Resolution in macroscopic measurements: progress and prospects

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The standard quantum limits on several macroscopic quantities and the conditions for attaining these limits are evaluated. Possibilities for exceeding them by means of quantum nondemolition measurements are examined. The best frequency stability in self-excited oscillators, the smallest mechanical displacements, and the highest Q values of mechanical and electromagnetic resonators which have been achieved are reviewed. The outlook for improving the sensitivity in several gravitational experiments is evaluated.

CONTENTS

1. INTRODUCTION

Macroscopic measurements, e.g., measurements in which it is necessary to observe the effect of a small impulse on a test mass, small variations in distances between masses, small changes in the permittivity of a sample, etc., continue to play a significant role in physical experiments aimed at fundamental problems. It has been more than a decade since a previous review[1] of such measurements was published. Over this time there has been a noticeable improvement in sensitivity, new methods have appeared, and the limiting sensitivity of several methods has been analyzed. Some interesting new results have been achieved in these macroscopic measurements, and several new programs have begun to materialize. The present review contains brief descriptions of the major advances in this field of physical experiments, a list of unresolved problems, and an attempt to predict the increase in sensitivity over the next few years. A theoretical section of this review is devoted to an analysis of quantum limitations on the sensitivity in macroscopic measurements.

2. QUANTUM NONDEMOLITION MEASUREMENTS

This section of the paper is based on two reviews[2,3] which were published in 1980 and which dealt with quantum nondemolition measurements and also several papers which have appeared since then.

Quantum nondemolition measurements (QNM) are generally understood as those measurements performed on a quantum entity in which the interaction of the entity with the instrument does not influence the results of repeated measurements. A repeated measurement does not demolish the quantum state of the entity. If no external agent (other than the measurement) acts on the entity in the interval between measurements, the results of repeated QNMs will be the same, within a small error. The QNM procedure is an approximate realization of an ideal quantum measurement as described by Von Neumann's reduction postulate: After the first QNM of an observable $\hat{y}$, the quantum entity goes into one of the eigenstates $|y\rangle$ of the operator y with a probability $\langle y|\hat{\rho}|y\rangle$ (where the operator ρ represents the state density of the system before the measurement). The result of the measurement is the corresponding eigenvalue y.

The possibility in principle of QNMs was essentially pointed out a long time ago: A paper by Landau and Peierls[4] regarding errors in measurements performed on free particles mentioned that if it were possible to choose a Hamiltonian in an arbitrary way then it would become possible to measure a momentum exactly, in an arbitrarily short time and without any change in velocity. Thirty years after that paper, the possibility of QNMs in connection with the relation $\Delta\mathscr{E}\cdot\Delta\tau \gtrsim \hbar/2$ became the subject of a debate between Aharonov and Bohm, on the one hand, and Fok, on the other.[5-9] A particular case of the conditions for the realization of QNMs was first formulated by Bohm[9] in 1962.

Practical interest in the QNM problem arose almost 40 years after the publication of Ref. 4 in connection with an analysis of the limiting sensitivities of gravitational antennas intended for observing gravitational bursts from astrophysical catastrophies. This analysis led to results which have turned out to be important not only for developing highly sensitive gravitational antennas but also for other macroscopic measurements. Two of these results will be discussed in detail here.

The first is that in experiments with macroscopic free particles or macroscopic oscillators the quantum properties of these entities may be manifested even at a relatively high reservoir temperature T, provided that the coupling with the reservoir is sufficiently weak. This position can be justified in the following way.

Let us assume that some instrument *continuously* measures the coordinate of a mechanical oscillator, $x(t)$, that the bandwidth of the instrument is $\Delta\omega_M \simeq \omega_M$ (ω_M is the resonant frequency of the oscillator), and that the averaging time is $\tau \simeq 1/\omega_M$. The smallest error in a determination of the coordinate is then given approximately by

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$$\Delta x_{osc} \approx \left(\frac{\hbar}{2m\omega_M}\right)^{1/2} \approx 6\cdot 10^{-18}\ \mathrm{cm}\cdot\left(\frac{10^3\ \mathrm{r}}{m}\right)^{1/2}\left(\frac{10^4\ \mathrm{rad/s}}{\omega_M}\right)^{1/2}, \qquad (1)$$

and the oscillator will be in a state which is approximately a coherent state.[11] Expression (1) is valid under the assumption that the oscillator is a *quantum* oscillator and has no coupling of any sort with a reservoir. On the other hand, the random amplitude change Δx_T of a *classical* oscillator which is weakly coupled with the reservoir will become progressively smaller as the observation time τ becomes smaller in comparison with the relaxation time $\tau^*_M = 2Q/\omega_M$ (Q_M is the quality factor of the oscillator):

$$\Delta x_T \approx \left(\frac{2kT\tau}{m\omega_M^2\tau^*_M}\right)^{1/2} = \left(\frac{kT\tau}{m\omega_M Q_M}\right)^{1/2}. \qquad (3)$$

In the case $\Delta x_{osc} \gtrsim \Delta x_T$, the macroscopic oscillator should obviously be treated as a quantum entity if its coordinate is being measured. Substituting the right sides of (1) and (3) into this inequality, we find a condition for the "quantumness" of an oscillator:

$$\frac{2kT\tau}{Q_M} \leqslant \hbar. \qquad (4)$$

For this condition to be satisfied at $T = 2$ K and $Q_M = 10^9$ (a quality factor of this magnitude has been achieved reliably for several types of mechanical oscillators at frequencies $\omega_M/2\pi \gtrsim 10^3$ Hz), we must have $\tau \lesssim 10^{-3}$ s. In other words, even a relatively low-frequency macroscopic oscillator will behave as a quantum entity under conditions attainable in the laboratory. Later in this paper, in the section on gravitational antennas, we will take a more detailed look at just how close the experimentalists have come to this threshold.

Corresponding arguments can be repeated for a free particle: If one continuously measures the coordinate Δx_{fp} of a free particle over a time τ, then the smallest error in the determination of the average value of the coordinate will be[2]

$$\Delta x_{fp} \approx \left(\frac{\hbar\tau}{2m}\right)^{1/2} \approx 2\cdot 10^{-17}\ \mathrm{cm}\cdot\left(\frac{\tau}{10^{-1}\ \mathrm{s}}\right)^{1/2}\left(\frac{10^3\ \mathrm{g}}{m}\right)^{1/2}. \qquad (5)$$

On the other hand, if the classical particle has a dissipative coupling with a reservoir (the relaxation time is $\tau^*_M = m/H$, where H is the coefficient of friction), its random classical displacement will be

$$\Delta x_T \approx \left(\frac{kT\tau^3}{m\tau^*_M}\right)^{1/2}, \qquad (6)$$

if $\tau \ll \tau^*_M$.

A macroscopic particle evidently must be treated as a quantum entity in a measurement of its coordinate if $\Delta x_{fp} \gtrsim \Delta x_T$ i.e., if

$$\frac{2kT\tau^2}{\tau^*_M} \leqslant \hbar. \qquad (7)$$

Condition (7) is relatively easy to satisfy, even for a reservoir at room temperature: At $\tau = 10^{-3}$ s, we would need $\tau^*_M \gtrsim 4\times 10^7$ s.

We wish to stress that quantumness conditions (4) and (7), like limits (1) and (5), which are called *standard quantum limits*, are valid only in measurements of a coordinate. If the observable is some other quantity, then the conditions and the limits will be different. For example, if the experimentalist has an instrument which can directly measure the energy of an oscillator (no such instrument has been developed; possible methods for realizing one will be discussed below), the discrete nature of the energy levels of the oscillator will be manifested if

$$\frac{4}{3}\,\frac{\pi}{Q}\left(nn_T + \frac{n + n_T}{2}\right) \leqslant 1, \quad \text{where } n_T = \left(\exp\frac{\hbar\omega}{kT} - 1\right)^{-1}. \qquad (8)$$

Here n is the number of quanta in the oscillator (see Refs. 10 and 11 for more details regarding this condition).

Conditions (4) and (8) also hold for electromagnetic resonators if the experimentalist wishes to measure, for example, the field amplitude of one of the modes or the energy of one of the modes in such a resonator.

We wish to emphasize an important consequence of this result: Under conditions (4), (7), and (8) and corresponding conditions, the experimentalist is dealing with a *single* quantum entity which has macroscopic dimensions. According to quantum mechanics, an entity can be prepared in a certain state as the result of a first measurement (or of an action on it). During the measurement, the experimentalist may, if he wishes, change the interaction of the instruments with the entity, observe the evolution of the parameters of the system, etc.

A second important result of the analysis of the limiting sensitivity of gravitational antennas (and a result which also applies to many other experiments with macroscopic entities) can be formulated as follows: Standard quantum limits (1) and (5) and similar limits do not limit the sensitivity in measurements of a small force $F(t)$ which is acting on a macroscopic oscillator or a free macroscopic particle. These standard quantum limits can be exceeded, in particular, if one uses QNMs. The basic idea of QNMs is simple: It is necessary to choose an observable whose operator commutes with itself in time so that one can keep the measurement errors small and repeat the measurements many times. For an oscillator, such observables are any of the two quadrature components of the coordinate $x(t)$ [see Eq. (11) below] and the energy of the oscillator, $\mathscr{E}$ (these variables correspond to integrals of motion[12]). For a free particle, the integrals of motion are its momentum (as was pointed out back in 1931 by Landau and Peierls, as we mentioned earlier) and its energy.

Let us take a more detailed look at the procedure for QNMs for an oscillator. During a continuous measurement of a coordinate, as we mentioned earlier, the error in this measurement, Δx_{osc}, is determined by (1). This quantity corresponds to the standard quantum limit on the impulse whose effect can be observed, $(F\times\tau_F)_{sql}$:

$$(F\tau_F)_{sql} \approx (\hbar m\omega_M)^{1/2}. \qquad (9)$$

If the experimentalist has an instrument which performs a QNM of energy of the oscillator (or of its amplitude, but without a measurement of the phase), then it is possible to detect the effect of the impulse $(F\tau_F)$ under the condition that there has been a transition from level n to level $n\pm 1$. For the transition probability to be of the order of unity, we would have to have

$$(F\tau_F)_n \approx \left(\frac{\hbar m\omega_M}{n}\right)^{1/2}. \quad (10)$$

By measuring the energy (or amplitude), one can evidently observe arbitrarily small values of $(F\cdot\tau_F)_n$, by first increasing the value of n. Clearly, the oscillator must be in state n before the impulse is applied. This situation can be arranged by connecting to the oscillator the same instrument as is used to measure the energy of the oscillator.[31] Some possible procedures for measuring the energy of an electromagnetic oscillator will be discussed later in this section of the paper.

A second possibility for achieving a sensitivity higher than the standard quantum limit (9) is to measure one of the quadrature components of the coordinate of an oscillator, X_1 or X_2 (see the reviews in Refs. 2 and 3; see also Refs. 15 and 16):

$$\hat{x}(t) = \hat{X}_1 \cos\omega_M t + \hat{X}_2 \sin\omega_M t. \quad (11)$$

A graphic way to realize this measurement procedure would be to carry out stroboscopic measurements of the coordinate of the oscillator over *short* time intervals $\tau \ll 2\pi/\omega_M$; these measurements would be repeated each oscillation period. In this case the measurement error would be determined by limit (5): $\Delta x_{fp} \simeq (\hbar\tau/2m)^{1/2}$. The instrument which measures the coordinate is "connected" to the oscillator for only a brief time interval during the oscillation period. In the pauses between these intervals of connections, the wave function initially spreads out and then contracts to a magnitude Δx_{fp}. This magnitude depends on the particular choice of instrument, but even in the optimum case it cannot be smaller than the limit (5). If the oscillator does not experience an impulse of an external force, then after an oscillation period (or an integer number of periods) the observer will detect the same value of the coordinate, with approximately the same error (Fig. 1). This quantity will be one of the quadrature components (the latitude in the choice of X_1 and X_2 is determined by the latitude in the choice of the phase of the stroboscopic measurement). The second quadrature component, on the other hand, is not measured; since

$$\Delta x \simeq \Delta X_1 \approx \left(\frac{\hbar\tau}{2m}\right)^{1/2}, \quad \Delta X_1\cdot\Delta X_2 \approx \frac{\hbar}{2m\omega_M},$$

we have

$$\Delta X_2 \gg \Delta X_1.$$

If an external impulse $(F\tau_F)$ acts on the oscillator in the interval between two stroboscopic measurements, it can be detected in this measurement procedure under the condition

$$\Delta x_F \approx \frac{F\tau_F}{2m\omega_M} \gtrsim \Delta X_1 \approx \left(\frac{\hbar\tau}{2m}\right)^{1/2}, \quad (12)$$

or, in a different form,

$$(F\cdot\tau_F)_{\rm strob} \gtrsim (\hbar m\omega_M)^{1/2}(\tau\omega_M)^{1/2}. \quad (13)$$

Clearly, the improvement in sensitivity increases with decreasing value of $(\tau\omega_M)^{1/2}$, which is equal to the ratio of limits (1) and (5).

The stroboscopic procedure for measuring the coordinate of an oscillator leads to a large uncertainty in $x(t)$ between measurements [it leads to a large perturbation $\Delta X_2 \gg (\hbar/2m\omega_M)^{1/2}$] and thus to a large mean square perturbation of the oscillator energy. It is not difficult to show that this perturbation of the energy is approximately equal to $n\hbar\omega_M (F\tau_F)_{\rm strob} \simeq (F\tau_F)_n$ [see (10)]. In other words, the price paid for the increase in sensitivity in this procedure is the same as that paid in measurements of transitions from level n to a neighboring level: a pronounced excitation of the oscillator (the extent of the excitation is approximately the same in the two cases). A rigorous equation relating the condition for the observation of a force to the initial state of the quantum entity was derived by Vorontsov and Khalili[17]: The effect of the force can be observed only if the quantity

$$\int_0^{\tau_F} F(t)x(t)\,dt$$

is greater than $\hbar/2$.

Several authors[18-20] have carried out detailed analyses of the possibility, suggested by Thorne *et al.*,[16] of *continuous* measurements of one of the quantities $X_1(t)$, $X_2(t)$ in an arrangement similar to a stroboscopic arrangement. These analyses led to the same result: One can achieve an improvement by a factor of $n^{1/2}$ in comparison with standard limit (9) if the oscillator has a mean square energy uncertainty $n\hbar\omega$. An attempt has been undertaken to carry out an approximate QNM of one of the quadrature components of a mechanical oscillator in order to detect responses significantly weaker than the standard quantum limit, (1) (Johnson and Bocko[20]). Those experiments have not yet been completed. Without going into detail on the difficulties which have arisen, we would point out that the basic problem turns out to be the need to develop a cryogenic parametric sensor of small values of Δx which has an electrical quality factor $\gtrsim 10^8$ with small mechanical gaps in the volume of the sensor.

It can be seen from this discussion that an analysis of the QNM of X_1 or X_2 of a mechanical oscillator has been pursued to the state of engineering estimates for specific experiments. No one has suggested methods for direct QNMs of the energy of a mechanical oscillator.

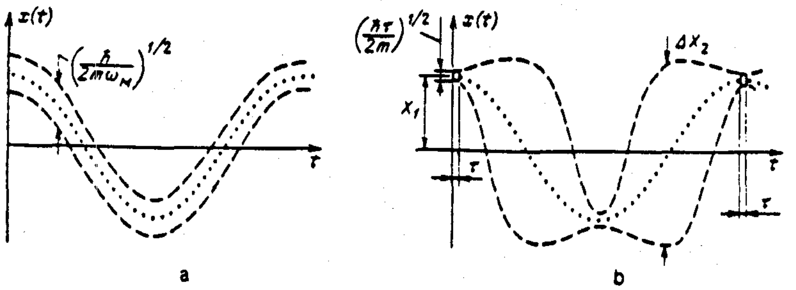

FIG. 1. Uncertainty in the coordinate of a mechanical oscillator in a coherent quantum state (a) and during stroboscopic measurements (b).

Turning to electromagnetic oscillators (resonators), we find the opposite situation: The possibilities of QNMs of energies have been studied in some detail.[21-26] The most graphic method for QNMs of the energy $\mathscr{E}$ in one of the oscillation modes ω_e of an electromagnetic oscillator is to measure the ponderomotive force F_p between the walls of the resonator or parts thereof: $F_p = (\mathscr{E}/l) \approx n\hbar\omega_e \cdot l^{-1}$ (where l is of the order of the dimensions of the resonator).[21,22] In this method we can clearly see the particular features of QNMs: The force to be measured is proportional to the energy $\mathscr{E} = n\hbar\omega_e$ (not to a charge or a field); detecting the force requires measuring a slow mechanical displacement (over a time $\tau \gg 2\pi/\omega_e$) Δx of one of the walls, which is caused by F_p. This displacement leads to a "reddening" or "lueing" of the entire set $n\hbar\omega_e$. As a result, there is a pronounced perturbation of the oscillation phase φ, which does not commute with the energy, while n does not change. A relatively straightforward calculation shows that in this measurement (provided that there are no additional perturbations during the detection of small values of Δx) the error in the measurement of the number of quanta, $\Delta n = \Delta\mathscr{E}/\hbar\omega_e$, and the perturbation of the phase, $\Delta\varphi$, will satisfy the condition

$$\Delta n \cdot \Delta\varphi \approx \frac{1}{2}. \qquad (14)$$

However, we have $\Delta n \approx 1/2\omega_e\tau \ll 1$ and $\Delta\varphi \gg 1$. In other words, the ponderomotive measurement instrument prepares oscillations in a mode in state n (a purely energy state) and provides information about the value of n within an error $\Delta n \ll 1$. If there is a significant excess noise during the measurements of the wall displacement Δx, the quantity $\Delta n \cdot \Delta\varphi$ may also be greater than unity. That circumstance, however, will not, in principle, interfere with the attainment of a resolution level $\Delta n \lesssim 1$ in the presence of such noise. An interesting feature of electromagnetic resonators with soft walls was pointed out by Vyatchanin[27]: Continuous measurements of the coordinate in these resonators lead to a progressive transition of the quantum states into states which are approximately energy states. Vorontsov[28] has shown that in an *indirect* quantum measurement in an arrangement similar to that which we just discussed the error Δn can in principle be even less than $1/2\omega_e\tau$ (Ref. 28).

In practice, the ponderomotive method for QNMs of the energy turns out to be not very convenient: Very large values of Q_e are required [in order to satisfy condition (8) in a real reservoir], as are extremely sensitive dynamometers (especially if ω is in the microwave, rather than optical, range). It is apparently for these reasons that attempts have been undertaken to develop other methods for QNMs of energies. These other methods are like the ponderomotive method in that the response of the resonator to the energy applied to it, $\mathscr{E} = n\hbar\omega_e$, is proportional to $\mathscr{E}$, rather than to a charge (or field). As a quadratic effect here one could use (a) the inverse Faraday effect,[23] (b) the optical Kerr effect in optical experiments,[26] or (c) the cubic nonlinearity of a substance in a resonator (a frequency shift which is proportional to $\mathscr{E}$ in an additional "measurement" mode of a resonator).[24,25] Implementlng these procedures will require a large nonlinearity along with a high quality factor (low damping).[41] If it does prove possible to find such substances (or to devise systems with an "amplified" nonlinearity,[29] e.g., by using capacitors whose plates have a set of sharp metal points surrounded by a nonlinear dielectric), then this direction for the development of methods for energy QNMs has the potential to lead to several important results, among which we will single out the following:

a) counting quanta without absorption and developing detectors with a noise temperature much lower than $\hbar\omega_e/k$, and

b) transmitting information by means of purely energy states (with a lower expenditure of energy at a given reliability level[30]). There are the further possibilities, pointed out by Whitten,[31] of using such methods to study new unknown mechanisms in genetic systems and of using such measurement procedures in quantum-mechanical computers, the formal theory for which, developed by Feynman, has features in common with energy QNMs (Ref. 32).

Among the methodological applications of energy QNMs, aside from their use for gravitational antennas, we should mention the possibility of detecting the stepped nature of the Brownian motion of an oscillator if $kT > \hbar\omega_e$ but the quantumness condition (8) holds. The size of each step is close to $\hbar\omega_e$, and a high-Q oscillator will in each step be in a state which is approximately a pure energy state. The step "thickness" ($\Delta\mathscr{E} = \Delta n \cdot \hbar\omega_e$) (the fluctuations in the readings of the instrument) will be greater, the higher the value of n and the higher the reservoir temperature[11]:

$$(\Delta n)^2 \approx \frac{1}{(2\omega_e\tau)^2} + \frac{4}{3}\frac{\tau}{\tau_e^*}\left(nn_T + \frac{n+n_T}{2}\right), \qquad (15)$$

where τ is the averaging time of an optimum QNM of the energy.

On the whole, one can say about these two examples of QNMs that for the quantities X_1 and X_2 of a mechanical oscillator and for the quantity $\mathscr{E}$ of an electromagnetic resonator the theoretical side of QNMs has been developed well, and "all" that remains to be done is to perform these measurements. We will not go into the details of the detection systems (e.g., systems for measuring small values of Δx for a mechanical oscillator), but we should emphasize that these systems will have to meet some fairly stiff requirements. For example, when capacitive or optical parametric sensors are used to detect small values of Δx, the frequency and amplitude fluctuations of the pump sources must be within certain ranges, and the pump power itself must be at an optimum level (which depends, in particular, on m and τ). The calculation details are given in the literature cited above and also in a paper by Caves.[33] Caves suggested the term "squeezed quantum states"; and that term is used along with the "two-photon coherent quantum states," proposed by Yuen,[34] to describe states of an oscillator or a radiation field which are greatly different from coherent states.[35] One example of squeezed states is in the stroboscopic measurement described above. Figure 2 illustrates the situation with various cases of the quantum states of a mechanical oscillator.

Methods for quantum nondemolition measurements of quantities analogous to X_1 and X_2 in an electromagnetic resonator have been developed to a considerably lesser extent. Vorontsov and Kolesov[36] have studied the possibility in principle of measuring an alternating electric field in a microwave resonator by making use of the scattering of an electron beam. If a continuous electron beam is used, and if the detection time is $\tau \lesssim 1/\omega_e$, one can reach a sensitivity corresponding to the standard quantum limit:

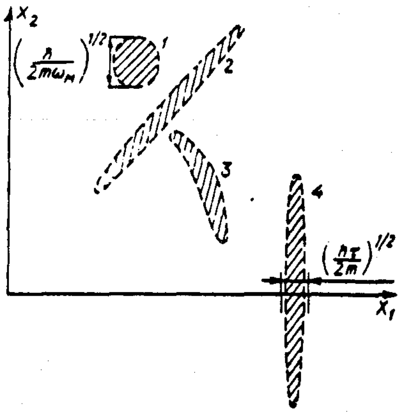

FIG 2 Various quantum states of an oscillator in terms of the coordinates X_1, X_2. 1—Coherent state ($\Delta n \approx n^{1/2}, \Delta\varphi \approx 1/n^{1/2}$); 2—phase-squeezed state ($\Delta n > n^{1/2}, \Delta\varphi < 1/n^{1/2}$); 3—energy-squeezed state ($\Delta n < n^{1/2}, \Delta\varphi > 1/n^{1/2}$); 4—state after a stroboscopic measurement corresponding to Fig. 1b. The area of each of the figures is $\pi\hbar/8m\omega_M$.

$$\Delta U_{\rm sql} \approx \left(\frac{\hbar\omega_e}{2C_e}\right)^{1/2} \approx 1\cdot 10^{-6}\,{\rm V}\cdot\left(\frac{\omega_e}{2\cdot 10^{10}\ {\rm rad/s}}\right)^{1/2}\left(\frac{1\ {\rm pF}}{C_e}\right)^{1/2}, \tag{16}$$

where $\Delta U_{\rm sql}$ is the error in the measurement of the voltage across capacitance C_e, which is part of a resonator with a frequency ω_e. Modulation of the electron beam (as in the case with X_1 and X_2) makes it possible to measure values smaller than $\Delta U_{\rm sql}$.

The situation regarding the quantum nondemolition measurements of the momentum of a free particle is roughly the same: Vorontsov[28] has proposed a method for the nondemolition measurement of a generalized momentum (the mass has an electric charge). Possibilities for a simple implementation of that method have not been studied.

Several authors[37–42] have discussed the possibility in principle of detecting the response of a free particle to an applied force within an error smaller than the standard quantum limit, (5), in *coordinate* measurements on a free particle. Caves[41] appears to have summed up the situation appropriately when he concludes that either the particle is not completely free in the course of the measurements or these measurements are not quantum nondemolition measurements in the sense of the definition given above. We see that the theory of quantum nondemolition measurements has been developed well for the examples of entities having one or two degrees of freedom. Certain aspects of quantum measurements in distributed systems were examined in Ref. 43.

In summarizing this brief description of the present state of affairs regarding methods for quantum nondemolition measurements, we find two comments to be important.

1. Quantum theory (as it is presented in textbooks today[44]) allows *exact* measurements of certain observables in an arbitrarily short time. As can be seen from the examples above, that assertion is *incorrect*. As we have already mentioned, if one attempts to detect an impulse of a force (or change in energy) with a progressively smaller error one will find it necessary to increase the energy in the oscillator and the power flux in the measuring instrument without limit. The impossibility of an unlimited increase in sensitivity thus stems from "breakdown" effects, which are determined by such quantities as the charge of an electron and the masses of an electron and a proton. These quantities do not enter the formal quantum theory in any essential way, and their presence in this world rules out the possibility of exact quantum measurements.

2. The scheme for quantum nondemolition measurements is clear in the simplest cases. A universal (necessary and sufficient) condition for such measurements is that the evolution operator of the pair consisting of the measuring instrument and the object of the measurements commute with the operator of the observable. Necessary and sufficient conditions for quantum nondemolition measurements or for approximate quantum nondemolition measurements have not yet been formulated in a way which allows for the finite measurement time and the finite bandwidth of the interaction between the instrument and the object. We might add that we do not yet have a systematic theory for quantum measurements in which the relationship between the object and the instrument is parametric or nonlinear.

In the rest of this review we will touch on some additional problems of quantum measurements, and we will compare the resolution which has actually been achieved for various measured quantities with their standard quantum limits.

3. DEVELOPMENT OF HIGH-Q OSCILLATORS AND FREQUENCY STABILIZATION OF SELF-EXCITED OSCILLATORS

High-Q mechanical oscillators and electromagnetic resonators are widely used in experimental physics, in particular, in macroscopic measurements. We will present three examples here to illustrate the governing significance of large values of the mechanical and electrical quality factors $Q_{\rm M}$ and Q_e, respectively.

a) In order to reach a sensitivity corresponding to the standard quantum limit (9) for the impulse $(F\tau_F)_{\rm sql}$, it is necessary to satisfy the inequality (4): $\hbar \gtrsim 2kT_{\rm M}\tau Q_{\rm M}^{-1}$. If this inequality *does not* hold, then purely classical Brownian fluctuations will determine the minimum observable impulse $(F\tau_F)_{\rm T}$, which is

$$(F\tau_F)_{\rm T} \approx \left(\frac{4kTm\omega_{\rm M}\tau}{Q_M}\right)^{1/2}, \tag{17}$$

where the measurement time satisfies $\tau \gtrsim \tau_F$

It obviously follows from (17) that increasing the sensitivity of galvanometers, accelerometers, etc., even in the classical approximation, requires the use of oscillators with a $Q_{\rm M}$ value as large as possible.

b) The smallest quasistatic displacement Δl (or oscillation amplitude) which can be detected with a parametric capacitive sensor is found from the simple condition

$$Q_e^2 W\left(\frac{\Delta l}{l}\right)^2 \geqslant 4kT_a\Delta f, \tag{18}$$

where Q_e is the quality factor of the circuit of which the capacitance is part, the capacitance gap l changes by a small amount Δl, W is that power of the pumping by an external source which "passes through" the circuit, T_a is the noise temperature of the amplifier (or detector), and Δf is the frequency band which contains the greater part of the spectrum of Δl. Condition (18) remains in force if we replace

$\Delta l/l$ by $\Delta\varepsilon/\varepsilon$, i.e., by a small relative change in the permittivity constant of the material in the dielectric. As is obvious from (18), we have $\Delta l/l$ or $\Delta\varepsilon/\varepsilon \sim Q_e^{-1}$.

c) The long-term instability of the frequency of secondary frequency standards (if we ignore the drift of the resonance frequency of the highly stable resonator) decreases with increasing quality factor of the resonator since we have

$$\left(\frac{\Delta\omega}{\omega}\right)_{\text{s-osc}} \approx \frac{\delta\varphi}{Q_e}, \qquad (19)$$

where $\delta\varphi$ is the phase instability in the regeneration circuit (ordinarily, we would have $\delta\varphi \sim 10^{-5}$–$10^{-6}$ rad).

In the rest of this review we will give some examples of measurement methods in which Q_M and Q_e play a governing role.

Data on methods for achieving large values of Q_M and Q_e are summarized in the review in Ref. 45, which reflects the "experimental culture" in this area in 1981. That review lists as record values $Q_M \simeq 5\cdot 10^9$ (a cylindrical resonator consisting of a sapphire single crystal with $\omega_M/2\pi \simeq 3\times 10^4$ Hz $T_M \simeq 4$ K) and $Q_e \simeq 5\times 10^{11}$ (Refs. 46–48; a superconducting Nb resonator with $\omega_e/2\pi \simeq 10$ GHz and $T_e \simeq 1.3$ K). In the years since the publication of Ref. 45, these record high quality factors have not been surpassed for *mechanical and microwave electromagnetic resonators*. On the other hand, a unique *optical* resonator of the Fabry-Perot type[49] has been fabricated with a quality factor $Q_e \simeq 10^{13}$ (a relaxation time $\tau_e^* \simeq 10^{-3}$ s for a frequency $\omega_e/2\pi \simeq 6\times 10^{14}$ Hz). This resonator is distinguished by not only its length ($l = 4\times 10^3$ cm) but also its mirrors: The difference between their reflectances from unity is $1 - R \simeq 3\times 10^{-5}$. We will go into more detail on the use of this resonator for measuring the small values of Δl in a gravitational antenna in the two sections of this paper which follow.

Among other achievements over the past five years, three deserve mention here:

1) The quality factor Q_M for the fundamental mode of a mechanical Nb resonator cooled to 4 K has reached 4×10^8 (Veitch *et al.*[50]).

2) The anomalously low level of dielectric loss in high-quality sapphire single crystals (α-Al_2O_3) which was observed[51] in 1976 has made it possible to develop disk dielectric microwave resonators ("whispering-gallery-mode") with quality factors $Q_e \simeq (5$–$6)\times 10^7$ at $T_e \simeq 77$ K and $Q_e \simeq 1.3\times 10^9$ at $T \simeq 4$ K at a frequency of 9–10 GHz (Refs. 52 and 53). As the $Q_e(T_e)$ dependence was changed, it was found possible to observe in such resonators the *intrinsic loss* in dielectrics, predicted by Gurevich.[54,55] Such a loss arises in an ideal dielectric single crystal only as a result of a lattice anharmonicity. This loss has a characteristic steep temperature dependence, which differs for different classes of crystals (for purely hexagonal crystals, for example, we have $\tan\delta = Q_e^{-1} \sim T^5$. In Fig. 3, taken from Refs. 52 and 53, we can clearly see a region of a "power-law" growth of the quality factor Q_e, in the temperature range 200 K > T_e > 50 K, which is characteristic of an intrinsic loss according to Gurevich. Below 50 K, the increase in Q_e with decreasing T slows down; it is determined here by the low impurity level and crystal defects. If it becomes possible to "extend" the region of intrinsic loss from 50 down to 4 K through refinements in

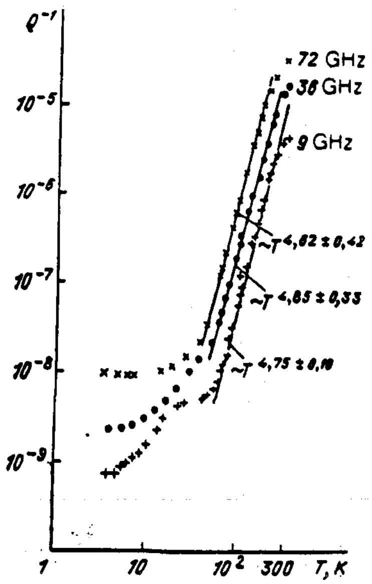

FIG. 3. Temperature dependence of quality factors of the modes of microwave dielectric ring resonators.[52,53]

techniques for growing leucosapphire single crystals, we can expect to reach values $Q_e \gtrsim 1\cdot 10^{13}$.

3) The use of a cladding of Pb of high chemical purity on a cylinder of high-quality sapphire with a low level of impurities and dislocations has made it possible to achieve $Q_e \simeq 2\cdot 10^9$ at $T = 1.5$ K in a resonator of this type ($\omega_e/2\pi \simeq 2.7$ GHz).[56] This figure is about an order of magnitude higher than the quality factor which had been found previously[52] for resonators of this type.

In completing this brief description of advances in the development of oscillators and resonators with high quality factors, we might note that in essentially none of the methods have the record values which have been achieved for Q_M and Q_e been determined by fundamental factors (except in a relatively narrow temperature range in the case of the dielectric disk resonators). At low temperatures, the values of Q_M of the mechanical resonators using single-crystal dielectrics are determined by the loss in the suspension and by the surface layer damaged by the processing. In the case of superconducting resonators the determining factor is the residual resistance in the superconductors, whose nature has received little study. In the case of optical resonators the determining factor has been the loss in the multilayer interference mirrors; the dissipation limit in these mirrors is also unknown. There is accordingly the hope that we will see future increases in both Q_M and Q_e.

The high frequency stability of a self-excited oscillator generating electromagnetic oscillations is one of the factors which determines the sensitivity in macroscopic experiments. If it is necessary to detect a small change Δl in the distance (l) between the plates of a capacitor in a parametric capacitive sensor a condition which must be satisfied in addition to (18) is that the instability of the self-excited oscillator, $(\Delta\omega/\omega)_{\text{s-osc}}$, satisfy

$$\left(\frac{\Delta\omega}{\omega}\right)_{\text{s-osc}} \leqslant \frac{\Delta l}{2l}, \qquad (20)$$

under the condition that a balanced arrangement is not used.

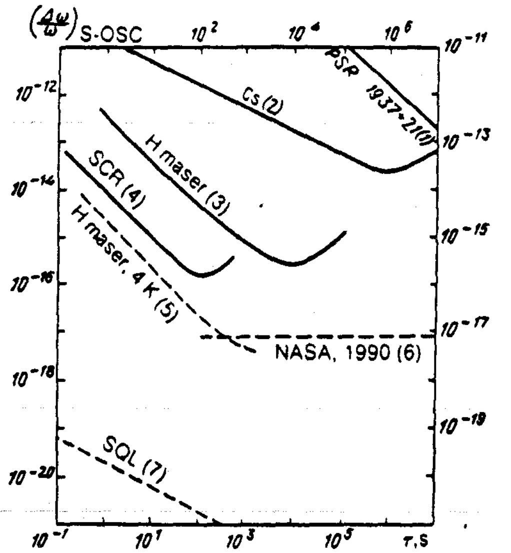

FIG. 4. Frequency instability of various sources of electromagnetic oscillations as a function of the averaging time. 1—Modulation of the radiation from the pulsar PSR 1937 + 21; 2—cesium frequency standard; 3—hydrogen frequency standard; 4—cryogenic self-excited oscillator using a niobium resonator. (SCR = superconducting resonator; SQL = standard quantum limit; NASA = National Aeronautics and Space Administration).

We could replace $\Delta l/2l$ in (20) by $\Delta L/L$ (if the propagation time of some radio or optical signal to a remote target changes) or $\Delta\varepsilon/\varepsilon$ (if one is measuring a small change in the permittivity of a material in a capacitor).

Figure 4 shows data characterizing the state of the art with regard to the relative frequency instability of various types of self-excited oscillators (in Allan variations), along with some promising estimates for new self-excited oscillators which are being developed. It can be seen from Fig. 4 that the smallest frequency instabilities, $(\Delta\omega/\omega)_{s\text{-}osc} \simeq 2\cdot 10^{-16}$, have been achieved with the help of secondary frequency standards based on superconducting resonators[57] (curve 4) and with the help of waveguide masers[58] (curve 3). The minimum values of $(\Delta\omega/\omega)_{s\text{-}osc}$ which have been achieved are not limits: With justified optimism we can expect to see a substantial reduction in $(\Delta\omega/\omega)_{s\text{-}osc}$ in the near future. The level $(\Delta\omega/\omega)_{s\text{-}osc} \simeq 1\cdot 10^{-17}$ at averaging times $\tau \simeq 10$–10^7 s (line 6 in Fig. 4) is the common 1990 goal of various new NASA-financed projects to develop secondary frequency standards.[59]

Let us take a brief look at the key features of three of these projects.

1) The possibilities for improving the frequency stability of a secondary standard which uses a superconducting resonator in the microwave range as a reference were not completely exploited in the project by Turneaure,[57] since the feedback circuit, using a relatively unstable Gunn diode, was outside the temperature-regulated cryostat. This circumstance may have been responsible for substantial frequency fluctuations. In the project of Dick *et al.*[56,57] a maser regeneration mechanism in ruby is to be used, and a completely cryogenic secondary frequency standard with an instability $(\Delta\omega/\omega)_{s\text{-}osc} \simeq 1\times 10^{-17}$ is to be developed in this manner. It is pertinent to note here that the drift (low-frequency) excursions of the resonant frequency of a superconducting resonator itself have essentially not been studied in detail at low temperatures. Such excursions could of course contribute to the instability $\Delta\omega_e/\omega_e$ at large values of τ. It may be that these drift phenomena are caused primarily by tunneling of atoms in solids.[60] If so, one can expect that the replacement of the superconducting resonators by dielectric (sapphire) ring resonators, which were discussed above, would make it possible to reduce this drift significantly, since the Debye temperature and Peierls barrier of sapphire are significantly higher than those of superconducting metals.

2) Line 5 in Fig. 4 shows the expected instability $(\Delta\omega/\omega)_{s\text{-}osc}$ of a hydrogen maser with a resonator cooled to liquid-helium temperature. The key idea of this refinement, which was proposed by Vessot,[58] is a substantial reduction of that frequency drift of the maser which is caused by the frequency drift of its resonator. For a hydrogen maser, the frequency which is generated differs slightly from the frequency of a hyperfine transition in the hydrogen atom, ω_{trans}, because of a difference between the resonator frequency ω_{res} and the transition frequency ω_{trans}:

$$\left(\frac{\Delta\omega}{\omega}\right)_{s\text{-}osc} \approx \frac{Q_{res}}{Q_{trans}}\,\frac{\omega_{res}-\omega_{trans}}{\omega_{trans}}, \tag{21}$$

where Q_{res} is the quality factor of the microwave resonator (about 104), $Q_{trans} \approx \tau^*\cdot\omega_{trans} \approx 10^9$ and τ^* is the time "spent" by a hydrogen atom in the resonator (about 1 s). Because of the temporal drift of the quantity ω_{res}, the second factor in (21) changes by about $\approx 10^{-9}$ over $\tau = 10^5$ s at $T \approx 300$ K. Drift effects decrease significantly at liquid-helium temperature. For a sapphire microwave resonator clad with a superconductor, for example, over $\tau \approx 3\cdot 10^7$ s we would have[61] $\Delta\omega_{res}/\omega_{res} \lesssim 3\cdot 10^{-9}$. In other words, the drift rate is two orders of magnitude smaller under these conditions.

3) Competing with the cooled hydrogen maser is a project to develop secondary frequency standards using mercury ions, $^{199}Hg^+$, in which, as in the hydrogen maser, use is made of the line of a hyperfine transition ($\omega_{trans}/2\pi = 40.5$ GHz). These ions have the obvious advantage of a large mass over hydrogen atoms: The frequency shift due to the second-order Doppler effect is smaller by a factor of 200. The $^{199}Hg^+$ ions are confined in an rf confinement system for $\tau \gtrsim 10^3$ s (three orders of magnitude longer than in a hydrogen maser). The effect is to reduce the linewidth substantially. It has been suggested that in the near future such frequency standards will have a relative frequency excursion $(\Delta\omega/\omega) \approx 10^{-12}(1\text{s}/\tau)^{1/2}$ (see the details in Ref. 62 and the literature cited there).

Optical standards are surpassed by standards in the microwave range by about an order of magnitude in terms of the frequency stability level under the condition $\tau > 1$ s (Refs. 63 and 64). It is pertinent to note here that in some cases it is convenient to switch from laboratory standards to the radio signals from pulsars, which, although low in intensity, are quite stable in frequency. As an example, Fig. 4 (line 1) shows the frequency instability of the fastest-rotating pulsar, PSR 1937 + 21 (Ref. 65). These radio signals have turned out to be convenient for use in radio-transmission studies of the plasma near the sun[66] and for some interesting gravitational-wave experiments which we will mention in §5 of this review.

We conclude this section by noting that beginning at a certain characteristic level attempts to achieve a higher frequency stability are confronted by purely quantum (measurement) factors analogous to those described in the preceding section. It can be shown[67,68] that there exists a standard quantum limit on the frequency instability of a self-excited oscillator, $(\Delta\omega/\omega)_{sql}$, analogous to the standard quantum limits for the coordinate of a free particle and a mechanical oscillator [see (1) and (5)]. If the resonator of a self-excited oscillator has a volume V, and if this volume is filled with a solid with a Young's modulus Y, the expression for $(\Delta\omega/\omega)_{sql}$ can be put in the simple form

$$\left(\frac{\Delta\omega}{\omega}\right)_{sql} \approx \left(\frac{\hbar}{VY\tau}\right)^{1/2}; \qquad (22)$$

this limit can be reached at the optimum self-excited oscillator power

$$W_{optimum} \approx \frac{VY\omega_e}{Q_e^2}. \qquad (23)$$

Expressions (22) and (23) contain as a particular case the Schawlow-Townes relation.

Line 7 in Fig. 4 shows, as an example, a plot of $(\Delta\omega/\omega)_{sql}$ as a function of τ for $V = 1$ cm^3 and $Y = 4\cdot 10^{12}$ dyn/cm^2. It can be seen from a comparison of lines 4 and 7 that even in the better cases the frequency instability level which has been attained today is still five orders of magnitude away from the standard quantum limit. This difference means that the frequency instability levels which have been achieved so far are determined primarily by technological imperfections and that experimentalists potentially have a great deal of room for improving the stability. It is important to emphasize here that neither limitations (1) and (5), on the one hand, nor $(\Delta\omega/\omega)_{sql}$, on the other, is an absolute limit: It is possible in principle to achieve a frequency instability smaller than $(\Delta\omega/\omega)_{sql}$, but doing so will require the use of specially selected nonlinear resonators.[68]

4. MEASUREMENT OF LENGTHS, SMALL VARIATIONS IN LENGTHS, AND CAPACITANCES

The instruments being used today to measure a distance l between two macroscopic objects or a variation Δl in this distance are essentially parametric converters. Such instruments usually include a stable-frequency self-excited oscillator which generates electromagnetic oscillations (in the rf or optical range) with a low power fluctuation level. The value of l or Δl *is converted into a change* in one of three quantities: a frequency, a phase, or a power. It is the latter change which is recorded. There are two obvious conditions which determine the error $\Delta l/l$ (these conditions were illustrated for the particular example of a capacitive sensor in the preceding section of this review):

a) The initial frequency stability of the self-excited oscillator must be sufficiently high.

b) The energy $\mathscr{E} = W\tau$ which "passes through" the converter over the measurement time τ must be sufficiently large.

The first of these conditions [see (20)], $\Delta l/l \gtrsim (\Delta\omega/\omega)_{s\text{-}osc}$, is common to converters of all types if it is necessary to know l with a metrological error Δl or if it is necessary to know only Δl, but a balanced (bridge) arrangement cannot be used. For metrological measurements of l, it is necessary to use a secondary frequency standard which has been calibrated against a primary reproducible standard. For differential measurements of Δl, on the other hand, all that is necessary is a high *short-term* stability of the self-excited oscillator: Only that spectral component $(\Delta\omega/\omega)_{s\text{-}osc}$ which corresponds to the basic part of the spectrum of the quantity Δl should be substituted into the right side of condition (20).

With some simplifications, the second condition can be put in the form

$$\frac{\Delta l}{l} \gtrsim \frac{1}{A}\left(\frac{kT_a}{\mathscr{E}}\right)^{1/2} \quad \text{or} \quad \frac{\Delta l}{l} \gtrsim \frac{1}{A}\left(\frac{2\hbar\omega_e}{\mathscr{E}}\right)^{1/2}, \qquad (24)$$

where T_a is the noise temperature of the amplifier, and the dimensionless factor A depends on the particular type of parametric converter which is used. For a capacitive sensor we would have $A \approx Q_e$; for an rf rangemeter we would have $A \approx \omega_e \cdot \tau_l$, $\tau_l = l/c$; for a Fabry-Perot resonator we would have $A = \omega_e \tau_l (1-R)^{-1} = Q_{opt}$; and for a multiple-pass Michelson interferometer we would have $A = \omega_e \tau_l N$, where N is the number of passes of the light between the mirrors. If the self-excited oscillator is working in the optical range, we should use the second of relations (24) for estimates, under the condition that the self-excited oscillator gives the radiation flux in the quantum coherent state (there is no significant excess noise in either the self-excited oscillator or the receiver). If l is the gap between two plates in a capacitor, then conditions (20) and (24) can also be used to estimate the resolution of small variations $(\Delta\varepsilon/\varepsilon)$ in the permittivity constant of the material in the capacitor.

Table I shows five examples of the resolution which has been attained in precise measurements of l and in measurements of small values of Δl, $\Delta l/l$, and $\Delta\varepsilon/\varepsilon$ in five different measurement methods. In the first two, metrological measurements of l were carried out, while in the others differential measurements of small values of $\Delta l/l$ and $\Delta\varepsilon/\varepsilon$ were carried out. In example 3 we used a balanced arrangement in which condition (20) was significantly weakened because of the small relative difference between the "arms" in the optical parametric converters. This circumstance allowed Shoemaker et al.[73] to come close to the threshold for Δl which is determined by quantum shot noise see the relation on the right in (24)], albeit at relatively small values of $\mathscr{E}$.

As a general assessment of these examples of resolutions and accuracies which have been achieved, we might say that although either extremely small values of Δl or small values of $\Delta l/l$ and $\Delta\varepsilon/\varepsilon$ have been achieved in all cases there is an extremely wide margin of sensitivity remaining in both the absolute values of Δl and the values of $\Delta l/l$ and $\Delta\varepsilon/\varepsilon$. The significant reserves in terms of frequency stability of the self-excited oscillators which are already available have not been fully utilized (in examples 1, 2, 4, and 5). The largest values of the factor A which have already been attained have not always been used for the particular parametric converters which have been selected. In many cases the resolution is determined not by conditions (20) and (24) but by other factors, which might be partially or completely eliminated. We would expect that experiments will be carried out in the near future in which the values of Δl and $\Delta l/l$ will be several orders of magnitude smaller than those listed in Table I. More-detailed numerical estimates of expected values of Δl and $\Delta l/l$ in some of the experimental projects which are

TABLE I.

Example	l,cm	Δl,cm	$\Delta l/l, \Delta\varepsilon/\varepsilon$	Measurement conditions	Reference
1	3×10^{13}	$2\cdot10^{2}$	6×10^{-12}	Metrological measurements of l; radio rangemeter measurements with an active repeater station on a satellite	69
2	$3.8\cdot10^{10}$	10	$2.5\cdot10^{-10}$	Metrological measurements of Δl, laser location of the moon	79–72
3	4×10^{3};	10^{-15};	$2.5\cdot10^{-18}$;	Differential measurements of $\Delta l, \bar{f}\simeq10^{3}$ Hz, $\Delta f\simeq10^{3}$ Hz; optical Fabry–Perot resonator; Michelson interferometer	49–73
4	$3\cdot10^{-4}$	$6\cdot10^{-17}$	$2\cdot10^{-13}$	Differential measurements of $\Delta l, \bar{f}\simeq8$ kHz, $\Delta f\simeq1$ Hz; cryogenic capacitive sensor	74
5	$3\cdot10^{-5}$	$3\cdot10^{-15}$	$1\cdot10^{-10}$	Differential measurements of $\Delta\varepsilon, \bar{f}\simeq0.1$ Hz, $\Delta f\simeq0.1$ Hz; cryogenic capacitive sensor	75

being planned will be presented in the following section of this paper.

This critical evaluation, which only emphasizes the existence of a large reserve of sensitivity for future macroscopic experiments, is not intended to diminish the physical significance of the results which have been obtained in several experiments, among which we have selected some examples for Table I. We might note three of them.

1. In the experiments by Shapiro and Reasenberg[69] the error did not exceed 0.1% of the magnitude of the effect in measurements of the general-relativity delay of an electromagnetic pulse in the gravitational field of the sun (example 1 in Table I). The results of the measurements agree with the predictions of the general theory of relativity.

2. In experiments by two groups of experimentalists,[70,72] the equivalence principle was tested for the gravitational mass defect within about 3% (example 2 in Table I).

3. Experiments by Panov and Sobyanin[75] yielded the observation and detailed measurements of the magnitude of the shift of the λ-point of liquid helium caused by a decrease in the dimensions of the vessel holding the helium. This shift, which had been predicted qualitatively by the Ginzburg-Pitaevskiĭ theory[76] and by quasimicroscopic theoretical models of second-order phase transitions,[77] arises when the finite ratio of the correlation length of the order parameter to the smallest dimension of the vessel is taken into account (example 5 in Table I). We should point out that in this experiment it was not a small value of $\Delta l/l$ but a small relative change in the permittivity of the liquid helium which was measured.

We conclude this section of the review by pointing out two important circumstances.

a) The smallest value of Δl (example 4 in Table I) is close to the estimates which were made for mechanical systems in §2 for standard quantum limits. This circumstance seems to justify the optimistic predictions regarding the implementation of projects in which this limit is to be "crossed."

b) Condition (24) is valid only in a case in which the self-excited oscillator radiates a quantum coherent state, as we mentioned earlier. For squeezed quantum states, the requirements on the magnitude of $\mathscr{E}$ are weaker.[33] Accordingly, there is even more margin for increasing the sensitivity, but this margin will be utilized only after simple methods have been developed for both preparing and detecting such states.

5. INCREASING THE SENSITIVITY OF GRAVITATIONAL ANTENNAS

In the early 1960s, Weber pointed out that it would be possible in principle to construct ground-based or near-earth gravitational antennas which would be capable of detecting bursts of gravitational radiation from certain astrophysical phenomena. This problem has yet to be solved, although much has been done to improve old antennas and to develop new types of antennas. It is pertinent to note in this regard that the efforts of the experimentalists have essentially been directed toward the development of a new channel of astrophysical information. The existence of gravitational radiation as a physical phenomenon is not in doubt; its existence has been unambiguously confirmed by observations of the evolution of the orbital period of two close compact stars which were carried out by Taylor and his colleagues.[78] Fifteen laboratories in several countries are involved in the development and refinement of various types of antennas.

We recall that a gravitational wave causes a time-varying acceleration gradient: The amplitude (a_{grav}) of the difference between the accelerations of two objects separated by a distance l is

$$a_{grav} \approx \frac{1}{2}\,\omega_{grav}^{2}\,lh, \qquad (25)$$

where h is the amplitude of the metric variations. One can detect either the force $F = ma_{grav}$, which excites mechanical vibrations in an extended object (Weber's initial idea), or the change in the distance between two free masses separated by a distance l. In either case, if the duration of the gravitational burst is $\tau_{grav} \simeq 2\pi/\omega_{grav}$ and if the relation $\omega_M \simeq \omega_{grav}$ holds, the displacement amplitude Δl_{grav} which must be detected will be given in order of magnitude by

$$\Delta l_{grav} \approx \frac{1}{2} lh, \tag{26}$$

where l is of the order of the length of the object in the case of Weber's antenna.

Over the past decade, theoretical astrophysicists have developed a fairly long list of scenarios for sources of bursts of gravitational radiation. These scenarios predict the duration τ_{grav}, the shape of the burst, the value of h near the earth, and the frequency of events (see the collection in Ref. 79, the review in Ref. 80, and also Ref. 81). The value of h and the event appearance frequency, however, are predicted only approximately. With $\tau_{grav} \simeq 10^{-4}$–$10^{-2}$ s, for example, an optimistic prediction would lead us to expect a burst with an amplitude $h \simeq 2\cdot 10^{-19}$, and a pessimistic prediction $h \simeq 10^{-22}$, about once every 10^7 s.

In the early 1970s, when series of tests of Weber's first experiments were completed in several laboratories, the sensitivity of solid gravitational antennas for $\tau_{grav} \simeq 10^{-3}$ s was of the order of $h \simeq 10^{-15}$–10^{-16}. Over the more than 10 yr since then, increases in the quality factor Q_M and the suppression of thermal mechanical noise through a lowering of the temperature have reduced the observable amplitude to $h \simeq 5\cdot 10^{-18}$–$1\cdot 10^{-18}$ in various laboratories. There is a great deal of diversity in the types of cryogenic parametric sensors of small vibrations[82-88] which are used in these antennas (one of the sensors was mentioned in the preceding section of this paper; see example 4 in Table I). Much effort has been devoted to an optimum matching of the sensors with high-Q mechanical oscillators. It is not difficult to evaluate the significance of the sensitivity level which has been reached. Here we can make use of the standard quantum limit for the coordinate of an oscillator [see (1)] and relation (25). Given the shape of the gravitational pulse, one can then easily derive a standard quantum limit, h_{sql} for the observable amplitude of the metric variation. If the pulse has a shape approximately that of 1 period of a sine wave, we find, after some straightforward calculations,

$$h_{sql} \approx \left(\frac{4\hbar}{\pi v^2 \tau_{grav} m_{eff}}\right)^{1/2}$$

$$\approx 2\cdot 10^{-21}\cdot \frac{5\cdot 10^5\ \text{cm/s}}{v}\left(\frac{10^{-3}\ \text{s}}{\tau_{grav}}\right)^{1/2}\left(\frac{2\cdot 10^6\ \text{g}}{m_{eff}}\right)^{1/2}, \tag{27}$$

where v is the sound velocity, and m_{eff} is of the order of a third of the total mass of the antenna. A comparison of this estimate with the estimates above shows that in antennas of the Weber type there is a sensitivity margin of about three orders of magnitude before we reach the standard quantum limit. One can hope that the use of mechanical resonators with the maximum quality factor Q_M which has already been achieved, a lowering of the regulated temperature below 2 K, and improvements in the sensitivity of parametric converters (a reduction of their inverse fluctuation effect) will make it possible to reach $h \simeq h_{sql}$ at these antennas in the near future.

Serious competition for solid cryogenic antennas of the Weber type is posed by free-mass antennas with a laser system for detecting small vibrations.[49,73,89] Their sensitivity today, $h \simeq 2\cdot 10^{-17}$ in order of magnitude (see example 3 in Table I), is slightly poorer than that of a Weber antenna. However, the laser gravitational antennas which exist today constitute only small prototypes of future large antennas, some of which are already in the construction process. For example, it is intended to increase l from $4\cdot 10^3$ cm (in the working prototype) to $4\cdot 10^5$ cm in the LIGO plan (Laser Interferometer Gravitational Observatory). At essentially the same sensitivity level of an optical parametric converter using a Fabry-Perot resonator, the sensitivity in units of the metric variation should be $h \simeq 10^{-20}$ by virtue of the increase in l alone. It is expected that this sensitivity will be achieved in 1990. The standard quantum limit for large free-mass antennas is considerably lower than that for Weber antennas, with their relatively modest dimensions:

$$h_{sql} \approx \frac{1}{l}\left(\frac{2\hbar_{grav}}{m}\right)^{1/2}$$

$$\approx 3\cdot 10^{-23}\cdot \frac{4\cdot 10^5\ \text{cm}}{l}\left(\frac{10^{-3}\ \text{s}}{\tau_{grav}}\right)^{-1/2}\left(\frac{10^4\ \text{g}}{m}\right)^{1/2} \tag{28}$$

Free-mass laser antennas have two advantages. First, in this case it is a relatively simple matter to achieve parameter values at which condition (7)—the necessary condition for reaching values close to h_{sql}—is satisfied, without lowering the temperature. Second, in such antennas it is a simple matter to use two mutually perpendicular arms (two pairs of masses), create a bridge arrangement, and thereby substantially relax the requirements on the frequency stability of the pump laser [see example 3 in Table I and condition (20)].

The possibility of putting a laser optical interferometer in an orbit rather distant from the earth and thereby increasing l to 10^{11} cm has recently been discussed (the LAGOS plan).[90] The authors of this plan suggest that it would be possible to achieve a sensitivity ranging from $h \simeq 3\cdot 10^{-20}$ for $\tau_{grav} \simeq 10$ s to $h \simeq 10^{-21}$ for $\tau_{grav} \simeq 10^4$ s. We note that achieving such a sensitivity will be possible only if the designers succeed in compensating for the nongravitational accelerations of the satellites which are caused by the solar wind and solar radiation, from the usual level of $5\cdot 10^{-8}$ cm/s to $5\cdot 10^{-17}$ cm/s^2. This is not a simple task. In particular, in order to solve this problem it will be necessary either to cancel or to correct for, within 10^{-5}, the radiation pressure exerted by the on-board laser on the mirrors of the interferometer, even for a laser power of the order of 1 mW. An indisputable advantage of this plan is the switch to longer bursts of gravitational radiation, for which h should be higher.

In addition to the improvements in the ground-based Weber and laser antennas, much has been accomplished toward the realization of a space version of the antenna, in which a pair of masses (a satellite and the earth) is used, and the response to a perturbation of the metric is detected not in a shift of Δl but in a Doppler frequency shift of a microwave electromagnetic signal relayed from the satellite back to the earth. This idea was proposed[91] back in 1967, but its practical realization began only relatively recently on Voyager satellites.[92,93] If the distance from the earth to the satellite is $l \gtrsim c\tau_{grav}$, a perturbation of the metric with an amplitude h should cause a variation of the order of

$$\frac{\Delta\omega}{\omega} \approx h. \tag{29}$$

in the Doppler frequency shift. The program of the Voyager satellites did not include back-and-forth multifrequency communications, which might have been of significant help in eliminating those perturbations on the earth-satellite path which were caused by the interplanetary plasma and also by the earth's ionosphere and troposphere. Accordingly, this gravitational antenna was able to do no more than establish an upper limit $h \lesssim 1\cdot 10^{-13}$ for relatively long bursts of gravitational radiation ($\tau_{grav} \approx 10^3$–10^4 s). This limit is at least two orders of magnitude above the frequency instability which has already been achieved in ground-based self-excited oscillators (see §3). Accordingly, there is reason to hope that the launch of the two satellites Galileo and Ulisis,[92] planned for the near future, with an improved system for communicating with the earth, will make it possible to realize the simultaneous operation of two such antennas in a coincidence arrangement with a sensitivity $h \lesssim 1\cdot 10^{-15}$ for bursts with $\tau_{grav} \approx 10^3$–10^4 s.

Sazhin[94] has suggested using the modulation of the emission of pulsars as a stable frequency source. Bursts of gravitational radiation should vary the period of the electromagnetic pulses detected by a ground-based antenna by a relative amount $\approx h$ (see also Ref. 95). This idea has been implemented by Taylor and his colleagues[65] in long-term observations of the millisecond pulsar PSR 1937 + 21. The stability of its frequency (after a monotonic drift is subtracted) is extremely high (Fig. 4)—close to the stability of primary cesium standards. It has thus proved possible to establish an upper limit $h \lesssim 1\cdot 10^{-13}$ for very long bursts of gravitational radiation, with $\tau_{grav} \approx 3\cdot 10^7$ s. This estimate is important if it is assumed that bursts with this amplitude are stochastically in a steady state and are caused by a background (relic) gravitational radiation. These bursts correspond to a mass density $\rho \approx 2\cdot 10^{-35}$ g/cm^3 (which is six orders of magnitude below the critical value). These experiments, like the measurements taken on the Voyager satellites, constitute "working" with a single gravitational antenna, so they can provide only an estimate of an upper limit on the spectral components of h with long periods. If two frequency-stable pulsars separated by a sufficiently small angle are ever observed, it would be possible to pursue this principle and develop a coincidence arrangement (two antennas). Only in this case could a positive result of observations (a statistically significant coincidence of two responses) serve as proof of the observation of long-wave bursts.

We will not go into detail here on certain other new versions of gravitational antennas which have been proposed; the interested reader might look in the original papers.[96,97] In concluding this section of the review we would like to emphasize a circumstance which is important for all types of free-mass gravitational antennas.

Zel'dovich and Polnarev[98] have pointed out that bursts of gravitational radiation might possibly exhibit a memory effect: A shift $\Delta l_{grav} \approx hl/2$ *would be conserved* for an arbitrarily long time after the passage of a gravitational burst. This effect is important because the value would be $\Delta l_{grav} \approx hl/2$ even in the case $l \gg c\tau_{grav}$ (Ref. 99). A second distinctive feature involved here is that the Doppler response (frequency shift) would last l/c at $l \gg c\tau_{grav}$ (Ref. 100). These two features might have the consequence that bursts "with a memory" would be the first to be observed.

6. SEARCHES FOR VIOLATIONS OF THE EQUIVALENCE PRINCIPLE AND FOR OTHER NEW PHENOMENA

We do not have space here to go into a detailed discussion of all the progress in the last few years in the field of macroscopic experiments in which small forces, small accelerations, displacements, etc., have been measured. Many of the experiments have taken elegant approaches, which the reader can learn about in the literature cited below.

Here we will simply give a brief list of what we regard as the most interesting directions and the results of several programs of research. We will also mention some new programs which have recently been proposed.

1) Numerous and diverse tests of the predictions of the general theory of relativity have failed to reveal any deviation from the predictions of the theory, within the measurement errors. As mentioned above, the best resolution which has been achieved has been achieved in measurements of the retardation effect. The experiments have been summarized in detail and compared with theory in a book[101] and a subsequent review[102] by Will. It should also be noted that it has been found possible to test effects only at small values of $\Delta\varphi/c^2$ ($\Delta\varphi$ is the gravitational potential), so alternative relativistic theories of gravitation which predict effects at small values of $\Delta\varphi/c$ which are the same as the predictions of the general theory of relativity are still alive. Apparently the only experiments which will qualify as critical experiments for disproving incorrect theories will be experiments in which the condition $1-(\Delta\varphi/c^2) \ll 1$ holds.

In the solar system, which has served for a long time as our testing ground for testing effects of the general theory of relativity, this condition cannot be satisfied. Apparently the only remaining potential possibility for "performing" experiments under the condition $1-(\Delta\varphi/c^2) \ll 1$ would be a detailed study of the amplitude and shape of bursts of gravitational radiation generated during the production of black holes.

2) Attempts to observe a time dependence of the gravitational constant G which may prevail (in accordance with Dirac's suggestion[103]) have yielded contradictory results from different groups of experimentalists: both the absence and the observation of an effect at a level $\dot{G}/G \lesssim 1\cdot 10^{-10}$ yr^{-1} (see Ref. 104 and the bibliography there).

3) Certain modern theories allow the existence of new types of light scalar or vector bosons. One manifestation of their existence would be a so-called "fifth force." Such a force can be detected if one can observe a dependence of the gravitational constant on distance in experiments like the Cavendish experiment or if one can observe a violation of the equivalence principle in experiments such as the Eötvös experiment. So far, there are no reliable data which verify the relation $\partial G/\partial R = 0$ or a violation of the equivalence principle (see Refs. 105–107 and the bibliographies there). The only exception would consist of indirect geophysical observations.[108] Two arrangements for differential experiments using ballistic gravimeters[109] and with an arrangement for measuring the acceleration due to gravity at various depths in Lake Baikal[110] have recently been proposed. These experiments could provide a positive answer to the hopes (which stand on rather shaky ground[111]) for such an effect. More probably, they could push back considerably the boundary for the possible existence of such an effect.

4) A "violation" of the equivalence principle can be

observed in the experiment proposed by Shvartsman.[112] The reason for the "violation" should be the existence of a cloud of thermalized background (relic) neutrinos near the earth. If a dumbbell consisting of two masses, one homogeneous and the other consisting of grains with dimensions of the order of the de Broglie wavelength of the neutrino, were placed in earth orbit, an observer should detect a difference of the order of 10^{-21} cm/s^2 in the accelerations of these masses. The difference would arise because of a difference in the elastic scattering of neutrinos in the granular and homogeneous masses. Unfortunately, one should note that although the estimate above is considerably larger than the standard quantum limit for an average taken over a long time, the state of the art in on-board satellite experiments is still far from such a resolution level.

5) The experiment program proposed by Schiff, which is usually called the "relativistic gyroscope" and which has been described in detail in several papers (e.g., Ref. 113), is near completion.[114] In the final experiment, the precession of a gyroscope due to a spin-spin gravitational interaction between the gyroscope and the earth, amounting to $5\cdot 10^{-2}$ arc second per year, is to be measured. The preparations for this experiment and preliminary measurements by the team of experimentalists have taken more than 20 yr.

While this work was being carried out, many elegant approaches and measurement methods were developed, and these other approaches and methods are already being used in other fields.

6) Among the new programs which might possibly see life over the next decade we should mention the POINTS program.[115] This program calls for the use of two rigidly coupled interferometers in earth orbit. These interferometers are to make it possible to resolve the angular displacement of one star from another with a resolution of $5\cdot 10^{-12}$ rad. A sensitivity at this level would be sufficient for measuring relativistic gravitational effects at the level of $\Delta\varphi^2/c^4$ and also for observing planetary systems around the stars closest to the sun.

7. CONCLUSION

The basic achievements of recent years toward improving sensitivity in macroscopic experiments and also the most important unresolved problems, in both the theory and the experimental methods, which have been described in this review reflect the point of view of the author and are therefore not exhaustive. We might add that we believe that the approach which has been taken in this review—comparing the resolution level which has been achieved with standard quantum limits—is a natural and convenient one. It would be difficult to predict with any certainty just how soon the standard quantum limits will be reached and surpassed in low-frequency mechanical and electromagnetic experiments with $kT \gg \hbar\omega$. One might hope that this will happen in the next few years. An argument in favor of this optimistic prediction comes from the quantum nondemolition measurements in the optical range which have just recently been carried out.[116,117] The elegant approaches which were taken in those experiments led to the demonstration that it is possible to perform a nondemolition measurement on one of the quadrature components of a wave in an optical fiber and to suppress fluctuations of a laser below the level of Poisson fluctuations. Although these experiments have been of the nature of *demonstration* experiments and have required some fairly complicated apparatus, one can hope that simpler approaches will be proposed and realized and that as a result quantum nondemolition measurements of quadrature components or energies will become ordinary laboratory tools. We can then expect to see the realization of photon-counting methods without absorption and, finally, measurements of the energy (again, without absorption) of individual photons with an error much less than the energy of a quantum. If such a program is implemented in optics comparatively rapidly, the experience acquired in "working" with relatively heavy quanta will then make it possible to accelerate the attainment of the standard quantum limits for relatively low-frequency electromagnetic and mechanical systems.

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[1)] Approximate derivations of expression (1), and also of corresponding expressions (5) and (9), are given in reviews[1-3] and in the earlier publications cited in those reviews. We should point out that in the Heisenberg picture of an oscillator the coordinate operator does not commute with itself in time:

$$[\hat{x}(t),\ \hat{x}(t+\tau)] = \frac{i\hbar}{m\omega_M}\sin\omega_M\tau. \qquad (2)$$

Equation (2) can be derived easily on the basis of the following simple calculations. The coordinate operator of the oscillator varies in time in accordance with the classical law

$$\hat{x}(t) = \hat{x}(0)\cos\omega_M t + \frac{\hat{p}(0)}{m\omega_M}\sin\omega_M t.$$

We can thus write

$$[\hat{x}(t),\ \hat{x}(t+\tau)] = [\hat{x}(0),\ \hat{x}(0)]\cos\omega_M t\cdot\cos\omega_M(t+\tau) + \frac{[\hat{x}(0),\ \hat{p}(0)]}{m\omega_M}\cos\omega_M t\cdot\sin\omega_M t + \frac{[\hat{p}(0),\ \hat{x}(0)]}{m\omega_M}\sin\omega_M t\cdot\cos\omega_M(t+\tau) + \frac{[\hat{p}(0),\ \hat{p}(0)]}{(m\omega_M)^2}\sin\omega_M t\cdot\sin\omega_M(t+\tau).$$

Using $[\hat{x}(0),\hat{x}(0)] = [\hat{p}(0),\hat{p}(0)] = 0$ and the equation $[\hat{x}(0),\hat{p}(0)] = -[\hat{p}(0),\hat{x}(0)] = -i\hbar$, we find (2)

A rigorous transformation from (2) to (1) was given in Ref. 17, but that derivation was carried out under the assumption $\Delta\omega_M \gg \omega_M$. An exact solution has not yet been derived for an instrument with an arbitrary bandwidth $\Delta\omega_M$.

[2)] This result, as in the case of an oscillator, is a consequence of the circumstance that the Heisenberg coordinate operator for a free particle does not commute with itself in time:

$$[\hat{x}(t),\ \hat{x}(t+\tau)] = \frac{i\hbar\tau}{m}.$$

[3)] It is pertinent here to mention a quantum "watchdog" effect which has been pointed out by Zurek.[11] This effect can be summarized as follows: An instrument which is measuring an energy with a high accuracy will cause a pronounced perturbation of the phase of an oscillator. As a result, the oscillator will respond progressively more weakly to $(F\cdot\tau_*)$ with a decrease in the time over which the optimum QNM of the energy is averaged. However, as was pointed out by Khalili,[14] Eq. (10) remains valid, and this effect can be ignored if the measurement time satisfies $\tau \gg \tau_*$.

[4)] The simple limiting condition for the energy measurement error Δn in method (c) can be written $\Delta n \leq Q^{-1}(n^*)^{1/2}$ if the nonlinearity of the resonator mode is written[24]

$$\omega_n = \omega_0\left(1+\frac{n}{n^*}\right). \qquad (14')$$

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