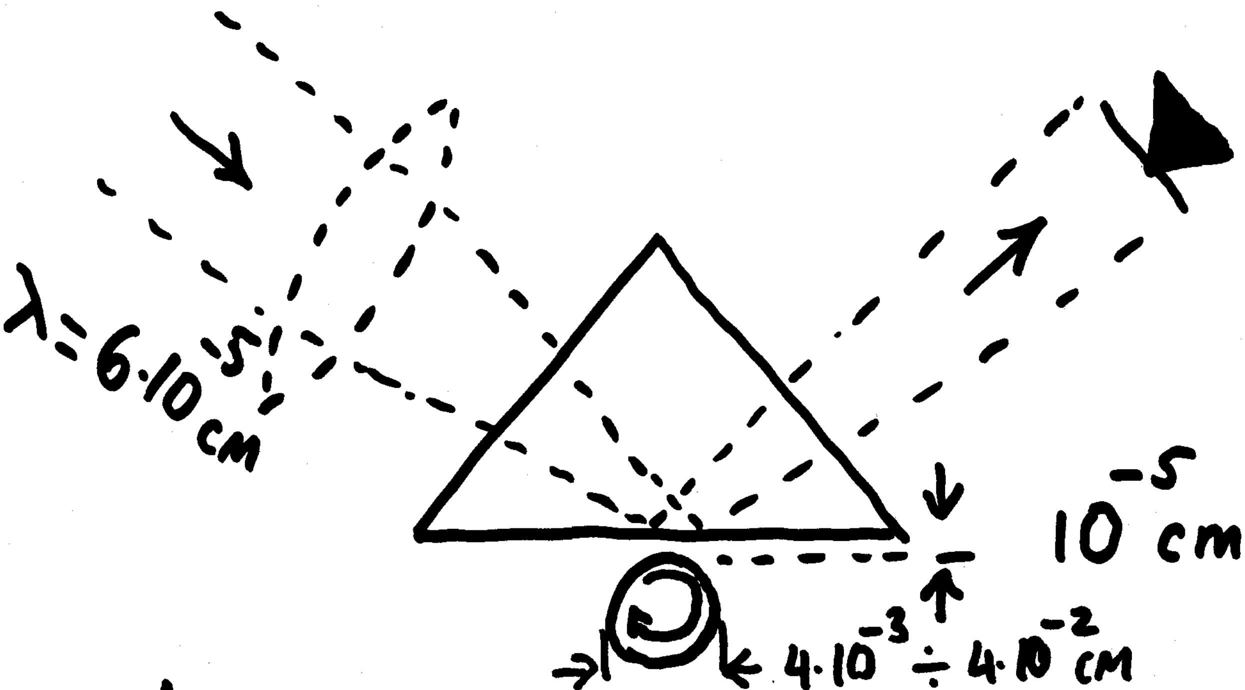
$$\boxed{\frac{\Delta \omega_2}{\omega_2} = \frac{12\pi^2 \chi^{(3)}(N_1+\frac{1}{2})\hbar\omega_1}{V}}$$

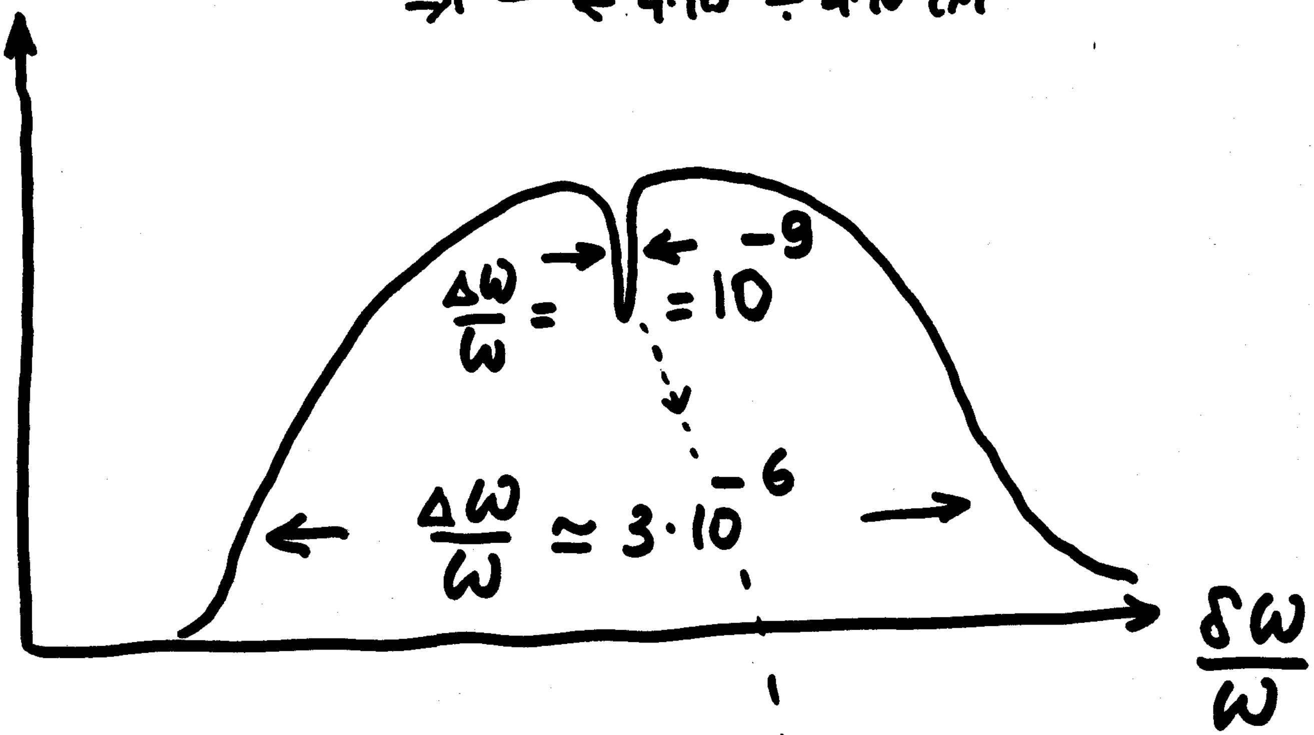
SIGNAL ω_1

RESONATOR MODES: $\omega_1, \omega_2, \chi^{(3)} \neq 0$

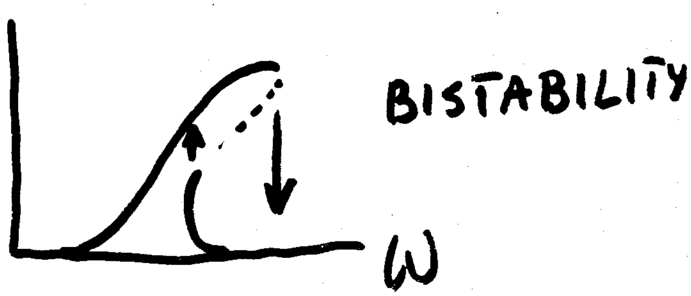
SOV. PHYS. DOKLADY 26(3) 686, 1982.

WHISPERING-GALLERY MODES OPTICAL RESONATORS





PHYS. LETT A
V. 137, 393,
(1989)



$$n = n_0 + n_2 E^2; \qquad n_2 = \frac{12\pi^2 \chi^{(3)}}{n_0}$$

$$\frac{\Delta\omega_2}{\omega_2} = \frac{N_1}{N^*}; \quad N^* = \frac{V n_0}{\hbar \omega_1 n_2} = 2\cdot 10^{15}; \quad \left\{ \begin{array}{l} SiO_2 \\ V = 10^{-9}\ cm^3 \\ Q = 10^9 \end{array} \right.$$

IF $\Delta N_1 < \Delta N_{SQL} = \sqrt{N_1}$

THEN $\underline{4\frac{N^*}{Q} < N_1}$

IF $\Delta N_1 \leq 1$

THEN $\underline{4\frac{N^*}{Q} < \frac{1}{N_1}}$

$$\Delta N_1 = \sqrt{3}\left(\frac{N_1 \cdot N^*}{2\sqrt{N_{PR}}\,\omega_{PR}\,\tau_1^*}\right)^{1/3}$$

$$\simeq 10^{+4}$$

IF $N_{PR} \simeq N_1 \simeq 10^{10}$

PHYS. LETT A
v. 137, 393
1989

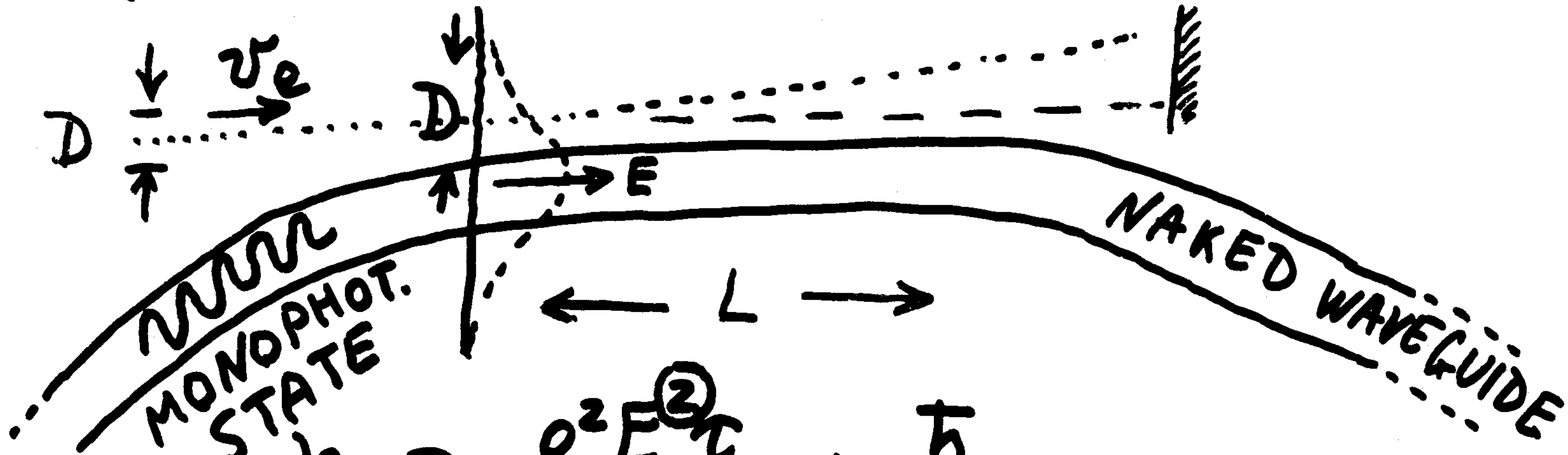
BLUE DREAM FOR N^*:

$N^* \simeq 10^8 \longrightarrow$ CdS_xSe_{1-x}, $\chi^{(3)} \simeq 10^{-9}$, $Q \simeq 10^{+7}$

$\longrightarrow V \simeq 10^{-11}\ cm^3$

REALISTC QND ε METER OF FLYING PHOTON

(SPECIAL VERSION OF COMPTON EFFECT)



$$P = \frac{e^2 E^{(2)} \tau}{2 m_e D \omega_e^2} > \frac{\hbar}{2D}$$

$$\omega_e = \omega_0\left(1 - \frac{v_e}{v_0}\right) \ll \omega_0$$

$\tau_{PHOT} \simeq 10^{-12}$ sec.

$\tau = 10^{-9}$ sec

$\omega_0 = 4 \cdot 10^{+14}$ sec^{-1} $(\lambda_0 = 5 \cdot 10^{-4}$ cm$)$

$\omega_e = 4 \cdot 10^{11}$ sec^{-1}

$L = 15$ cm

$v_0 = 1.5 \cdot 10^{10}$ $\frac{cm}{sec}$

$D = 10^{-4}$ cm

$$\frac{\Delta \mathcal{E}}{\mathcal{E}} \simeq 0.5$$

$\mathcal{E} = \hbar\omega$

ONE ELECTRON IS REPULSED

$\chi^{(3)} \simeq 10^{-3}$ CGSE $= 10^{+11} \chi^{(3)}_{SiO_2}$

IF 10^3 ELECTRONS ARE USED

PH. LETT. A V. 132, p206, 198?

THE LINE OF GOALS FOR QND ENERGY MEASUREMENTS

1. $\Delta\mathcal{E} < \hbar\omega\sqrt{N}$; $\Delta N < \sqrt{N}$

2. $\Delta\mathcal{E} \simeq \hbar\omega$

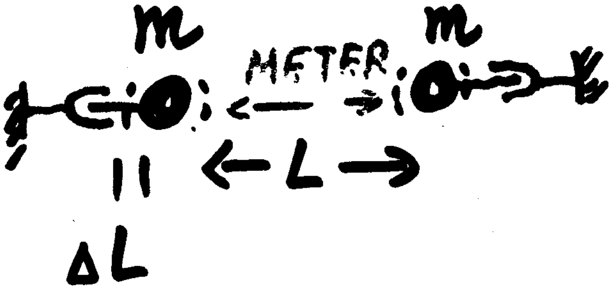
3. $\Delta\mathcal{E} \ll \hbar\omega$; $\Delta\mathcal{E} \simeq \frac{\hbar}{\tau}$

OPTICS, MICROWAVE, MECH. OBJ.

QND MEA. USE FOR

1. INFORMATION TRANSFER
2. FEYNMAN-LANDAUER QUANTUM COMPUTER
3. SENSITIVE GRAV. WAVE ANTENNAE.
4. ?

THE SENSITIVITY OF THE GRAV. WAVE ANTENNAE (CLASSICAL LIMITATION)



$$\Delta L \simeq \frac{1}{2} h L$$

$$\Delta v \simeq \frac{1}{2} h L \omega_{grav}$$

PERTURB. OF THE METRIC

$$\frac{1}{2} h_{THERM} \cdot L \omega_{gr}^{2} m \gtrsim \sqrt{\frac{4 k T m}{\tau^{*}_{MECH} \cdot \tau}} \, .$$

$$\left.\begin{array}{l} m = 10^{4}\,gr, \; T = 300, \\ \omega_{gr} \simeq 3 \cdot 10^{3}\,sec, \\ L = 3 \cdot 10^{5}\,cm, \; \tau^{*}_{MECH} \simeq 10^{2}\,sec \end{array}\right\} h_{THERM} \simeq 1 \cdot 10^{-20}$$

$$\left.\begin{array}{l} L \simeq 10^{2}\,cm, \; T \simeq 2\,^{\circ}K \\ \tau^{*}_{MECH} \simeq 4 \cdot 10^{6}\,sec \end{array}\right\} h_{THERM} \simeq 1 \cdot 10^{-20}; \text{ BUT } h_{SQL} \simeq 5 \cdot 10^{-20}$$

$$\left.\tau^{*}_{MECH} \simeq 5 \cdot 10^{9}\,sec\right\} h_{THERM} \simeq 3 \cdot 10^{-22}$$

FREQUENCY ANTICORRELATED QUANTUM STATES (FACQS)

INTRODUCTION

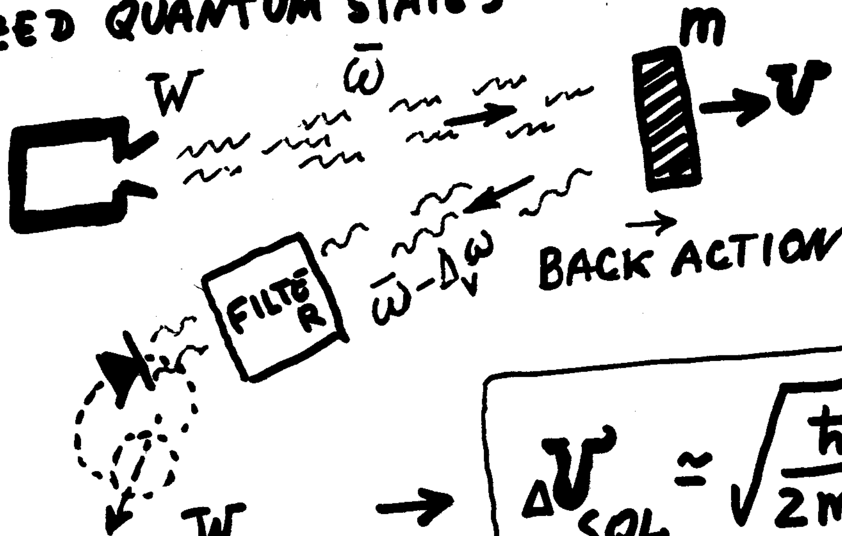
$$\frac{\Delta \mathcal{E}_i}{\mathcal{E}_i} \simeq \frac{1}{\omega \tau^*}$$

$$\frac{\Delta \bar{\omega}}{\omega} = \frac{\Delta \bar{\mathcal{E}}}{\mathcal{E}} \simeq \frac{1}{\omega \tau^* \sqrt{N}}$$

SCHAWLOW-TOWNES FORMULA FOR SELF-EXITED OSCILLATOR

$$\frac{\Delta \bar{\omega}}{\omega} \simeq \frac{1}{2Q\sqrt{N}}; \quad N = \frac{W\tau}{\hbar\omega}$$

COHERENT QUANTUM STATE
SQUIZED QUANTUM STATE } RADIATION



$$\boxed{\Delta v_{SQL} \simeq \sqrt{\frac{\hbar}{2m\tau}}}$$

STANDARD QUANTUM LIMIT

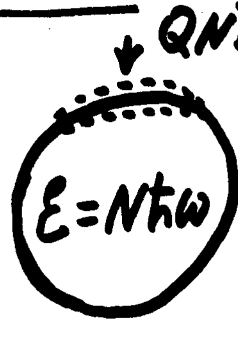
(1932) J. von NEUMANN:
DOPPLER MEASUREMENT WITH ONE PHOTON →

$$\boxed{\Delta v \simeq \frac{c}{\omega\tau}}$$

AVERAGING (PREPARATION) TIME

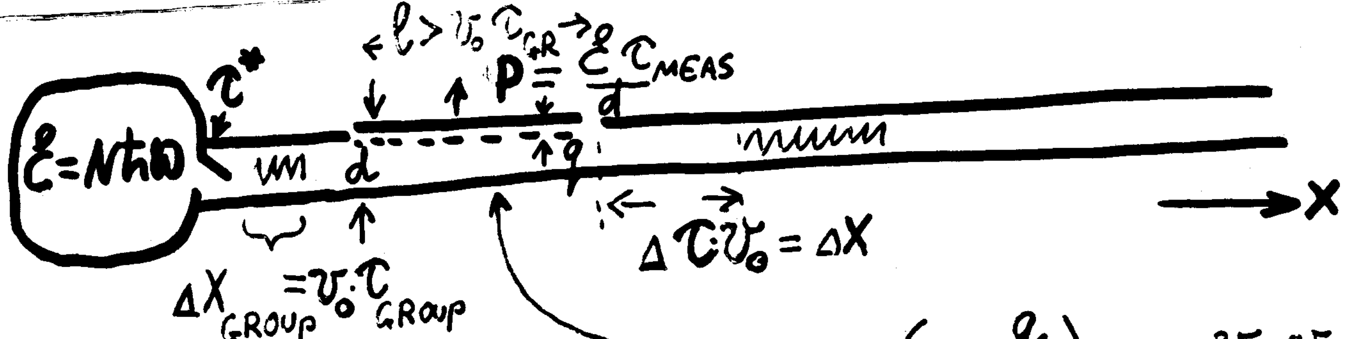
PREPARATION OF QUANTUM-ANTICORRELATED STATES

ω - EIGEN FREQUENCY OF ONE MODE



$$\boxed{\Delta\mathcal{E}\cdot\Delta\varphi \geq \frac{\hbar\omega}{2}};\quad \underbrace{\Delta\mathcal{E} \ll \hbar\omega,\ \Delta\varphi \gg 1}_{\text{ALMOST PURE } N\text{-STATE}}$$

$\chi^{(3)}$ - INSTEAD OF PONDERMOTIVE METER



$$v = v_0\left(1 - \frac{q}{d}\right) \rightarrow \Delta v = v_0\frac{\Delta q}{d}$$

$$\left.\begin{aligned} \Delta\mathcal{E} &= \frac{d}{\tau_{MEAS}}\cdot\Delta P \\ \Delta\tau &= \frac{\Delta X}{v_0} = \tau_{MEAS}\cdot\frac{\Delta q}{d} \end{aligned}\right\} \rightarrow \Delta P\cdot\Delta q \geq \frac{\hbar}{2} \rightarrow \boxed{\Delta\mathcal{E}\cdot\Delta\tau \geq \frac{\hbar}{2}}$$

$$\boxed{\frac{\Delta\mathcal{E}}{\mathcal{E}} = \frac{\overline{\Delta\omega}}{\omega} \geq \frac{\hbar}{2\Delta\tau\mathcal{E}} = \frac{1}{2\omega\Delta\tau N}}$$

$$(\Delta\mathcal{E}\cdot\Delta\tau)^2 = \frac{\hbar^2 N}{4}\cdot\frac{(N-2)\beta+1}{1-\beta};$$ β - FACTOR OF THE PHOTONS' FREQUENCY CORRELATION

$$\beta = \frac{\langle\Delta\omega_i\,\Delta\omega_k\rangle}{(\Delta\omega)^2};\quad \text{IF } \underbrace{\beta = -\frac{1}{N-1}}_{\text{MINIMUM}},\ \text{THEN } \Delta\mathcal{E}\cdot\Delta\tau = \frac{\hbar}{2}$$

SOV. PHYS. JETP v. 67, (1) 84, 1988

DOPPLER MEASUREMENTS WITH FREQUENCY ANTICORRELATED QUANTUM STATES

IF $\frac{\Delta \mathcal{E}}{\mathcal{E}} = \frac{\Delta \bar{\omega}}{\omega} = \frac{1}{2\omega \Delta \tau N} \rightarrow \boxed{\Delta v = \frac{c}{4\omega \Delta \tau N}}$

FOR $v \ll c$, $\mathcal{E} \ll mc^2$

$$P = \frac{2\mathcal{E}}{c} \rightarrow \Delta x_m = \frac{P}{m} \cdot \Delta \tau$$

$$\Delta v \cdot \Delta x_m = \frac{\hbar}{2m}.$$

THE INFLUENCE OF THE DISSIPATION:

$\underbrace{(1-R)}$ - THE PROBABILITY OF THE ABSORPTION,

$$\boxed{\Delta_R v \simeq \frac{c\sqrt{1-R}}{R\,\omega\, \tau_{GROUP} \sqrt{N}}}$$

IF ΔN IS CONTROLLED

$$1-R \neq 0 \rightarrow N_{OPTIM} \simeq \frac{mc^2}{\hbar \omega} \rightarrow \boxed{\Delta v_{min} \simeq \frac{1}{\tau_{GROUP}} \cdot \sqrt{\frac{\hbar(1-R)}{m\omega}}}.$$

THE SENSITIVITY OF THE BEAM GRAV. WAVE ANTENNAE

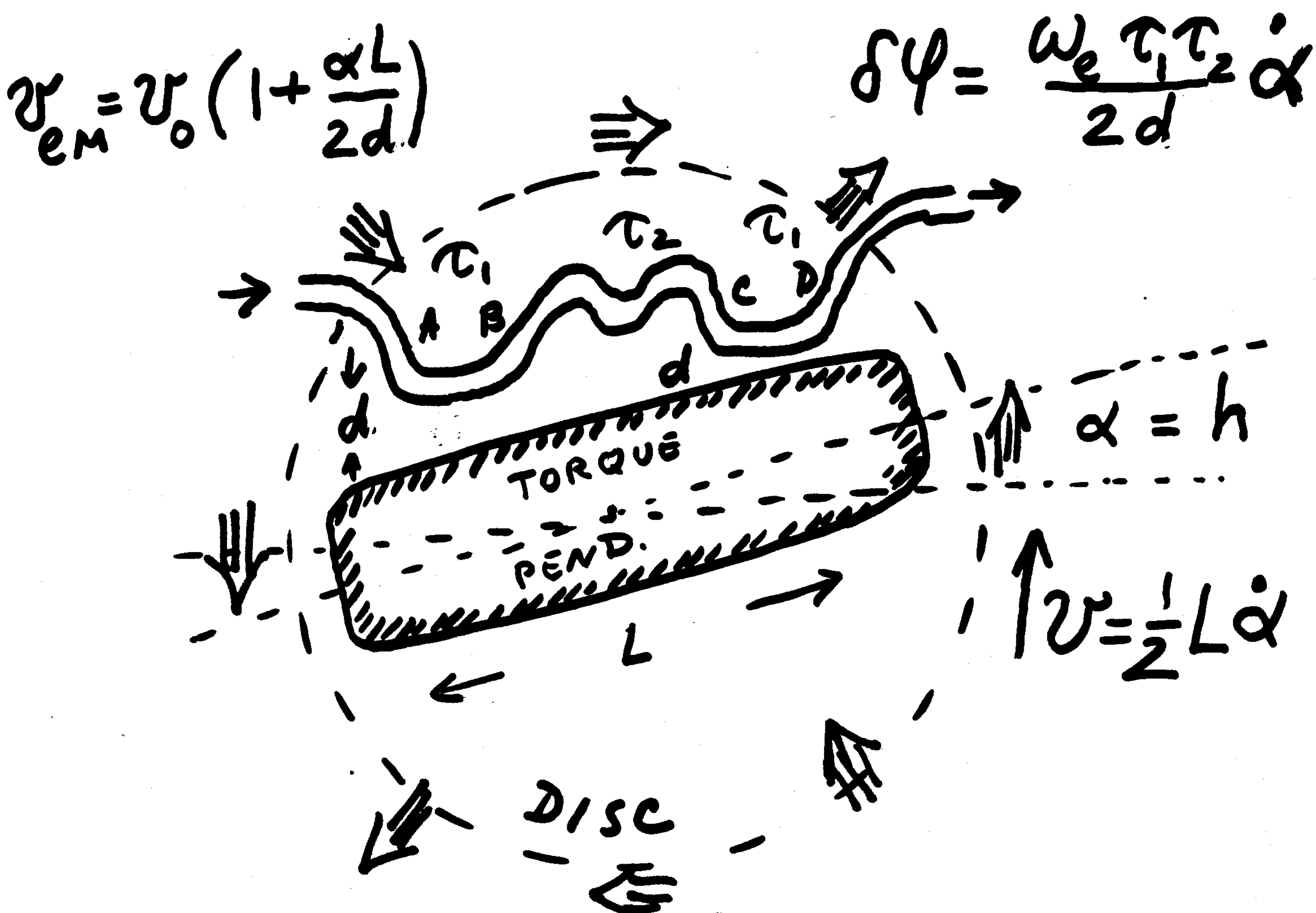
1) WITH COHERENT OR SQUIZED QUANTUM STATES PUMPING

$$h_{SQL} \simeq \frac{1}{L}\sqrt{\frac{2\hbar \tau_{GRAV}}{m}} \simeq 3\cdot 10^{-23} \left(\frac{L}{4\cdot 10^5 cm}\right)^{-1} \left(\frac{\tau_{GRAV}}{10^{-3} s}\right)^{1/2} \cdot \left(\frac{m}{10^4 gr}\right)^{-1/2}.$$

2) WITH FREQUENCY ANTICORRELATED STATE DOPPLER TRACKING

$$h_{FAC} \simeq \frac{c\sqrt{1-R}}{2\pi \omega L \sqrt{N}} \simeq 1\cdot 10^{-23} \left(\frac{1-R}{10^{-7}}\right)^{1/2} \cdot \left(\frac{\omega}{4\cdot 10^{15} s^{-1}}\right)^{-1} \cdot \left(\frac{L}{4\cdot 10^5 cm}\right)^{-1} \cdot \left(\frac{N}{10^{16}}\right)^{-1/2}.$$

REALISTIC (?) VERSION OF GRAV. WAVE ANTENNA WITH QND SPEED METER



COHERENT STATE → PEC STATE +

+ POSITIVE CUBIC NONLINEARITY ⇒

⇒ COHERENT STATE

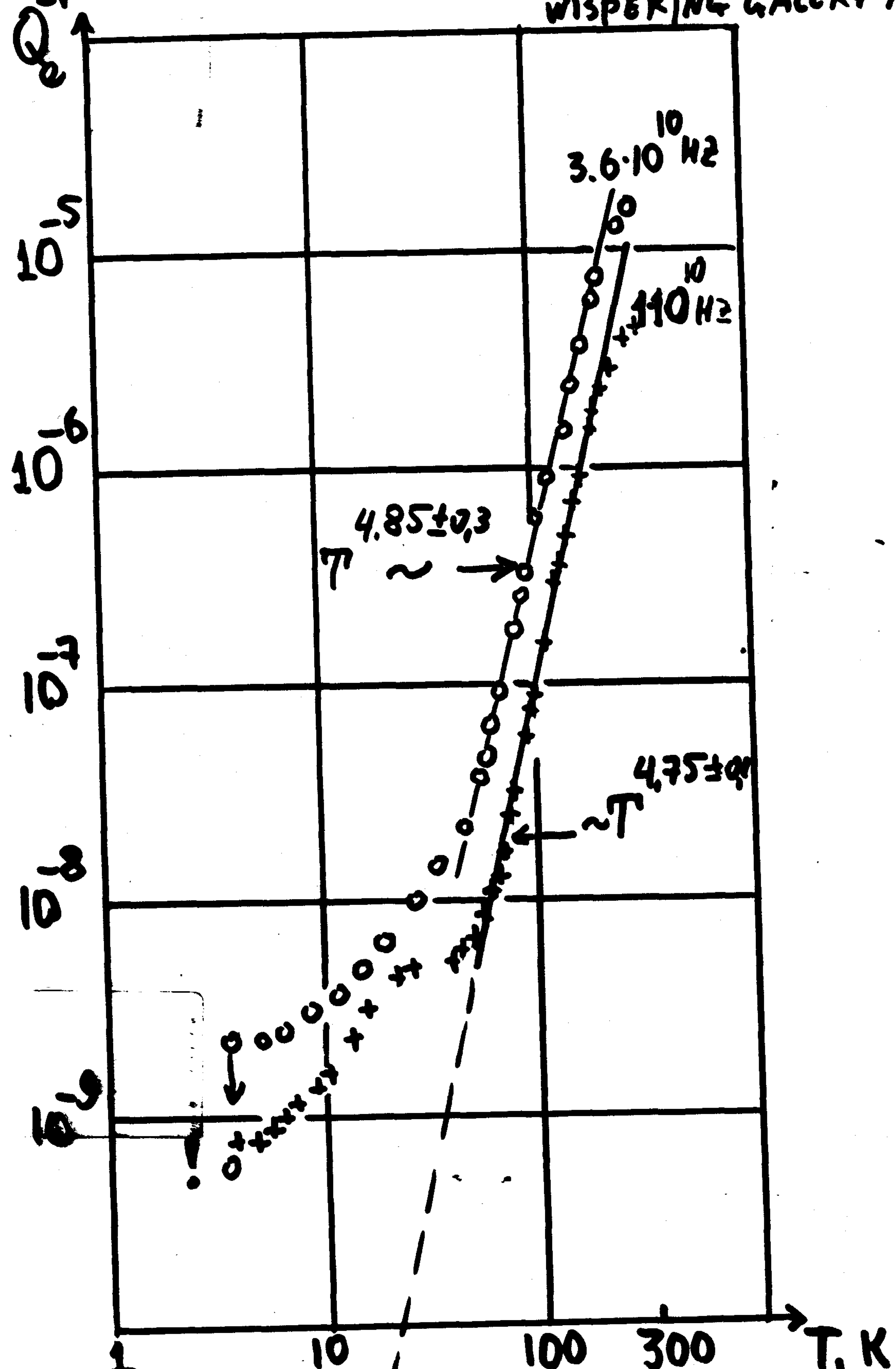
PHYS. LETT. A
v. 147, 251, 1990

DISSIPATION IS THE ENEMY OF QND MEASUREMENTS

$$h \simeq \underbrace{h_{SQL}}_{5\cdot 10^{-20}} \sqrt{\frac{\overset{10^{-3}\,\text{sec.}}{\tau}}{\underset{50\cdot 10^{-3}\,\text{sec}}{\tau_e^*}}} \simeq 1\cdot 10^{-20}$$

IF ONE EAGERS TO REACH $h \simeq 3\cdot 10^{-22}$, THEN ONE HAS TO HAVE $\tau_e^* \simeq 50\,\text{sec}$, $(Q_e \simeq 10^{13})$

Al_2O_3 RING MICROWAVE RESONATORS WISPERING GALERY MODES



PHYS. LETT A v.120 300, 1987

$Q_e V$

$Q \approx 10^{13} \div 10^{14}$ $T = 4$ K