Note about small angle scattering in the GEO 600 GW detector (LIGO-T070259-00-Z)

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1 Measurement of the small angle scattering of a GEO 600 core mirror

Due the installation of a new view port (of larger diameter) at the vacuum tank in the north building for the first time we have recently be able to perform a direct measurement of the actual small angle scattering of one of the GEO 600 core mirrors.

Figure 1 shows an image of the light transmitted through the 200 mm view port. One can clearly see the north mirror (MFn), with the ears and the fused silica fibres of the monolithic suspension at the left edge. Some parts of the metal catcher structure (which should gently catch the mirror in case of a suspension failure) are visible below the mirror. In the middle of the mirror one can see the transmitted main interferometer beam (strongly attenuated by the HR coating of the mirror). The main beam shape consists of a fringe pattern originating from the interference of the upper and lower beam inside the folded north arm.

The red circle in the upper left corner of Figure 1 indicates the position of the measurement. An optical power of $\delta P = 910 \pm 1\mu$W was measured. The measured light power δP comprises half of light scattered by MCn (upper beam path of the folded north arm) and half of light scattered by BS and/or MPR (lower beam path of the folded north arm). The fact that the fringe pattern of the scattered light has the same contrast as the fringe pattern of the main beam indicates that the scattered light measured is made of equal amounts of scattered light from the upper and lower beam path. This means that the scattered light from MCn contributes with $\delta P_{\mathrm{MCn}} = 455\mu$W to this measurement.

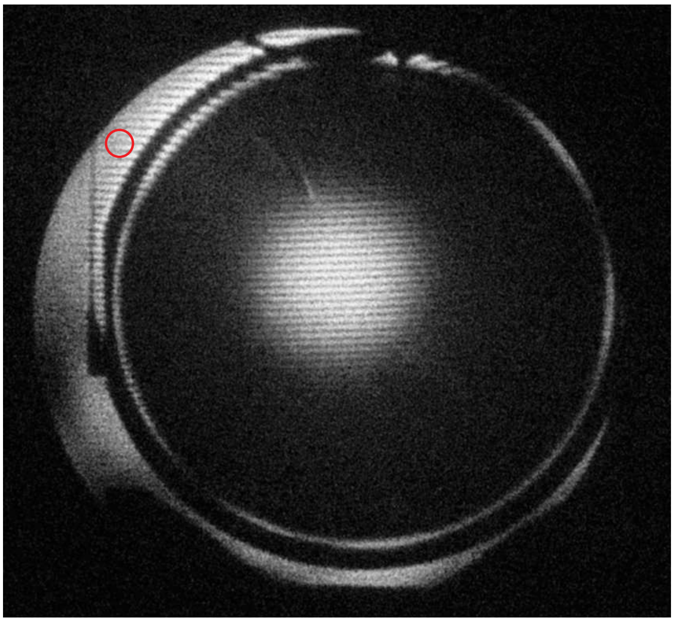

Figure 1: Image of the light transmitted through the 200 mm view port. One can clearly see the north mirror (MFn), with the ears and the fused silica fibres of the monolithic suspension at the left edge. Some parts of the metal catcher structure (which should gently catch the mirror in case of a suspension failure) are visible below the mirror. In the middle of the mirror one can see the transmitted main interferometer beam (strongly attenuated by the HR coating of the mirror). The main beam shape consists of a fringe pattern originating from the interference of the upper and lower beam inside the folded north arm. The red circle indicates the measurement point for the small angle scattering.

Using this measurement we can calculate the scattering function $f(\theta)$ of MCn into the solid angle $\delta\Omega$, with θ being the geometrical angle of the scattering:

$$f_{\mathrm{MCN}}(\theta) = \frac{\delta P_{\mathrm{MCN}}}{P_{\mathrm{MCN}} \cdot \delta\Omega} \qquad \text{with} \qquad \delta\Omega = \frac{A}{r^2}, \tag{1}$$

where A is the area of the sensor, P the power impinging on MCN and r the distance between scattering source and the power sensor. Using $r = 600\,\mathrm{m}$, $A = 4.9 \cdot 10^{-5}\mathrm{m}^2$ and $P = 1.3\,\mathrm{kW}$ we get:

$$f_{\mathrm{MCN}}(0.17\mathrm{mrad}) = 2.57 \cdot 10^3. \tag{2}$$

Figure 2: Image of the scattered light hitting on of the catchers at the far end. The intensity of the light on the catcher was estimated to be about 1.8 mW/cm^2. Overall 90 mW of light are assumed to hit the front surface of the catcher.

2 Estimating the scattered light noise from the far end catchers

2.1 Introduction

Each of the core optics elements is surrounded by a metal structure, its so-called catcher, which has the purpose to gently catch the mirror and absorb the shock in case of a suspension failure. As shown in figure 2, it is observed that these catchers are illuminated by stray light during nominal operation of the interferometer. The combination of circulating laser powers of the order kilowatts and the fact that even the best optics exhibit small imperfections cause this illumination of the catchers. From the measurement described in Section 1, we can estimate the intensity of the light on the catcher to be about 1.8 mW/cm^2. Overall the light hitting the front surface of the catcher is assumed to be $P_{\rm cat} = 90\,\mathrm{mW}$.

The problem about the light hitting the catchers is that (in contrary to other GW detectors like for instance LIGO) the catchers in GEO 600 are not seismically isolated, but rigidly connected to the base plate of the corresponding vacuum tank. Light that hits the catcher is experiencing a phase shift corresponding to the movement of the catcher. If this scattered light with non stationary phase reenters the main interferometer mode and is detected at the output of the interferometer it looks similar to a GW signal and can therefore decrease the sensitivity of the instrument.

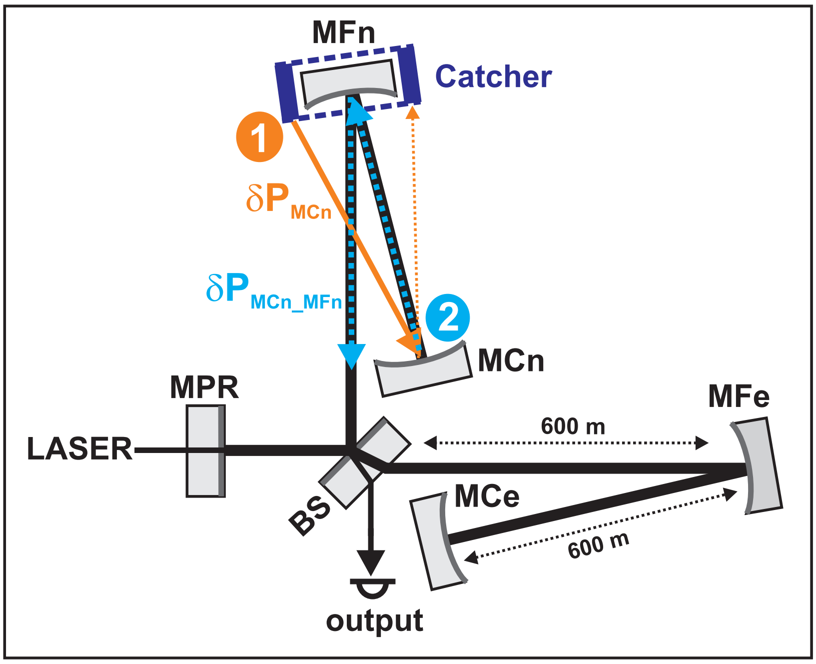

Figure 3: Simplified schematic of the scattering processes necessary to scatterer light from the MFN catcher (blue) back into the main interferometer mode. A fraction of light hitting the catcher, δP_{MCn} (orange solid arrow), is scattered towards MCN and hits the main interferometer beam spot on MCn. Since the scattered light hitting MCn is not reflected back towards MFn, see orange dashed arrow, a second scattering process is required to bring the catcher light back into the interferometer mode. This second scattering process is indicated by the cyan arrows. The fraction of the light scattered from the catcher of MFn that reenters the interferometer mode is given by $\delta P_{\mathrm{MCn\_MFn}}$.

2.2 Calculation of the catcher light that reenters the main interferometer mode

In order for the catcher light to reenter the main interferometer mode two scattering processes are required as indicated by figure 3. First a fraction of light hitting the catcher is scattered towards the solid angle of the interferometer beam spot on MCN, δP_{MCn} (orange solid arrow). Since the direct reflection of the orange arrow is not hitting any other optical component this light is not sensed at the output of the interferometer. A second scattering process, indicated by the cyan arrows, is required to bring a fraction, $\delta P_{\mathrm{MCn\_MFn}}$, of the catcher light via MFn back into the interferometer mode.

The amount of light that is scattered from the MFn catcher onto the effective area of MCn can be calculated as follows:

$$\delta P_{\mathrm{MCn}} = P_{\mathrm{cat}} \cdot f_{\mathrm{cat}}(\theta) \cdot \delta\Omega_1, \tag{3}$$

where $f_{\mathrm{cat}}(\theta)$ describes the scattering function of the catcher material and $\delta\Omega_1$ is the solid angle in which the light has to be scattered to hit the effective mode diameter at MCn. With the beam radius at MCn, ω_{MCn}, of about 1 cm and a separation of MFn and MCn of $L = 600\,\mathrm{m}$ we get

$$\delta\Omega_1 = \frac{2 \cdot \omega_{\mathrm{MCn}}^2 \cdot \pi}{L^2} = 1.7 \cdot 10^{-9}. \tag{4}$$

The factor 2 in the numerator accounts for the fact, that the effective mode diameter for this kind of scattering is 2 times the beam diameter of the main interferometer beam. (This was found in the PhD thesis of Walter Winkler.)

Unfortunately for $f_{\mathrm{cat}}(\theta)$ no direct measurement is available. However, from the early experiments at the Garching prototype interferometer we can get the following two pieces of information:

- For very small angles θ it was found that $f(\theta)$ is independent of the angle ($f(\theta) = \text{constant}$).
- Walter Winkler measured with an Ar^+-Laser ($\lambda \approx 500\,\mathrm{nm}$) the small angle scattering function of unpolished oxidized aluminium to be $f_{\mathrm{Al}} = 6.4$.

Using $f_{\mathrm{cat}} = 6.4$ for very small angles θ, Equation 3 gives

$$\delta P_{\mathrm{MCn}} = 1 \cdot 10^{-9}\,\mathrm{W} \tag{5}$$

Only a fraction of the light scattered onto the surface of MCn is actually scattered back into the interferometer mode. A second scattering process, this time at the surface of MFn, is required. Analogous to Equation 3 we can describe this scattering process and the light power scattered back into the interferometer mode, $\delta P_{\mathrm{MCn\_MFn}}$, by

$$\delta P_{\mathrm{MCn\_MFn}} = P_{\mathrm{MCn}} \cdot f_{\mathrm{MCn}}(\theta) \cdot \delta\Omega_2, \tag{6}$$

where $\delta\Omega_2$ is the solid angle in which the light has to be scattered to hit the effective mode diameter at MFn. With $\omega_{\mathrm{MFn}} = 2.5\,\mathrm{cm}$ we get:

$$\delta\Omega_2 = \frac{2 \cdot \omega_{\mathrm{MFn}}^2 \cdot \pi}{L^2} = 1.1 \cdot 10^{-8}. \tag{7}$$

Putting all information together the total light power that is scattered back into main interferometer mode amounts to

$$\delta P_{\mathrm{MCn\_MFn}} = 2.8 \cdot 10^{-14}\,\mathrm{W}. \tag{8}$$

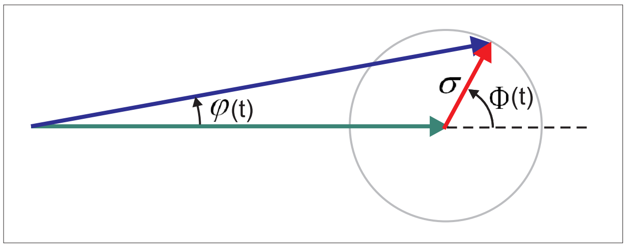

Figure 4: Simple schematic explaining how scattered light influences the differential arm length measurement. The green arrow represents the amplitude of the carrier light inside the interferometer. The scattered light (indicated by the red arrow) has the amplitude σ and a non-constant phase $\phi(t)$. The resulting light is represented by the blue arrow. The observable actually sensed by Michelson interferometer is the phase, $\varphi(t)$, of the blue arrow.

2.3 Transforming the scattered light power to phase fluctuations

Since the Michelson interferometer performs a phase measurement, we have to translate the scattered light power $\delta P_{\mathrm{MCn\_MFn}}$ into a phase. Figure 4 shows the principle of how scattered light influences the measurement inside a GW detector. The scattered light vector (red arrow) with a phase different from the carrier (green arrow) rotates around the tip of the green arrow, resulting in an slightly varying phase, $\varphi(t)$, of the the resulting blue arrow.

The length of the scattered light vector is determined by the amplitude σ of the scattered light. For σ^2 we find the following relation

$$\sigma^2 = \frac{\delta P_{\mathrm{MCn\_MFn}}}{P_{\mathrm{cav}}} = \frac{2.8 \cdot 10^{-14}\,\mathrm{W}}{1300\,\mathrm{W}}, \tag{9}$$

where P_{cav} describes the carrier light power in one interferometer arm. The scattered light amplitude is found to be

$$\sigma = 4.7 \cdot 10^{-9} \tag{10}$$

Using σ we can now calculate the amplitude spectral density (ASD) of the displacement noise, $\hat{N}_{\mathrm{scat}}$, caused by the scattered light from the MFn catcher:

$$\hat{N}_{\mathrm{scat}} = \hat{X}_{\mathrm{cat}} \cdot \sigma, \tag{11}$$

where $\hat{X}_{\mathrm{cat}}$ is the ASD of the catcher movement.

2.4 Movement of the catcher

Due to the fact that neither accelerometer nor seismometer are attached to the MFn catcher, there is no measurement of the actual catcher movement

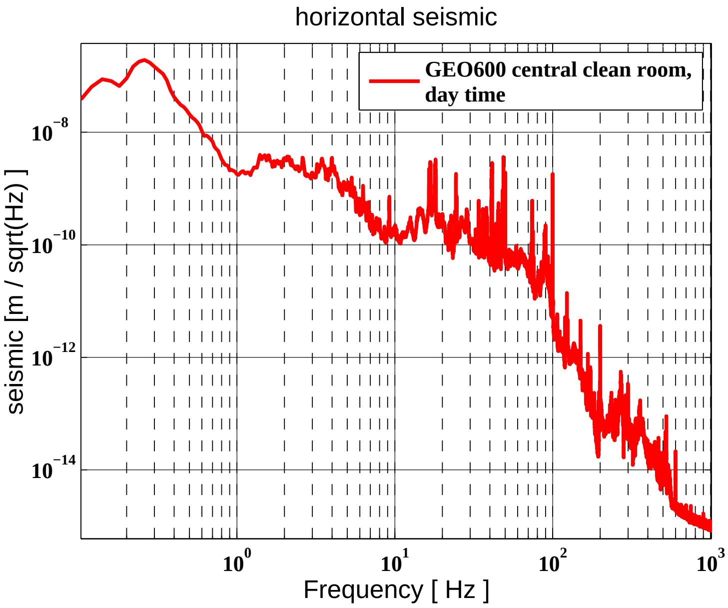

Figure 5: ASD of the horizontal seismic in the central cleanroom of the GEO 600 detector. The data was measured with a 'Streckeisen' STS2.

accessible. Figure 5 shows the a measurement of the horizontal seismic in the GEO 600 central cleanroom. Neglecting all potential mechanical resonances of the catcher structure, we can suppose the seismic data to be a reasonable approximation of the actual catcher movement.

2.5 Projection of the scattered light noise from the MFN catcher

Now we can put all information collected above together and project the scattered light noise from the MFn catcher to $h(t)$. Following Equation 11 we can now calculate $\hat{N}_{\mathrm{scat}}$, which can be transformed from displacement to strain as follows:

$$\hat{h}_{\mathrm{scat}} = \frac{\hat{N}_{\mathrm{scat}}}{1200\,\mathrm{m}} \tag{12}$$

The result of this projection is depict in Figure 6. Only in a very narrow band around 90 Hz the projection (blue trace) comes close to the actual strain sensitivity of the GEO 600 detector (black trace). At all frequencies above 100 Hz the projection is at least a factor of 100 below limiting. Already at 200 Hz the projection is 3 orders of magnitude below $\hat{h}(t)$.

Please note that this projection can only be considered as a rough estimation. In order to allow for a more accurate projection it would be required to get better measurements of:

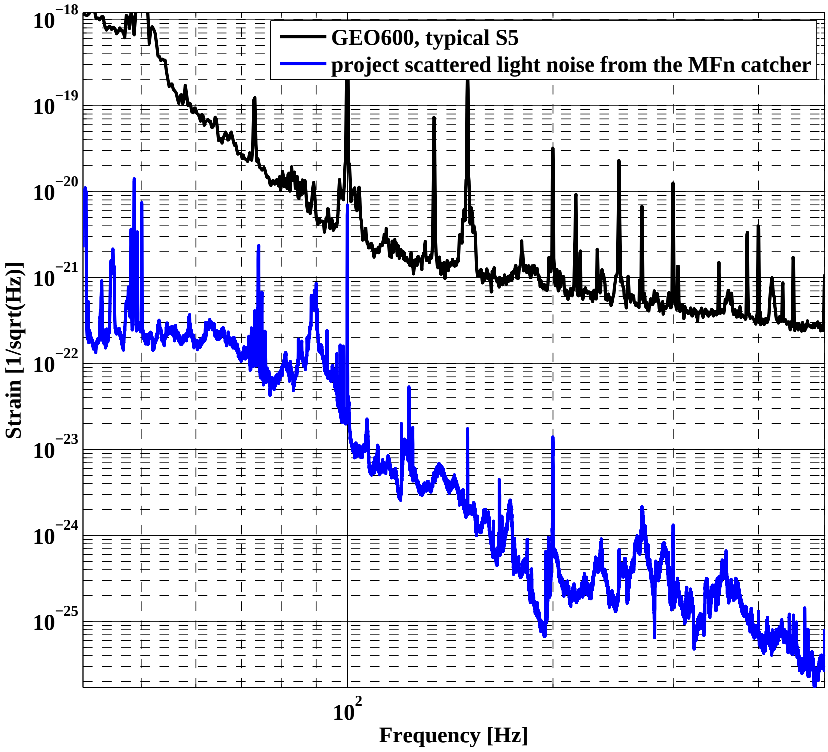

Figure 6: Projection of the scattered light noise from the MFn catcher $\hat{h}_{\mathrm{scat}}$. Only around 90 Hz the projection is less than a factor ten below limiting the GEO sensitivity. For frequencies above 100 Hz the projection is at least a factor of 100 below limiting $h(t)$.

- $f_{\mathrm{cat}}(\theta)$ (Using the same catcher material, the same surface quality and measured for light of $\lambda = 1064\,\mathrm{nm}$.)
- $\hat{X}_{\mathrm{cat}}$ (Using the seismometer in the north end station)

Both of these measurements are currently in progress.

3 The Matlab script used for the projection

This is a small Matlab script I used for the calculations:

```
% Stefan Hild   16.10.2007
%%%%
% This calculates the noise contribution of scattering from the
% far end  catchers of the GEO600 detector.

clear;

dP = 4.55e-4          % light power measured at MFN  [Watt]
A = (0.0079/2)^2*pi   % area of the sensor [m^2]
r = 600               % distance between MCN and sensor [m]
P = 1300              % light impinging on MCN [Watt]
d = 0.1               % distnace of the measurement point from
                      % the center of the main beam

P_cat = 0.09          % light hitting the catcher [Watt]
w_mcn = 0.01          % beam radius at MCn [m]
w_mfn = 0.025         % beam radius at MCn [m]
f_cat = 6.4           % scattering function of the catcher
                      % material
arml = 600            % arm length of IFO

f_mir=dP*r^2/(P*A)    % scattering function f(\theta) of a test
                      % mass
theta = d/r           % theta = angle of scattering

omega1 = 2*(w_mcn^2*pi/arml^2)    % solid angle of effective mode
                      % diameter at MCn seen from 600m distance

P_MCn = P_cat*f_cat*omega1       % scattered light hitting MCn

omega2 = 2*(w_mfn^2*pi/arml^2)   % solid angle of effective mode
                      % diameter at MFn seen from 600m distance

P_MFn = P_MCn*f_mir*omega2       % scattered light reentering
                                 % the main IFO mode

sigma = sqrt(P_MFn/P)           % amplitude of scattering vector
```

```
%%% loading seismic data %%%%
data = fig2data('geo_seis.fig');
x = data.line.x;                %frequency vector
seis = data.line.y;             %seismic vector

%%% Projection of the scattered light noise %%%
scatter_noise = sigma.*seis./1200;

%%% Plotting of the result %%%
figure;
loglog(x,scatter_noise);
grid on;
legend('project scattered light noise from the MFn catcher');
xlabel('Frequency [Hz]');
ylabel('Strain [1/sqrt(Hz)]');
 xaxis(40,500);
```