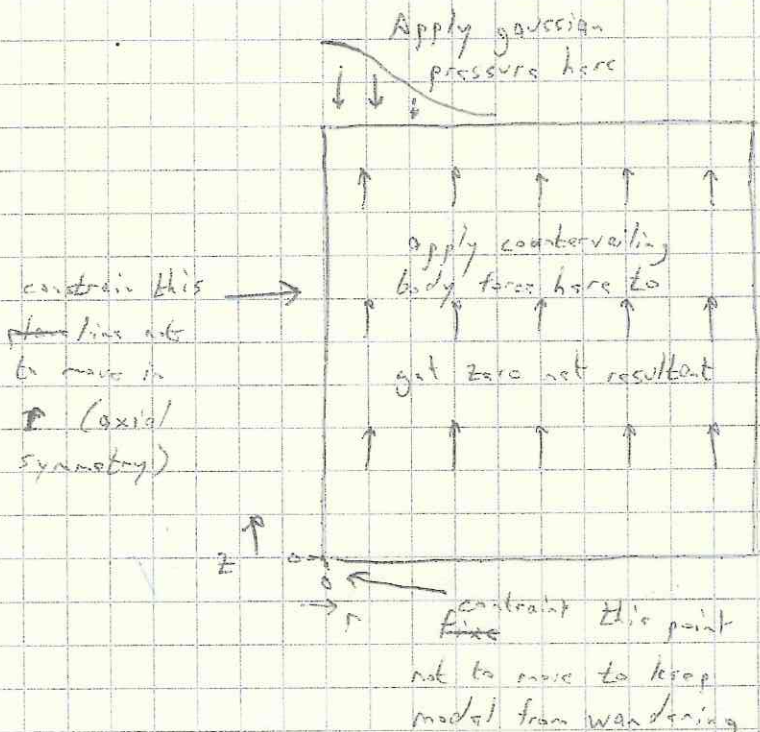
Oct. 11, 2006

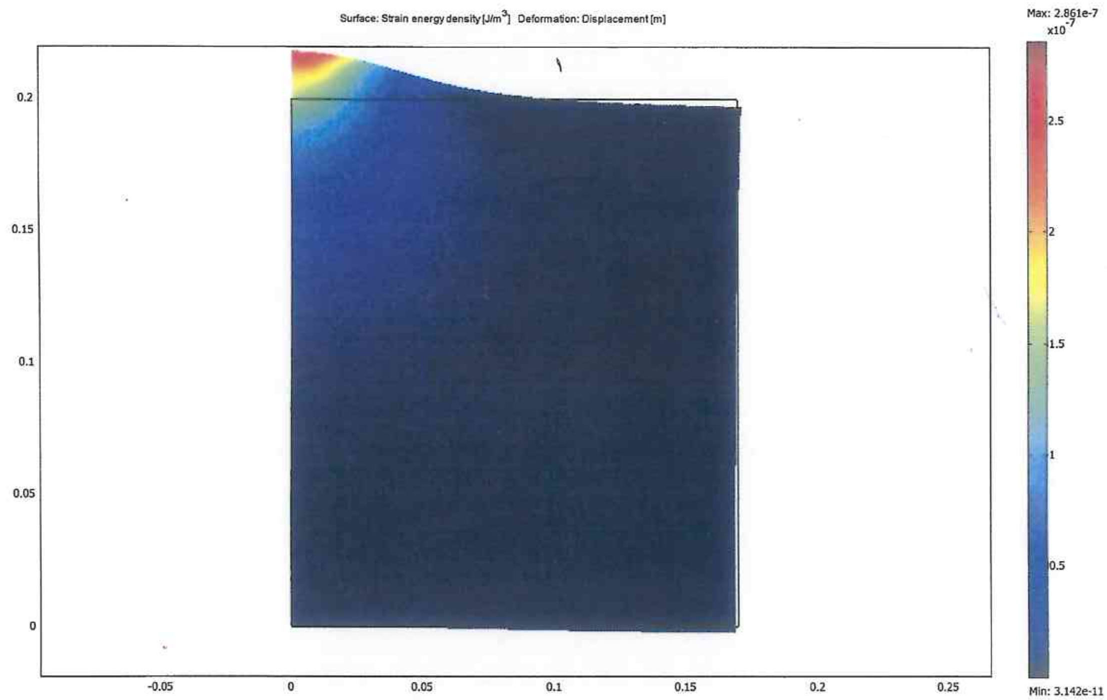
Test Mass Barrel Coating Thermal Noise

I may need to ~~too~~ insulate the barrel of the ITMs in Advanced LIGO to deal with the CP/ITM heating problem. If so, a thin gold coating protected with a silica overlayer would be a very convenient way. What would this do to thermal noise?

Gold has the ≈same Young's modulus as fused silica (78 GPa vs. 72 GPa), although different Poisson ratios (.44 vs. .17), so probably it is safe to model a monolithic test mass.



Above is the basic model I programmed into the COMSOL 3.2 Axial Symmetry, Stress-Strain Structural Mechanics Module. At the first I make the quasistatic assumption since I worry about 100 Hz & the resonant modes are much higher.



This figure shows the deformed shape & strain energy density distribution for this model.

By expanding the colorbar scale I find that the average strain energy density at the barrel is $1.0 \pm .2 \times 10^{-10}$ J/m^3. The total strain energy density is found by subdomain integration to be 4.24×10^{-11} J.

Assume the barrel coating is 1 μm thick (I think < 1/2 this would be sufficient) so the volume is

$$\cong h \cdot 2\pi r \cdot t = .2m \cdot 2\pi \cdot .17m \times 10^{-6} m = 2.1 \times 10^{-7} m^3$$

The fraction of the elastic energy in the barrel coating is

$$\frac{1.0 \times 10^{-10} \text{ J/m}^3 \times 2.1 \times 10^{-7} m^3}{4.24 \times 10^{-11} \text{ J}} = 5.0 \times 10^{-7}$$

Assume the gold has a loss angle of 10^{-2} (probably it is ~5-10x lower). The fraction of the energy lost is

$$\frac{\Delta E}{E} = 5\times10^{-7}\times10^{-2} = 5\times10^{-9}$$

By contrast, the fused silica itself has internal friction of $\cancel{\sim10^{-8.3} \Rightarrow 6.3\times10^{-9}}$, 10^{-9}

So, <u>if</u>: 1) the coating is 1 μm thick, and
2) the Q of gold is only 100,

then the barrel coating will contribute ~~slightly less thermal noise than the substrate~~ $\sqrt{5}$ more thermal noise than the substrate. If the coating is 0.1 μm thick, or has Q of 1000, then it contributes $\sqrt{0.5}$ as much thermal noise as the substrate. If it is 0.1 μm thick, <u>and</u> has higher Q, then the thermal noise is insignificant.

Model saved as "gold coating on barrel.mph"

What about thermal expansion?

Gold: $\alpha = 14\times10^{-6}/K$
$\rho = 19{,}300\ kg/m^3$
$C = 128\ J/kg\,K$

$$\frac{\alpha}{\rho C} = \frac{5.66\times10^{-12}}{J/m^3}$$

Silica: $\alpha = 5.5\times10^{-7}/K$
$\rho = 2202\ kg/m^3$
$C_v = 739\ J/kg\,K$

$$\alpha = \frac{3.37\times10^{-13}}{J/m^3}$$

Rough estimate of thermoelastic damping. Compare heat capacities of glass & gold

$$\rho C_{gold} = 19{,}300\ kg/m^3 \times 128\ J/kg\,K = 2.5\times10^6\ J/m^3K$$
$$\rho C_{glass} = 2{,}202\ kg/m^3 \times 739\ J/kg\,K = 1.6\times10^6\ J/m^3K$$

So for equal temperatures, 3/2 as much glass holds the same heat as gold. ~~So if~~ We assume the loss peak occurs when 1/2 the heat in the gold layer has time to diffuse into the glass.

The diffusion length in glass is $d_{th} = \sqrt{\dfrac{K}{2\pi f \rho C}}$

$$\Rightarrow d_{th} = 30\,\mu m \left(\frac{150\,Hz}{f}\right)^{1/2}$$

Assume a 0.1 μm layer of gold, so we want 0.15 μm diffusion into glass. This happens at frequency

$$0.15\,\mu m = 30\,\mu m \left(\frac{f}{150\,Hz}\right)^{1/2}$$

$$\Rightarrow f = 150\,Hz \times \left(\frac{30}{0.15}\right)^2 = 6\,MHz.$$

The relaxation strength is about $\left.\dfrac{Y\alpha^2 T_0}{\rho C}\right|_{gold}$

$$\frac{78\times 10^9 \cdot (14\times 10^{-6})^2 \cdot 300}{19{,}300 \cdot 128} = 1.9\times 10^{-3}$$

Since Zener damping tends to fall like ~~f²~~ f below the peak, the internal friction at 100 Hz will be roughly

$$\phi \approx \frac{150\,Hz}{6\,MHz} \times 1.9\times 10^{-3} = 4.8\times 10^{-8}$$

So thermoelasticity is not really an issue.

Looking at Marty Fejer's article, I see that I got the peak frequency right but the slope wrong. In fact, the loss falls as $\sqrt{f}$. So

$$\phi(150\,Hz) \sim \sqrt{\frac{150\,Hz}{6\,MHz}} \times 1.9\times 10^{-3} = 9.5\times 10^{-6}$$

Still ignorably small.