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Technical Note **LIGO-T080137-00-I**

Signal Read-Out for Lock Acquisition in AdvLIGO

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This is an internal working
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1 Signal Read-Out for Lock Acquisition

The design of signal read-out is typically done in order to produce signals shot noise limited, that is signals in which the read-out noise (amplifier noise, Johnson noise, etc..) is well below the shot noise. However, this constraint is not really necessary during lock acquisition, where we are not concerned about sensitivity issues. Since the lock acquisition scheme currently designed for AdvLIGO [1] involves signals at quite high frequency (around 140 MHz), it is useful to know the tolerance that we have on the read-out noise in order to possibly simplify the read-out design for lock acquisition. The analysis which follows is done in this way:

- for each photo-detector, the maximum shot noise level $\widetilde{P}_{shot}$ in $W/\sqrt{Hz}$ is computed;
- by assuming the photo-detector to be shot noise limited, and knowing the transfer function from shot noise to actuators, the resulting correction signal shot noise limited is calibrated in $N/\sqrt{Hz}$. As example, figure shows the transfer function from shot noise to actuators for the DARM loop, together with its open loop and closed loop transfer functions and the filter shape;
- the RMS of the correction signal is directly compared with the relative actuator limit: the only requirement that we impose is that the RMS of the correction signals sent to each mirror is at least a factor 10 lower than the mirror actuator limit, in order to be safely far from saturations;
- if the RMS of the correction signal shot noise limited is already only a factor ten lower than the actuation limit, the read-out noise must be lower than the shot noise, otherwise we can afford to have higher read-out noise, as long as the RMS of the correction signal is within the requirement (the read-out noise has the same transfer function to actuators as the shot noise);
- possible read-out topologies currently under investigations for AdvLIGO [2] have been analyzed to see if they would meet the requirement also for lock acquisition.

With a factor 10 margin required with respect to the full range of the actuators, the maximum correction signal (RMS) acceptable on each mirror is summarized in table 1:

Mirror	**Max correction signal (RMS)**
ETMX, ETMY	20 μN
PRM, SRM	1 mN
BS	60 mN

Table 1: Summary of the maximum correction signal (RMS) allowed for each mirror.

The control filters used for the analysis which follows are the ones shown in figure 2. A maximum incident DC power of 10 mW on each diode and a diode efficiency $\eta = 1$, will be considered.

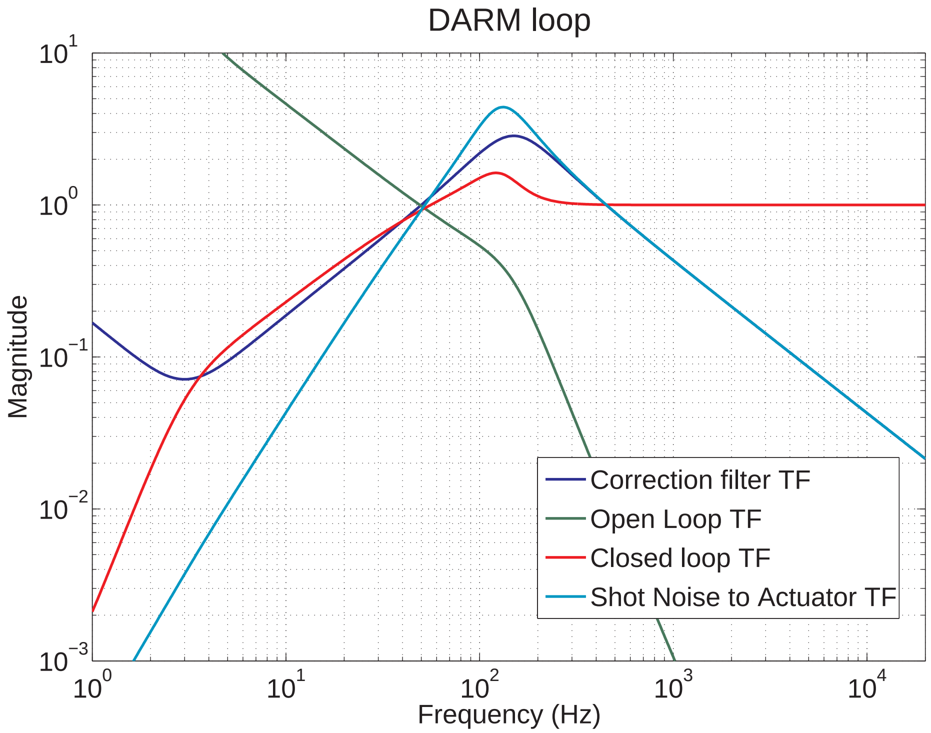

Figure 1: Some transfer function for the DARM loop: filter shape, open loop (unity gain is set around 50 Hz) and closed loop transfer function, and the shot-noise to actuators transfer function, given by the product between the controller filter TF and the closed loop TF (arbitrary unit in the plot).

1.1 Read-out noise requirements

1.1.1 RF Photodiodes at the reflection port: REFL31 and REFL32

Two modulation frequencies are designed for AdvLIGO:

$$f_1 = 9,399,566 \text{ Hz} \tag{1.1}$$

$$f_2 = 46,997,832 \text{ Hz} \tag{1.2}$$

At the reflection port signals are extracted both at $3 \times f_1$ (28 MHz, REFL31) and at $3 \times f_2$ (141 MHz, REFL32). At the beginning of the locking sequence, all the input power is reflected back from the IFO, so that a 99% attenuator is needed on this path in order to have 10 mW on each diode. This condition corresponds to the worse case in terms of shot noise. The power impinging upon the diode is mostly carrier power, which contributes only to increase the shot noise, but not to making signal (produced by the beating between the sideband). The maximum shot noise level, both in power and in current, is therefore:

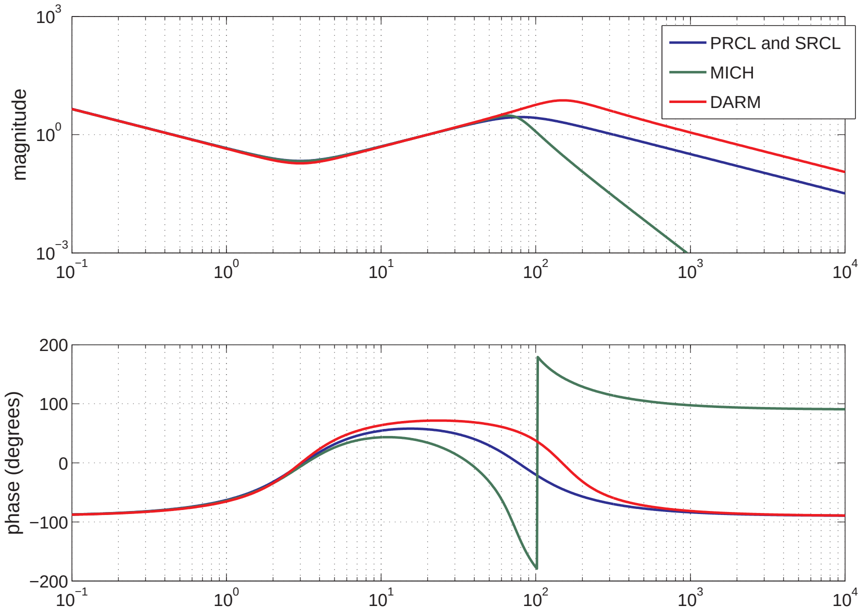

Figure 2: Control filters for DARM and the central cavity degrees of freedom. The corrections sent to the BS are further filtered in order to compensate for the double pendulum in the penultimate mass to mirror mechanical transfer function.

$$\widetilde{P}_{shot} = \sqrt{1hvP_0/\eta} = 4.5 \times 10^{-11}\ \mathrm{W}/\sqrt{\mathrm{Hz}} \tag{1.3}$$

$$\widetilde{I}_{shot} = \frac{\eta e}{hv}\widetilde{P}_{shot} = 3.6 \times 10^{-11}\ \mathrm{A}/\sqrt{\mathrm{Hz}} \tag{1.4}$$

where $P_0 = 10$ mW. The most stringent constraint for setting the requirements on the read-out noise performance of REFL31 and REFL32 is expected to come by the BS control. This is because the actuation for the BS is done from the Penultimate Mass (PM), so an f^3 controller is needed in order to compensate for the PM to mirror transfer function (instead of an f controller). Since the noise is propagated to the mirror through the control filter, this makes the re-injected noise grow by a factor f^2 faster than in the other loops. On the other hand, the BS actuator limit is only a factor 60 bigger than PRM and SRM's one. The possibility of controlling the BS position using both REFL31 and REFL32 is considered in the analysis which follows.

I) REFL31 (MICH controlled by REFL31)

For each mirror of the central cavity, the RMS of the correction signal shot noise limited is shown in table 2.

As said above, the BS actuator limit is 60 times bigger than for PRM and SRM, while the RMS of its correction signal is more than a factor 100 bigger. It is therefore the BS control which sets the requirement for the maximum allowed read-out noise (called generically electronic noise), which can be a factor 15 above shot noise:

Mirror	**Corrections shot noise limited (RMS)**
PRM:	4.0 μN
BS:	**4.0 mN**
SRM:	14 μN

Table 2: RMS of the correction signal (shot noise limited) sent to each mirror when MICH is controlled by REFL31.

Mirror	**Corrections shot noise limited (RMS)**
PRM:	2.0 μN
BS:	**1.7 mN**
SRM:	11 μN

Table 3: RMS of the correction signal (shot noise limited) sent to each mirror when MICH is controlled by REFL32.

$$\widetilde{P}_{elect} = 6.75 \times 10^{-10}\ \mathrm{W}/\sqrt{\mathrm{Hz}} \tag{1.5}$$

$$\widetilde{I}_{elect} = 5.40 \times 10^{-10}\ \mathrm{A}/\sqrt{\mathrm{Hz}} \tag{1.6}$$

II) REFL32 (MICH controlled by REFL32)

For each mirror of the central cavity, the RMS of the correction signal shot noise limited is shown in table 3:

With the same argument as before, the maximum electronic noise allowed can be about factor 30 above shot noise:

$$\widetilde{P}_{elect} = 1.35 \times 10^{-9}\ \mathrm{W}/\sqrt{\mathrm{Hz}} \tag{1.7}$$

$$\widetilde{I}_{elect} = 1.1 \times 10^{-9}\ \mathrm{A}/\sqrt{\mathrm{Hz}} \tag{1.8}$$

1.1.2 DC Photodiodes in transmission to the arms: TRX and TRY

DC signals transmitted to the arms are used during lock acquisition. The maximum power on the diodes is achieved at the end of the locking sequence: 31.5 mW. An about 75% attenuator is therefore needed in order to have a maximum power of about 10 mW on the diodes. The signal to shot noise ratio in this case is proportional to the square root of the power impinging upon the diode, so that the worse condition arises at the beginning of the locking sequence, when the power on the diode is the lowest one (about $P_0 = 3.7\mu W$). The shot noise level, both in power and in current, is:

$$\widetilde{P}_{shot} = \sqrt{2hvP_0/\eta} = 1.2 \times 10^{-12}\ \mathrm{W}/\sqrt{\mathrm{Hz}} \tag{1.9}$$
$$\widetilde{I}_{shot} = \frac{\eta e}{hv}\widetilde{P}_{shot} = 9.8 \times 10^{-13}\ \mathrm{A}/\sqrt{\mathrm{Hz}} \tag{1.10}$$

The RMS of the correction signals sent to the end mirrors is 3.4x10^{-6} N. Since the maximum RMS allowed for the end mirrors is 20x10^{-6} N, the maximum photodiode noise allowed can be about factor 5 above shot noise:

$$\widetilde{P}_{elect} = 6.0 \times 10^{-12}\ \mathrm{W}/\sqrt{\mathrm{Hz}} \tag{1.11}$$
$$\widetilde{I}_{elect} = 4.9 \times 10^{-12}\ \mathrm{A}/\sqrt{\mathrm{Hz}} \tag{1.12}$$

1.1.3 RF Photodiode at the anti-symmetric port: AS2

During the locking sequence, when the arm transmitted power is about 100 times bigger than the single Fabry-Perot cavity power, the DARM control is moved form a DC to the RF signal at the second modulation frequency f2, about 47 MHz. The power at the anti-symmetric port is about 4.6 mW. Since this is the gravitational wave signal port, not more than 1% is likely to be available for lock acquisition photodiodes. For this reason, we consider having a power on the diode of about $P_0 = 46\ \mu$W. This power is dominated by the f2 sideband power, since the carrier and the f1 sidebands are not resonant inside the signal recycling cavity, and they are poorly transmitted to the anti-symmetric port. The f2 sideband power is also contributing to make signal, so that the shot noise level, both in power and in current, is given by:

$$\widetilde{P}_{shot} = \sqrt{1.5hvP_0/\eta} = 3.7 \times 10^{-12}\ \mathrm{W}/\sqrt{\mathrm{Hz}} \tag{1.13}$$
$$\widetilde{I}_{shot} = \frac{\eta e}{hv}\widetilde{P}_{shot} = 3.0 \times 10^{-12}\ \mathrm{A}/\sqrt{\mathrm{Hz}} \tag{1.14}$$

The maximum photodiode noise can be up to a factor 3000 above shot noise:

$$\widetilde{P}_{elect} = 1.1 \times 10^{-8}\ \mathrm{W}/\sqrt{\mathrm{Hz}} \tag{1.15}$$
$$\widetilde{I}_{elect} = 9.0 \times 10^{-9}\ \mathrm{A}/\sqrt{\mathrm{Hz}} \tag{1.16}$$

1.1.4 Summary

Table 4 summarizes all the results obtained before.

Photodiode	**Signal**	**Power**	**Shot noise limit**(A/$\sqrt{\text{Hz}}$)	**Max elect noise**(A/$\sqrt{\text{Hz}}$)
REFL 31	RF	10 mW	3.6×10^{-11}	(×**15**) 5.4×10^{-10}
REFL 32	RF	10 mW	3.6×10^{-11}	(×**30**) 1.1×10^{-10}
TRX , TRY	DC	3.7 μW	9.8×10^{-13}	(×**5**) 4.9×10^{-12}
AS2	RF	46 μW	3.0×10^{-12}	(×**3000**) 9.0×10^{-9}

Table 4: Summary of the specifications for each photodiode.

2 Signal Read-Out Topology

2.1 RF photodiodes

Different topologies of RF photodiodes have been investigated for Advanced LIGO [2]. The design has been focused on the extraction of signals used during science mode. Since these signals are required to be shot noise limited, the characterization has been done in terms of the ratio between the shot noise resulting from the photo-current flowing in the diode and the electronic noise of the read-out. This value, shortly called SNR, turns out to be between 2 and 6, depending on the topology, for 50 mA of DC photocurrent and a read-out frequency of 50 MHz. Here we apply the same analysis but focusing on possible read-out for lock acquisition. According to the results reported in section 1.1, the shot noise limit is not strictly needed for signals only used for lock acquisition, so that we can tolerate an SNR lower than 1. On the other hand, the power impinging upon each diode can be much lower than in science mode, and this contributes to decrease the SNR. The following discussion aims to show if the noise performance of the proposed topologies are compatible with the requirements for lock acquisition, or if different solutions are needed. Two of the proposed topologies which allow simultaneous extraction of different frequencies (needed in particular at the reflection port), have been studied:

- TOPOLOGY A: *Sandberg-GEO-Virgo style design*;
- TOPOLOGY B: *Variant 1.*

The two topologies are described in figures 3 and 4. Here the basic assumption and formula used in the analysis will be reported for completeness, while all the details can be found in the original document. A standard photodiode model assumes a constant current source of DC photocurrent I2, a diode capacitance $C_3 = 100$ pF and a diode series resistance $R_5 = 10\ \Omega$. This implies a maximum frequency of 160 MHz, which is compatible with the largest frequency detected, 141 MHz. Standard values for the input referred voltage noise and current noise of the op-amp are respectively:

$$\widetilde{V}_n = 2\ \mathrm{nV}/\sqrt{\mathrm{Hz}} \tag{2.1}$$

$$\widetilde{I}_n = 3\ \mathrm{pA}/\sqrt{\mathrm{Hz}} \tag{2.2}$$

The SNR in these two configurations can be computed as summarized in the following:

Topology A

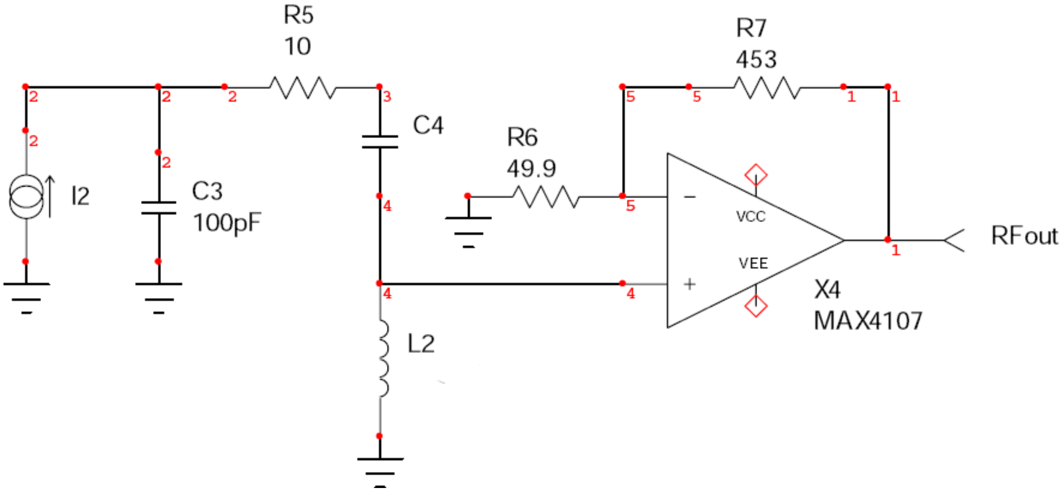

Figure 3: Topology A (Sandberg-GEO-Virgo style design)

$$SNR = \frac{\widetilde{V}_{shot}}{\widetilde{V}_{elect}} \tag{2.3}$$

$$\widetilde{V}_{shot} = \widetilde{I}_{shot}\omega L_2 \sqrt{\frac{1}{1+(\omega R_5 C_3)^2}} \quad \mathrm{V}/\sqrt{\mathrm{Hz}} \tag{2.4}$$

$$\widetilde{V}_{elect} = \sqrt{\widetilde{V}_n^2 + (\widetilde{I}_n Z_{network})^2 + \widetilde{V}_{therm}^2} \quad \mathrm{V}/\sqrt{\mathrm{Hz}} \tag{2.5}$$

where $Z_{network}$ and $\widetilde{V}_{therm}$ are:

$$Z_{network} = \sqrt{\frac{R_5^2 + (\omega L_2)^2}{(1-\omega^2 L_2 C)^2 + (\omega R_5 C)^2}}, \quad C = C_3||C_4 \tag{2.6}$$

$$\widetilde{V}_{therm} = \sqrt{4kTR_5} \quad \mathrm{V}/\sqrt{\mathrm{Hz}} \tag{2.7}$$

Topology B

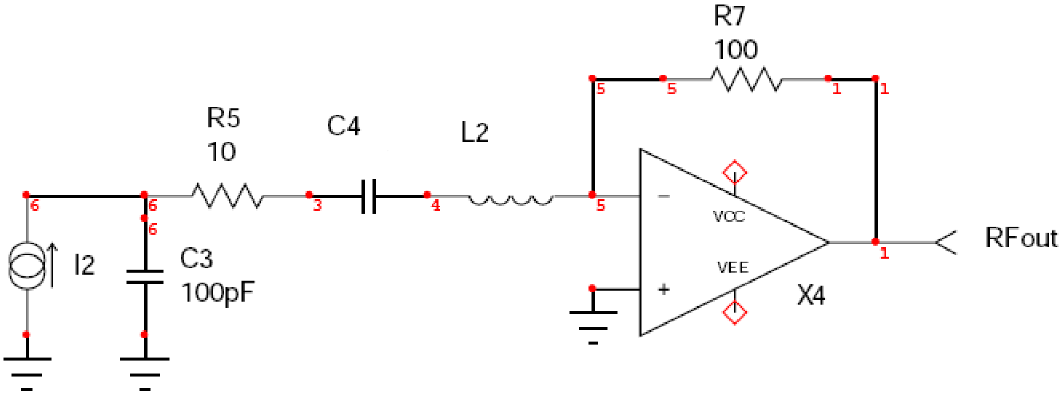

Figure 4: Topology B (Variant 1)

$$SNR = \frac{\widetilde{V}_{shot}}{\widetilde{V}_{elect}} \tag{2.8}$$

$$\widetilde{V}_{shot} = \widetilde{I}_{shot} R_7 \sqrt{\frac{1}{1+(\omega R_5 C_3)^2}} \quad \mathrm{V}/\sqrt{\mathrm{Hz}} \tag{2.9}$$

$$\widetilde{V}_{elect} = \sqrt{(\widetilde{V}_n \frac{R_7}{Z_{source}})^2 + (\widetilde{I}_n \frac{R_7 Z_{source}}{(Z_{source}+R_7)})^2 + \widetilde{V}^2_{therm}} \quad \mathrm{V}/\sqrt{\mathrm{Hz}} \tag{2.10}$$

where Z_{source} is:

$$Z_{source} = \sqrt{R_5^2 + \frac{1}{(\omega C_3)^2}} \tag{2.11}$$

and $\widetilde{V}_{therm}$ in this case is:

$$\widetilde{V}_{therm} = \sqrt{4kTR_7} \quad \mathrm{V}/\sqrt{\mathrm{Hz}} \tag{2.12}$$

Results from the analysis shown in section 1.1 are summarized in the following table:

2.1.1 Reflection port: signal extraction at 28 MHz (REFL 31)

Topology A

Photodiode	**Signal**	$\widetilde{\mathbf{I}}_{\mathbf{shot}}(\mathrm{A}/\sqrt{\mathrm{Hz}})$	**Min SNR tolarable**
REFL31	RF (28.1987 MHz)	3.6×10^{-11}	**0.067**
REFL32	RF (140.9935 MHz)	3.6×10^{-11}	**0.033**
AS2	RF (46.9978 MHz)	3.0×10^{-12}	$\mathbf{3.3 \times 10^{-4}}$

Table 5: Summary of the specifications for each photodiode.

In order to set the resonance frequency of the LC circuit at 28 MHz, and have an RF transimpedance of 100 Ω , the resulting values for L_2 and C_4 are:

$$L_2 = 570 \text{ nH} \tag{2.13}$$

$$C_4 = 56 \text{ pF} \tag{2.14}$$

By considering the value of the shot noise current reported in table 5, the equivalent shot noise voltage and the electronic voltage at the positive input to the op-amp are:

$$\widetilde{V}_{shot} = 3.57 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}} \tag{2.15}$$

$$\widetilde{V}_{elect} = 2.20 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}} \tag{2.16}$$

and the resulting SNR is:

$$\mathbf{SNR = 1.62}$$

Topology B

L_2 and C_4 set the resonance frequency of the circuit, but not the RF transimpedance at the resonance, which is set by R7. One possible choice is:

$$L_2 = 318 \text{ nH} \tag{2.17}$$

$$C_4 = 100 \text{ pF} \tag{2.18}$$

The equivalent shot noise voltage and the electronic voltage at the RFout port are:

$$\widetilde{V}_{shot} = 3.54 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}} \tag{2.19}$$

$$\widetilde{V}_{elect} = 3.70 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}} \tag{2.20}$$

and the resulting SNR is:

$$\mathbf{SNR = 0.95}$$

2.1.2 Reflection port: signal extraction at 141 MHz (REFL 32)

Topology A

In order to set the resonance frequency of the LC circuit at 141 MHz, and have an RF transimpedance of 100 Ohm, the resulting values for L2 and C4 are:

$$L_2 = 110 \text{ nH} \tag{2.21}$$
$$C_4 = 10 \text{ pF} \tag{2.22}$$

10 pF might be a too small value compared with the diode capacitance of 100 pF. By setting the transimpedance to 50 Ohm, we can end up with:

$$L_2 = 55 \text{ nH} \tag{2.23}$$
$$C_4 = 20 \text{ pF} \tag{2.24}$$

By considering the value of the shot noise current reported in table 5, the equivalent shot noise voltage at the positive input to the op-amp is:

$$\widetilde{V}_{shot} = 1.3 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}} \tag{2.25}$$
$$\widetilde{V}_{elect} = 2.1 \times 10^{-9} \text{V}/\sqrt{\text{Hz}} \tag{2.26}$$

The resulting SNR is:

$$\mathbf{SNR = 0.63}$$

Topology B

L_2 and C_4 set the resonance frequency of the circuit, but not the RF transimpedance at the resonance, which is set by R7. One possible choice is:

$$L_2 = 50 \text{ nH} \tag{2.27}$$
$$C_4 = 25 \text{ pF} \tag{2.28}$$

The equivalent shot noise voltage and the electronic voltage at the RFout port are:

$$\widetilde{V}_{shot} = 2.69 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}} \tag{2.29}$$
$$\widetilde{V}_{elect} = 1.33 \times 10^{-8} \text{ V}/\sqrt{\text{Hz}} \tag{2.30}$$

and the resulting SNR is:

$$\mathbf{SNR = 0.2}$$

2.1.3 Anti-symmetric port: signal extraction at 47 MHz (AS2)

Topology A

In order to set the resonance frequency of the LC circuit at 46 MHz, and have an RF transimpedance of 100 Ohm, the resulting values for L2 and C4 are:

$$L_2 = 339 \text{ nH} \tag{2.31}$$
$$C_4 = 34 \text{ pF} \tag{2.32}$$

By considering the value of the shot noise current reported in table 5, the equivalent shot noise voltage and the electronic voltage at the positive input to the op-amp are:

$$\widetilde{V}_{shot} = 2.86 \times 10^{-10} \text{ V}/\sqrt{\text{Hz}} \tag{2.33}$$
$$\widetilde{V}_{elect} = 2.34 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}} \tag{2.34}$$

and the resulting SNR is:

$$\mathbf{SNR = 0.12}$$

Topology B

L_2 and C_4 set the resonance frequency of the circuit, but not the RF transimpedance at the resonance, which is set by R7. One possible choice is:

$$L_2 = 314 \text{ nH} \tag{2.35}$$
$$C_4 = 36 \text{ pF} \tag{2.36}$$

The equivalent shot noise voltage and the electronic voltage at the RFout port are:

$$\widetilde{V}_{shot} = 2.86 \times 10^{-10} \text{ V}/\sqrt{\text{Hz}} \tag{2.37}$$
$$\widetilde{V}_{elect} = 5.80 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}} \tag{2.38}$$

and the resulting SNR is still:

$$\mathbf{SNR = 0.05}$$

Results from previous paragraphs are summarized in the following table:

Photodiode	**Topology A (SNR)**	**Topology B (SNR)**	**Minimum SNR**
REFL31	1.62	0.95	0.067
REFL32	0.63	0.20	0.033
AS2	0.12	0.05	$\sim 10^{-4}$

Table 6: Summary of SNRs for RF signal read-out.

2.2 DC photodiodes

DC signals are used during lock acquisition as error signals for CARM (REFL_DC) and DARM (TRX_DC and TRY_DC). Specifications for REFL_DC have not been investigated yet. On the other hand, they are not expected to be critical, since the CARM control is done through the frequency servo. On the contrary, because of the weak actuation force available on the end mirrors, the requirements on the noise performance of the diodes in transmission to the cavities are quite stringent, as summarized in table 7.

The DC read-out scheme is expected to be similar to that of Initial LIGO. The photo-current flows to ground through a resistor R, which is connected to ground and to the input of the

Photodiode	**Signal**	$\widetilde{\mathbf{I}}_{\mathbf{shot}}$(A/$\sqrt{\text{Hz}}$)	**Min SNR tolarable**
TRX,TRY	DC	9.8×10^{-13}	**0.2**

Table 7: Summary of the specifications for the DC diodes in transmission to the arm cavities.

amplifier. With the same photodiode model described before, the shot noise and electronic noise voltage at the input of an amplifier are:

$$\widetilde{V}_{shot} = \widetilde{I}_{shot} R \quad \text{V}/\sqrt{\text{Hz}} \tag{2.39}$$

$$\widetilde{V}_{elect} = \sqrt{\widetilde{V}_n^2 + (\widetilde{I}_n R)^2 + 4kTR} \quad \text{V}/\sqrt{\text{Hz}} \tag{2.40}$$

It can easily be shown that in order to have an $SNR = 0.2$ the minimum value required for the resistance is:

$$\mathbf{R = 2\ k\Omega}$$

2.3 Conclusions

Two RF read-out topologies investigated for AdvLIGO (science mode) have been analyzed in terms of their noise performance during lock acquisition. The analysis shows that the noise performance are largely compatible with lock acquisition requirements, as summarized in table 6: even in the worst case (REFL32, Topology B) the SNR is still a factor 6 higher than the minimum acceptable. It is worth to observe that Topology A provides a SNR which is about 3 times better than Topology B for each signal.

References

[1] Rich Abbott et al., *AdvLIGO Interferometer Sensing and Control Conceptual Design*, LIGO-T070247-01-I

[2] Rich Abbott, *RFPD Topology COmparison*, LIGO-T060268-03-C