QUADRATIC SCATTERING OF AN ELECTRON FOR QND MEASUREMENT OF ENERGY

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It is shown that the scattering of an electron, flying along a dielectrical waveguide without cover, is proportional to the amplitude squared of the electrical field of the wave. The scattering will be greatly increased, if the relative difference between the speeds of the electron and the wave in the waveguide is small. This opens a new possibility for experimental realisation of quantum nondemolition measurement of energy.

It is well known that for the realisation of quantum nondemolition (QND) energy measurement, it is necessary to have a meter, whose hamiltonian of interaction with the object (an oscillator or an ensemble of oscillators) has to be proportional to the *squared* generalized coordinate of the object [1]. It was proposed to obtain such a hamiltonian with either ponderomotive effects [1] or quadratic dependence of the dielectrical permeability on the electrical field [2]. In each case both a high nonlinearity and a low level of dissipation must be provided simultaneously. So far the convenient combination of these two values, required for the realisation of QND energy measurement (or at least counting quanta without absorption) has not been found. Resonators and waveguides available in the optical and rf region do not have a high enough nonlinearity with small dissipation. The only achievement in this area: a successful solution of a more simple problem – QND measurement of the quadrature components in an optical fiber [3]. It is appropriate to note here that so far there are no formulated theoretical limitations for the level of dissipation when the nonlinearity is high.

The goal of this paper is the analysis of the quadratic scattering of single electrons in the field of a monophoton state in a waveguide. It shows that this scattering permits to solve the problem of QND measurement of one or several photons.

Let us suppose, that an electron flies along a straightforward section of a dielectrical waveguide with length L. Let the speed of the electron be $V_e < V_0$, $(V_0 - V_e)/V_0 \ll 1$, where V_0 is the speed of the em wave and the phase speed is equal to the group speed in the working bandwidth. The beam of electrons has an aperture d which is less than D, the scale length of the transverse slope of the outer field E: $E \sim \exp(-x/D)$. Within the time interval of interaction $\tau \lesssim L/V_e$ the electron receives from the field of the traveling wave the transverse momentum $P_\perp$, which consists of two components. The first one P_{lin} is proportional to E, the amplitude of the electrical field of the em wave and its sign depends on the phase in the begining of the interaction. The second one P_{quadr} is proportional to E^2 and is caused by Miller's force F_M [4], which repels oscillating electrons from the waveguide in the nonhomogeneous electrical field:

$$P_{\text{quadr}} = F_M \tau \approx e^2 E^2 \tau / 2mD\omega_e^2 , \quad (1)$$

$$\text{if} \quad F_M \tau \leqslant P_{\max} = \sqrt{mDF_M} , \quad (2)$$

where e, m are the charge and mass of an electron, ω_e is the frequency of electron oscillations. Condition (2) is due to a considerable decrease of F_M at a distance $\sim D/2$ from the waveguide. We shall suppose further that the interaction time τ is small enough, so that $F_M \tau < P_{\max}$. It may be shown that if the waveguide smoothly "approaches" and "departs" from the beam at a distance of about $L \approx V_e \tau$ then $P_{lin} < P_{max}/\omega_e \tau$. So if $\omega_e \tau \gg 1$, the scattering is fully quadratic and the corresponding hamiltonian of the electron–wave interaction $H = e^2 E^2 / 4m\omega_e^2$ is proportional to the squared amplitude E of the field, matching the condition of the QND measurement of energy. It should be stressed here, that the electron oscillation frequency $\omega_e = (1 - V_e/V_0)\, \omega_0$ (ω_0 is the wave frequency) may be much smaller than ω_0. This allows a great increase of the electron–field interaction (see (1)).

Let us suppose that the wave (in the guide) is in a monophoton state [5], with one energy quantum $\epsilon = \hbar\omega_0$ ($\hbar$ Plank constant), uncertainty of frequency $\Delta\omega_0$ and duration τ_0: $\Delta\omega_0 \tau_0 \gtrsim 1$. Everywhere below we call such a state a photon. If $V_0 \tau_0 \approx L\omega_e/\omega_0$, then, in the synchronous flight of the electron and the photon during time $\tau = L/V_e$, the photon will outdistance the electron by its length $V_0 \tau$ and gives to the electron the momentum P_{quadr} (1). The error of the energy measurement will consist of two parts:

$$(\Delta\epsilon/\epsilon)^2 = (\Delta P/P_{quadr})^2 + (2\Delta x/D)^2 ,$$

where ΔP is the diffractional uncertainty of the transverse momentum, Δx is the uncertainty of the transverse coordinate. A minimal error is achieved at optimal aperture $d = \hbar/4P_{quadr}$:

$$\frac{\Delta\epsilon}{\epsilon} = \sqrt{\frac{2mc^2}{e^2} \frac{L}{\omega_0 \tau} \frac{\omega_e^3}{\omega_0^3}} , \qquad (3)$$

where c is the speed of light. It is accepted here and everywhere below for simplicity, that the effective cross section square of the waveguide is $S = 4\pi D^2$, $D = c/\omega_0$. If we put $\omega_0 = 4 \times 10^{14}\ s^{-1}$ ($\lambda = 5 \times 10^{-6}$ m), $\omega_e = 4 \times 10^{11}\ s^{-1}$, $L = 15$ cm, $V_0 = 1.5 \times 10^{10}$ cm/s then we obtain the estimate $\Delta\epsilon/\epsilon = 0.5$ (in this case $\omega_0 \tau = 400 \gg 1$). It is important to note, that this relatively large error $\Delta\epsilon/\epsilon = 0.5$ corresponds to a rather high reliability of registration, because with the parameters mentioned above, $\Delta P/P_{quadr} = 0.1$.

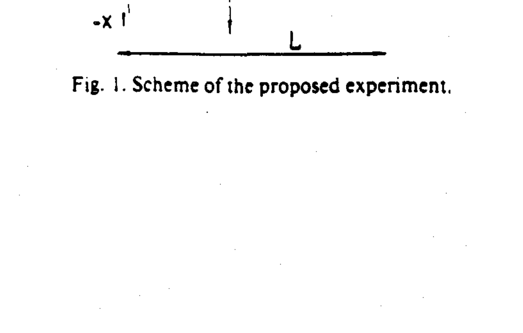

Fig. 1. Scheme of the proposed experiment.

The ratio ω_e/ω_0 cannot be made too small because of the condition $\omega_e \tau \gg 1$, as well as due to the limitation of the amplitude of electron oscillations $x_0 = eE/m\omega_e^2 \ll c/\omega_0$.

Let us consider the back action of the electron on the photon. And let the part of kinetic energy $\epsilon_\perp$, corresponding to the transverse movement of the electron, be changed after the interaction by a value $\Delta\epsilon_\perp$. Then using only the energy and longitudinal momentum conservation law, it may be shown that the photon energy must be changed by the value $\hbar\omega_0 \approx -\Delta\epsilon_\perp\, \omega_0/\omega_e$. It is clear, that at least in principle $\Delta\epsilon_\perp$ can be made equal to zero (if the transverse momentum $P_\perp$ changes sign, preserving its absolute value) and then the *mean* photon energy will not change. In the most unfavourable case ($\Delta\epsilon_\perp = +P_{quadr}^2/2m$), the relative reduction of photon energy (red shift) is about $\Delta\omega_0/\omega_0 = 10^{-2}$ (with the parameters used above). But the principal initial uncertainty of the transverse momentum $\Delta P \geqslant \hbar/2d$ causes an additional uncertainty of energy (and frequency) of the photon after interaction: $\Delta\omega_0/\omega_0 \approx \hbar/4d^2 m\omega_e$ and at $D = 10^{-4}$ cm and with the parameters used above $\Delta\omega_0/\omega_0 = 0.8 \times 10^{-4}$.

The radiation of the oscillating electron backward into the waveguide is considerable, but if the guide is used in a one-mode regime, there is no reradiation into parasitic modes. In this case strong reradiation of the electron to a signal mode results in a time shift of the photon by a certain time τ_e. At the same time the uncertainty of the transverse coordinate Δx of the electron produces the time shift uncertainty $\Delta\tau_e$, so that $\Delta\epsilon\, \Delta\tau_e \geqslant \hbar/2$.

We do not discuss technical aspects of the proposed measurement procedure (the necessary degree of monokinecity of the electron $\Delta V_e/V_e < \omega_e/\omega_0$, the equality of group and phase speeds, the compensation of mirror attraction of electrons to the waveguide, etc.) believing that all of them may be overcome.

It is interesting to note, that when flying along the guide the beam of electrons is equivalent to a dielectric medium whose receptivity $\chi = \chi^{(1)} + \chi^{(3)} E^2$ turns out to be nonlinear due to the repulsion of electrons from the guide by Miller's force. If the beam density is $n=k/LD^2$ (i.e. if there are k electrons in the interaction area), then

$$\chi^{(3)} \approx \frac{kL}{6m^3V_e^2}\left(\frac{e\omega_0}{c\omega_e}\right)^4 . \qquad (4)$$

With $k=10^3$ and the other parameters used above, we obtain $\chi^{(3)}=0.7\times10^{-3}$ cgse. This large value of $\chi^{(3)}$ (for example, quartz has $\chi^{(3)}=10^{-14}$ cgse) coexists with a rather small dissipation. Whereas nonlinear receptivity depends on the polarization of the traveling wave, i.e. it is anisotropic.

The situation analysed in this paper is, in fact, a sort of Compton scattering of the electron on the longwave monophoton state. As soon as the monophoton state is localized in space and time and attached to the waveguide, in our case the Compton effect shows two specific features: (a) there exists an impact parameter, (b) the value of the scattering is greatly enhanced due to adequate synchronisation of the electron's flight with the photon propagating in the guide.

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