LIGO-T880012

#06

List of 3/1/89

Wind-induced Motion of Unprotected LIGO Vacuum Pipes

§1 Summary

I sketch a method for calculating the amount of motion of the walls of LIGO beam pipes exposed to fluctuating forces from the wind. The model of the force spectral density comes from an article in the Shock and Vibration Handbook by Davenport and Novak. I use a simple model for the response of the pipe, treating a section between supports as a beam with pinned boundary conditions at both ends. For representative parameters, I calculate the amount of wall motion as a function of the average wind speed.

This calculation is of interest because motion of the pipe walls could modulate the light scattering in the interferometer, causing noise. (The transfer function from wall motion to interferometer noise is being calculated by others.)

§2 Model of Force Power Spectrum

The calculation follows the Shock and Vibration Handbook's chapter 29, part II, "Vibration of Structures Induced by Wind" by A.G. Davenport and M. Novak. I consider two mechanisms by which the wind applies a fluctuating force. First, the direct buffeting of the pipe due to turbulent fluctuations in the wind speed. Second, I will consider noise forces due to the shedding of vortices in the wake of the pipe.

I begin with Davenport and Novak's analytic approximation to the power spectrum of the wind speed. This can be written as

$$v^2(f) = 1.6 \times 10^{-5} v_0^{8/3} f^{-5/3} \tag{1}$$

where v_0 is the steady wind speed. (All quantities are in cgs units.) They also give an expression for another important quantity, the coherence length

$$z = v_0/10f. \tag{2}$$

We next need to be able to calculate the force due to the wind. The mean force F_0 due to a steady wind of speed v_0 is given by

$$F_0 = \frac{1}{2}\rho C_D A v_0^2 = 6.0 \times 10^{-4} A v_0^2. \tag{3}$$

Here ρ is the density of air and C_D is the drag coefficient, about equal to unity. A represents the area over which the force acts. The power spectrum of the force is related to the power spectrum of the wind speed by

$$F^2(f) = 4(F_0/v_0)^2 v^2(f). \tag{4}$$

The force is proportional to the area only for areas smaller than the square of the coherence length. I want to consider what happens to a segment of 48 inch pipe 20 meters long. Such a large structure contains many incoherent patches. To find the rms generalized force, I replace A by $A^{1/2}z$. After performing some algebra, we can write an expression for the power spectrum of the wind force as

$$F^2(f) = 4.7 \times 10^{-8} v_0^{20/3} f^{-11/3}. \tag{5}$$

A graph of this spectrum for a wind speed of 3 mph is shown in Figure 1.

Another mechanism, also discussed in the same article, by which wind may apply a force to the vacuum pipes is the shedding of vortices in the wake on the leeward side of the pipe. (These vortices make up the famous 'Karman vortex street'.) The nature of the flow in the wake depends on the Reynolds number, given by

$$R = v_0 D/\nu \tag{6}$$

where the kinematic viscosity of air is $\nu = 1.5 \times 10^{-1}$ cm ^{2}/ sec. For wind speeds below 8 miles per hour (R below 3×10^5), the driving force is nearly harmonic, with a frequency given by

$$f = 0.2 v_0/D. \tag{7}$$

The power spectrum is actually best modelled as a Gaussian centered at this frequency with fractional bandwidth of about 0.1. This means that at frequencies large compared with the center frequency, the power is very small. Thus the mechanism of turbulent velocity fluctuations, evaluated in the preceding paragraphs, dominates the noise spectrum at low wind speeds. At 8 mph, the driving frequency for 48 inch pipes is around 0.5 Hz. The lowest resonant frequency for the pipe section considered here is about 10 Hz.

At higher wind speeds, the nature of the spectrum changes profoundly. When the Reynolds number exceeds 3×10^5, the flow is in the so-called supercritical range. The power spectrum is then a power law with index -2, compared with -3.7 for the turbulent pressure fluctuation model evaluated in the first part of this section. The spectrum is given by

$$F^2(f) = 2.7 \times 10^{-2} v_0^5/f^2. \tag{8}$$

At all frequencies in our signal range, the noise will be dominated by this vortex mechanism when the wind speed is between 8 mph and 80 mph. Finally, above 80 mph (R greater than 3×10^6), the spectrum returns to a Gaussian, and the noise drops at high frequencies. Figure 2 shows the force power spectrum from the mechanism we have been discussing in this paragraph.

§3 Response of the Pipe to the Wind Force

Before calculating the noise spectrum of pipe motion due to fluctuating forces, it is worthwhile to consider the DC deflection due to the steady wind. The deflection at the middle of a simply supported beam loaded uniformly by a force per unit length w_a is given by

$$x_0 = 5w_a l^4/384EI \tag{9}$$

where l, E, and I are the length, Young's modulus, and moment of inertia of the beam, respectively. The force per unit length comes from equation (3), where we can use the actual area of the pipe, lD, with no worries about the coherence length since we are considering a DC problem. Then we find

$$x_0 = (5\rho l^4/96\pi ED^2 t)v_0^2 = 1.7 \times 10^{-8} v_0^2 \tag{10}$$

where I have computed the moment of inertia for $t = 1/4$ inch$= 0.64$ cm, with no stiffening rings. The motion is small. It takes winds of 100 mph ($= 4.5 \times 10^3$ cm/sec) to move the pipe .35 cm.

To compute the response of the pipe to the fluctuating force, we need to consider its normal modes of vibration. The motion of any part of the pipe is the superposition of the excitation of each of the normal modes, weighted by the modal amplitudes at that point. Each mode can be treated as an independent harmonic oscillator, excited by a force with the same power spectrum as we calculated in the previous paragraph for the lowest mode. The motion of a typical point on the wall is

$$x^2(f)/F^2(f) = \frac{1}{m^2} \sum_i \frac{a_i^2}{(\omega_i^2 - \omega^2)^2 + (\omega_i \omega/Q)^2}. \tag{11}$$

The amplitude of the motion varies from point to point on the pipe, but except at the nodes of an individual mode the dependence is not strong. (Another way of saying this is that a sine wave has at almost all points a value of order unity.) The spectrum won't be grossly in error if we treat the amplitude of each mode as unity everywhere. I chose to evaluate this expression for the normal modes of a beam with pinned boundary conditions. (This choice may be a good model, and it also has the virtue that the modal frequencies are easy to calculate, just the square of the mode number times the lowest frequency, about 10 Hz in this case.) I ignored the modes related to distortions of the pipe cross-section, which also fall in the same frequency range. (The lowest of these modes has a frequency of about 11 Hz.) Including them would add more peaks and increase the overall level by roughly a factor of two. A graph of the transfer function is shown in Figure 3.

The predicted motion of the pipe is given by the product of the force power spectrum with the transfer function. A graph is shown in Figure 4 for a wind speed of 3 mph, and Figure 5 for 10 mph.

The motion predicted by this model is surprisingly small. The spectrum for 3 mph is comparable to background seismic motion at a quiet site. The motion at 10 mph is a few orders of magnitude higher. An explanation of the small size of the response is perhaps to be found in the fact that a pipe of the dimensions we have been studying is actually quite stiff. The impedance of a beam in flexure, according to White and Walker's *Noise and Vibration*, is given by

$$Z = (1+i)A\rho(\frac{EI}{A\rho})^{1/4}\sqrt{\omega}. \tag{12}$$

The impedance of our pipe has a magnitude of about 3×10^8 dyn-sec/cm at 100 Hz. This is comparable to or larger than the impedance of a patch of ground 40 cm in diameter, according to Gutowski *et al.* (Noise Control Engineering, vol. 10, no. 3, p. 94, 1978). Since it is well known that wind can raise the seismic spectrum substantially above background levels, a pipe as stiff as the LIGO beam pipe should not be expected to move wildly. The large increase in motion at wind speeds of 10 mph and greater comes from the especially large forces due to vortex shedding, a mechanism not involved in coupling wind to the ground.

§4 Discussion

The next step in a calculation of how this motion would affect the noise spectrum of an interferometer depends on the evaluation of the amount of scattering. That is not finished yet. But it seems that baffles could be designed so that motion of the walls makes a negligible contribution to the scattering noise. Only the baffle motion would enter. If the baffles can all be placed at strong anchors for the pipe then the wind noise calculated here will not have any impact on the scattering. Wind is still a factor to the extent that the ground vibration spectrum is greater on windy days, and the baffle motion should be tied to the vibration of the ground. But if the baffling can be carried out in the way envisioned here, with due consideration given to all sources of DC misalignments, then wind-driven noise coupled through scattered light is not in itself a sufficient reason to enclose the vacuum pipe. (A way that this argument could fail is if scattered light which is diffusely reflected off the baffle faces and pipe walls is able to make a significant contribution to the interferometer signal.)

Note added 2 Feb 1989: Recent work by Thorne and by Weiss has in fact shown that the dominant processes by which scattered light causes noise in an interferometer do involve the interaction of light with the walls. Therefore, the calculation of pipe motion is more relevant than the argument given in the previous paragraph suggests.

In case the pipe motion does become relevant, I should point out what I think are the aspects of the calculation most in need of checking. The most important thing to verify is that the wind force models that I have used are realistic. The article I used as a source cites only obscure references, and is lax about indicating things such as the frequency range over which the models are valid. An assumption which would benefit from further investigation is that anchors can easily be made stiff compared with the pipe. If it should turn out that the anchors dominate the compliance, then the pipe motion would be larger than we have predicted here. An experimental check would be the most direct. It is rather easy to imagine how to set up the experiment, but it would require some time and effort to carry it out well. I have also been rather rough-and-ready with the modal model of the pipe. This can be done more carefully when engineering details such as stiffeners, supports, and bellows are well specified. I took $Q = 100$ for the modes, which is probably close. The numbers will certainly change some from those I have used, but probably not by enough to worry about now.

Peter R. Saulson

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FIGURE 1: FORCE POWER SPECTRUM, 3 MPH

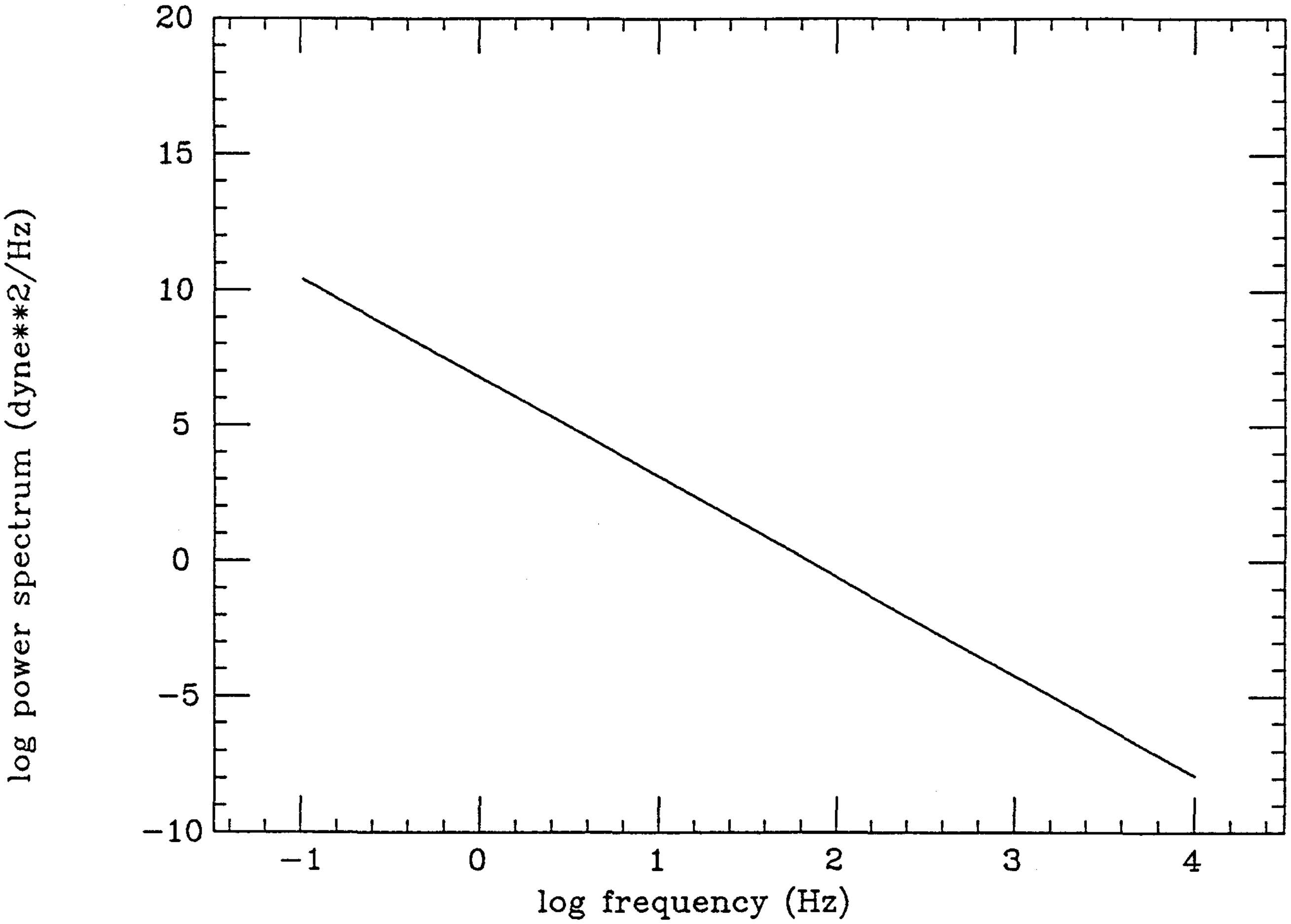

FIGURE 2: FORCE POWER SPECTRUM, 10 MPH

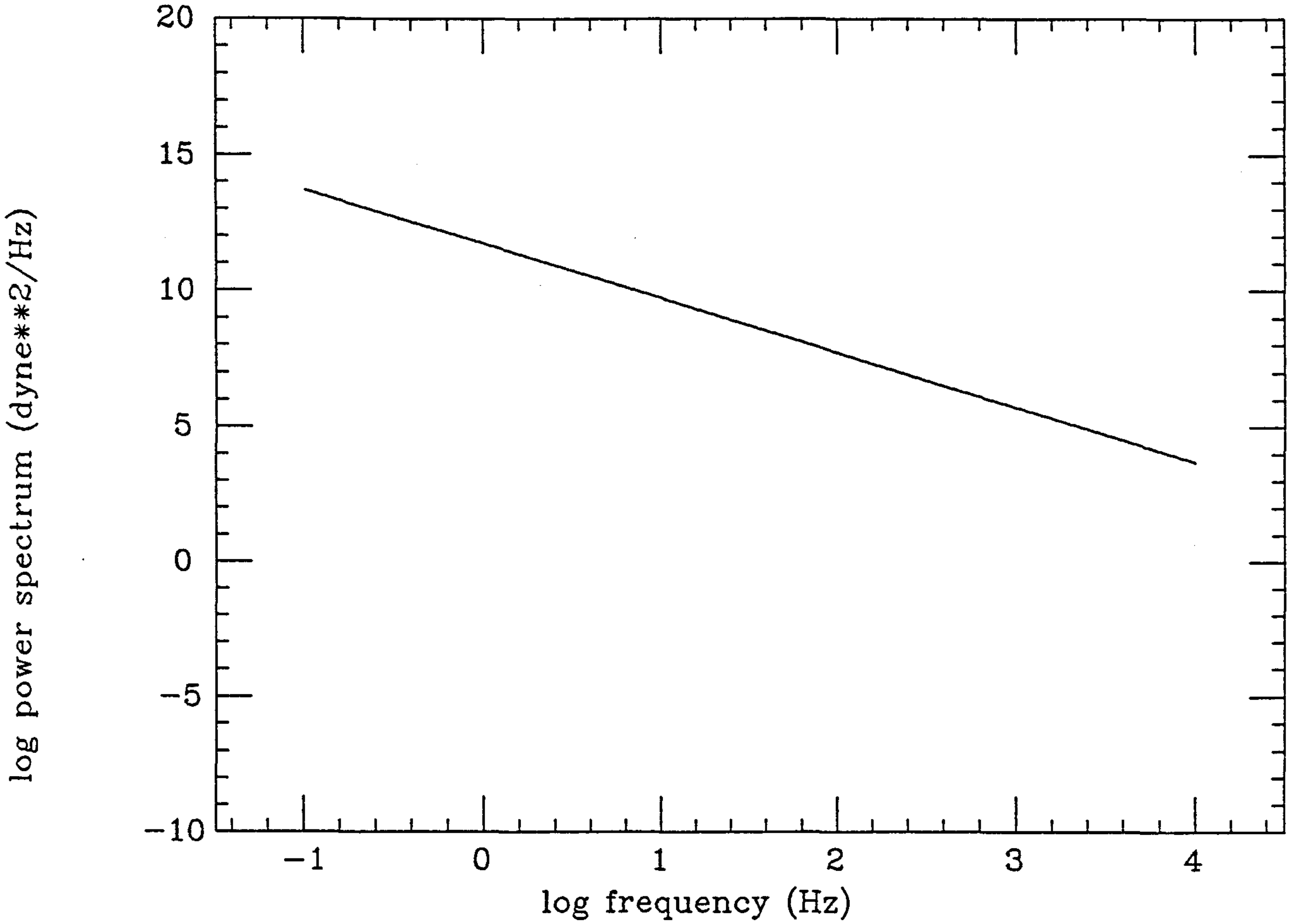

FIGURE 3: PIPE TRANSFER FUNCTION

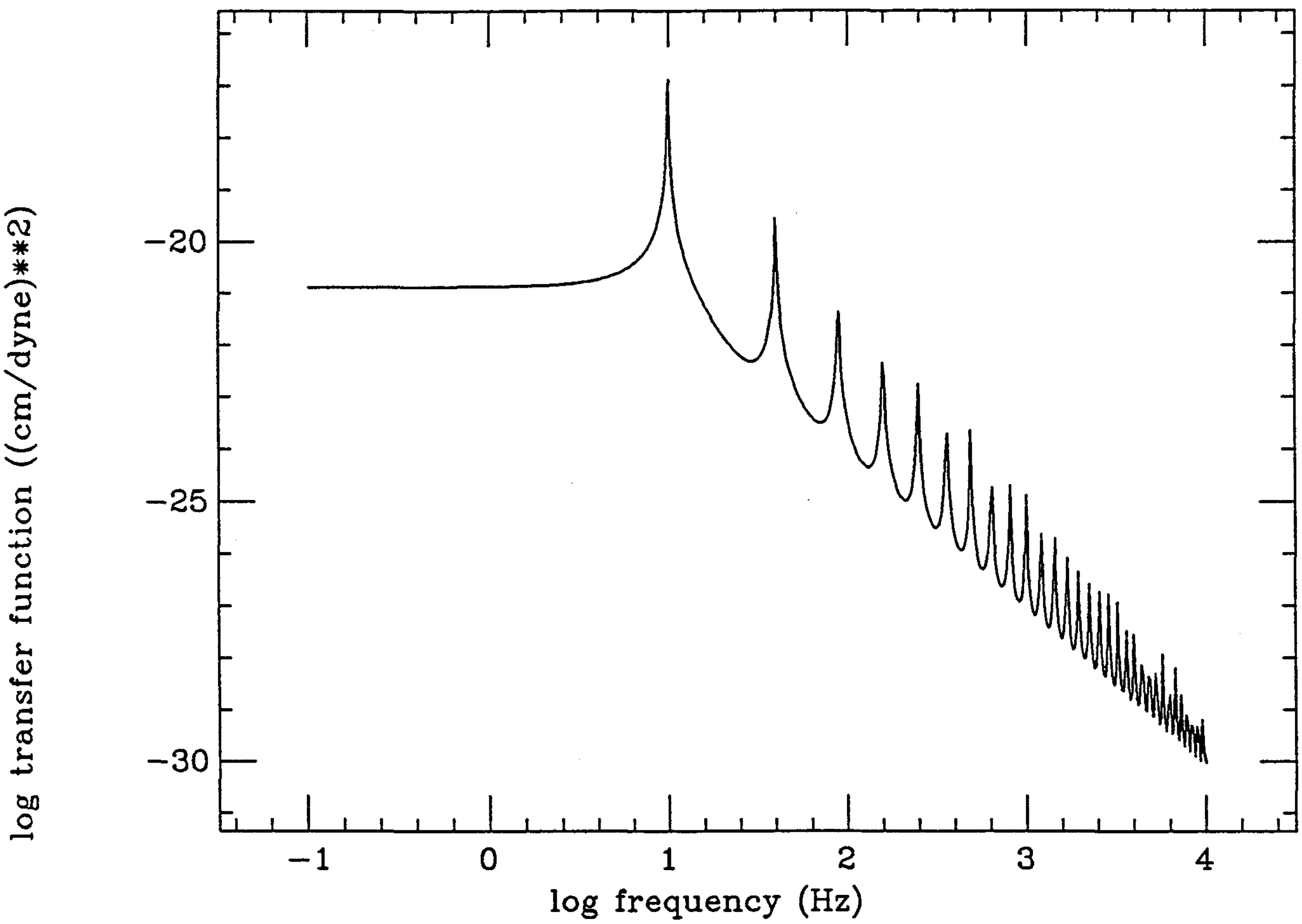

FIGURE 4: PIPE MOTION, 3 MPH

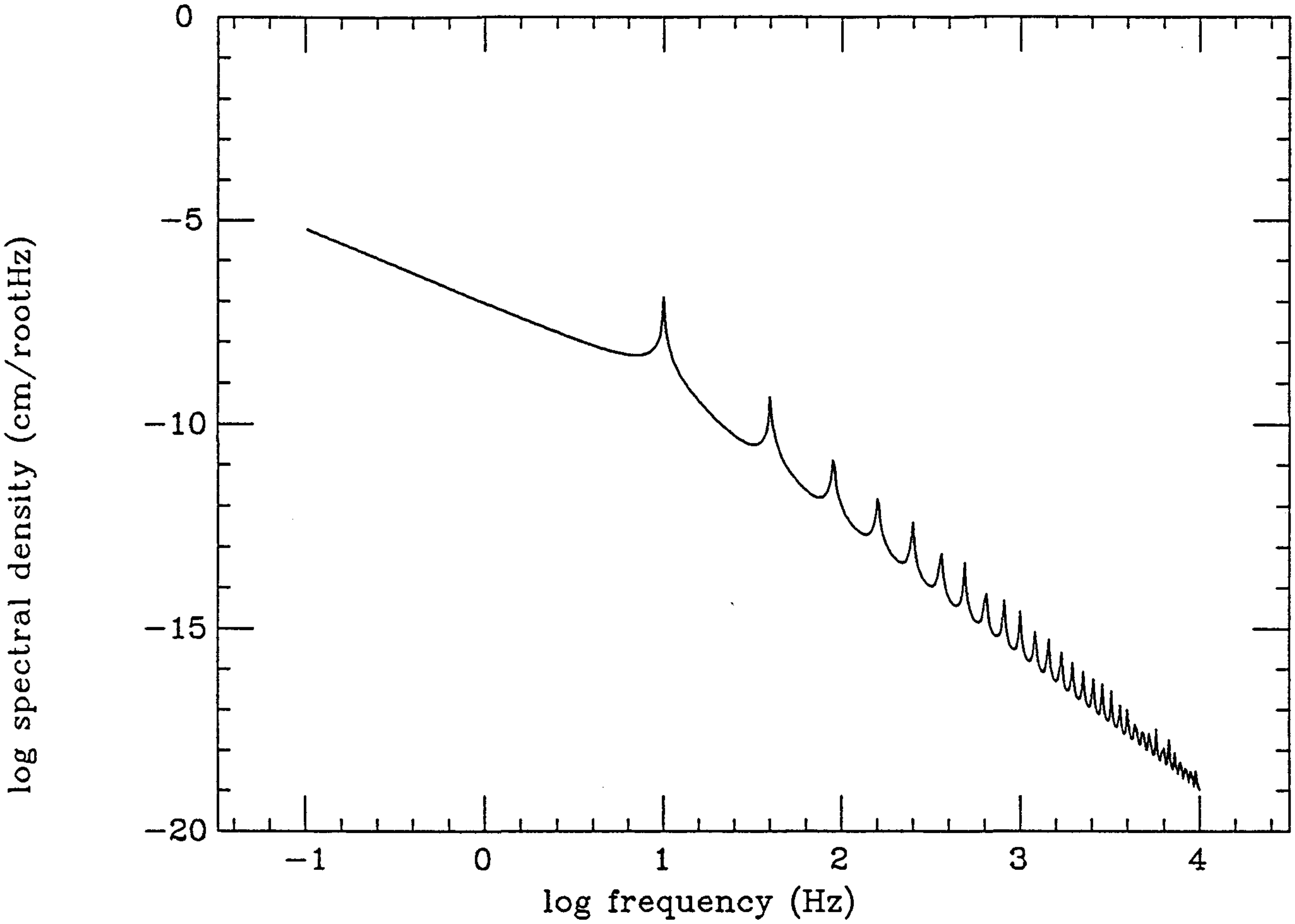

FIGURE 5: PIPE MOTION, 10 MPH