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LIGO Laboratory / LIGO Scientific Collaboration

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Constraints on advanced LIGO cavity lengths and modulation frequencies

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Abstract

The lengths of the input mode cleaner (L_{imc}), of the power recycling cavity (L_{prc}), of the signal recycling cavity (L_{src}) and of the arm cavities (L_{arm}) in advanced LIGO are considered in light of constraints imposed by existing vacuum chamber dimensions. A set of lengths which meet the constraints, and the corresponding modulation frequencies, are given in this document.

1 Introduction

The input mode cleaner, power and signal and recycling cavities span vacuum chambers whose separations determine the cavity lengths. There is some flexibility on account of the size of the optical tables in these chambers. The mode cleaners for the straight instruments (H1 and L1) occupy HAM-2 and HAM-3; the power recycling mirrors are in HAM-3; the input test masses are in BSC-1 and BSC-3. The signal recycling mirrors are in HAM 4.

The input mode cleaner for the folded interferometer (H2) occupies HAM-8 and HAM-9; the power recycling mirror is in HAM-9; the input test masses are in BSC-7 and BSC-8; the signal recycling mirror is in HAM-10.

The input mode cleaner (IMC), power recycling cavity (PRC), signal recycling cavity (SRC), and arm cavity lengths are all interrelated. The arm cavity length determines the PRC/SRC and IMC cavity lengths. Moreover, the arm cavities have only +/- 30 mm adjustability. The requirement is that the input mode cleaner and power-recycling cavity resonate the carrier and both locking sidebands but that the arms resonate only the carrier. The sideband is to be 5-6 Hz away from being maximally antiresonant in the arms.

The PRC length is of course quite different depending on whether marginal or stable configurations are chosen, and also differs between the straight (H1/L1) and folded (H2) interferometers.

We highlight one issue: G*iven the present location of the HAM chambers, there is no solution lower than 27.3 MHz for the folded interferometer with marginal PRC.* For the stable PRC, the lowest frequency is 17.6 MHz. The minimum frequencies could be reduced to 16.4 MHz (marginal) and 8.4 MHz (stable) by adding 1.8 m to the HAM2/HAM3 separation.

1.1 Cavity frequency/length relations

To resonate in the power recycling cavity, the sideband frequencies must satisfy

$$f_{prc} = (k + 1/2)c/2L_{prc} \qquad (1)$$

where (k = 0,1,2 ...) and L_{prc} is the power recycling cavity length. (Here the length is measured from the PRM to the average of the two ITMs.) The factor of 1/2 occurs because the carrier is resonant in the arm cavities whereas the sidebands are not resonant in the arms, giving an extra 180 degree phase shift in the reflectivity of the arms. In advanced LIGO, there are two modulation frequencies. Both resonate in the PRC.

Both sidebands must be equal to one of the input mode cleaner resonances, because the RF modulation is imposed before the input mode cleaner. The resonant frequencies of the input mode cleaner are:

$$f_{imc} = nc\,/2L_{imc} \qquad (2)$$

where n is an integer (1,2,3 ...) and L_{imc} the input mode cleaner length.

The signal recyling cavity length is also tied to these frequencies; its length is[1]

$$L_{src} = (p + \delta_\phi/2\pi)c/2f_{src} \qquad (3)$$

where p is an integer (1,2,3 ...), δ_ϕ is the signal recycling detuning (0.1 rad), and f_{src} is the frequency used to sense the position of the SRM.

There is a modest range of adjustments available for L_{prc} and L_{imc}, determined by the dimensions of the LIGO vacuum envelope, the size of the optical tables, and considerations of other articles that must reside on these tables. In turn, this determines a range of frequencies for both f_{prc} and f_{imc}.

[1] "Optical Layout for Advanced LIGO," D. Coyne, LIGO-T010076-010D (7/1/2001)

2 The Vacuum chambers

2.1 BSC Chambers

The BSC Chambers hold the ETM, ITM, FM, and BS optics. The optics table is at the top of the chamber and the suspensions hang from the bottom of the table. Figure 1 shows this, as well as the hexagonal ring and support tubes that limit the locations of the suspensions.

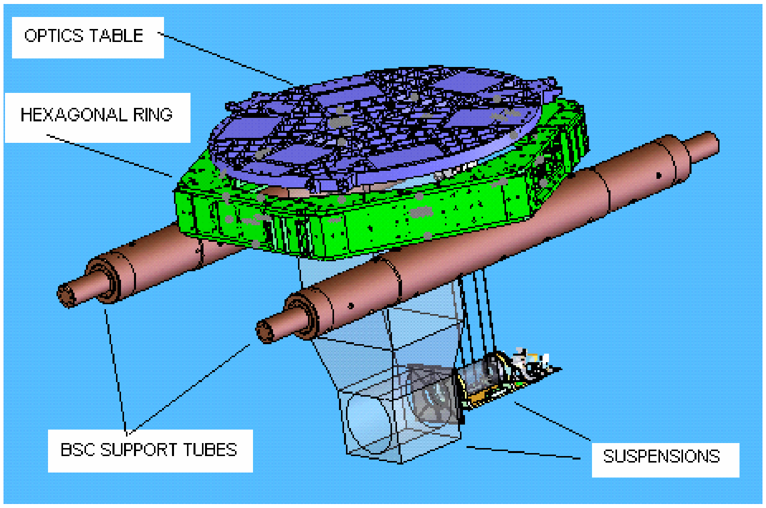


Figure 1. BSC Table and Suspensions

Figure 2 shows an envelope of the BSC Chamber around the table and suspensions.

Figure 2. BSC Chamber

2.2 HAM Chambers

The HAM chambers hold the input and output optics, including IMCs, MMTs, the PRM, and the SRM. The suspensions sit on top of the optics table. Figure 3 shows a typical HAM chamber with suspensions mounted on top of the table.

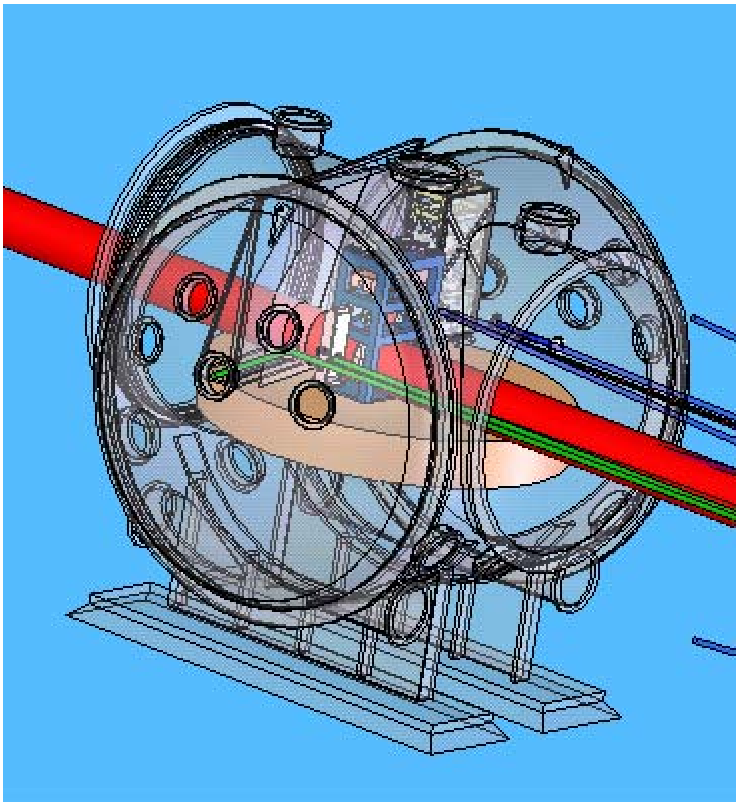


Figure 3. HAM Chamber with Suspensions

3 Fabry-Perot Arm Cavities

3.1 Straight FP Arm Cavity

The length of the FP arm cavity in the straight IFO is constrained by a great variety of boundaries in 3D space. There are multiple suspensions in each chamber that must not interfere with each other, the suspensions and their clamps must remain on the table, and the suspensions must not foul other equipment in the chamber, such as the BSC support tubes and the hexagonal ring. The maximum and minimum lengths of the FP arm cavities are limited by the y-arm, shown below in Figure 4. Table 1 gives the lengths.

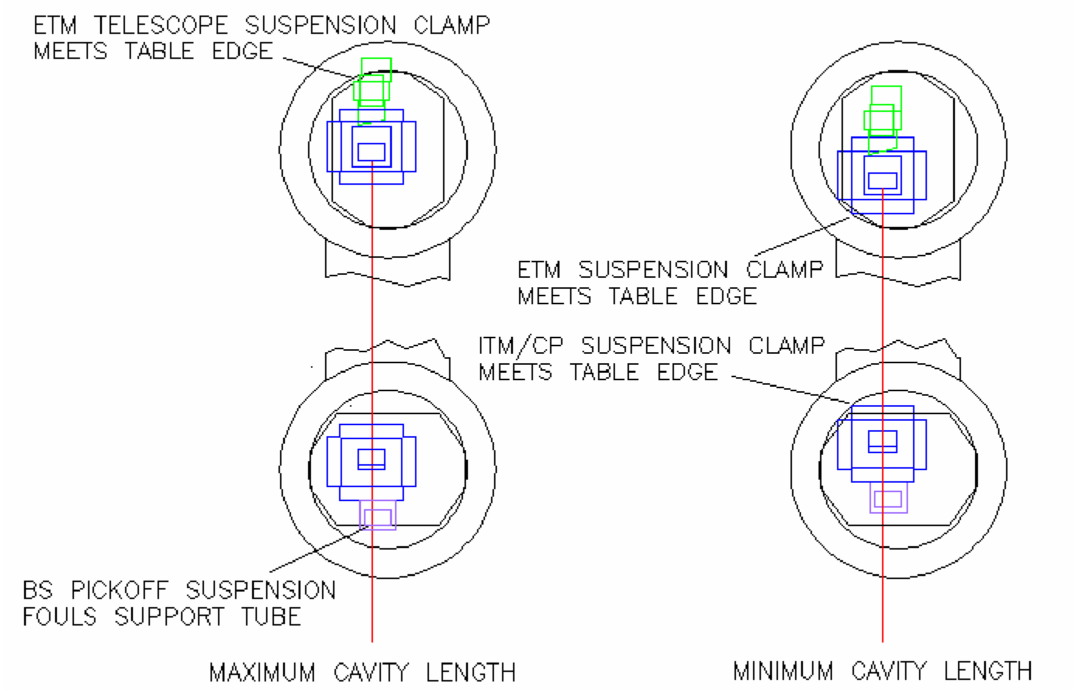


Figure 4. Maximum and Minimum Cavity Length Configurations.

Table 1. FP Cavity Lengths for Straight IFO

Max Length	3,995,033 mm
Min Length	3,994,455 mm
Average Length	3,994,744 mm
Recommended Length	3,994,750 mm

The y-arm FP Cavity is located by setting the center of the HR side of the ITM at (-200 mm, 4950 mm) in global coordinates, with the beam in the cavity parallel to the global y-axis. The provisional Schnupp Asymmetry is 200 mm, which determines the location of the x-arm FP cavity.

This sets the center of the HR side of the x-arm ITM at (4885.9mm, 200mm) in global coordinates, with the beam in the cavity parallel to the global x-axis.

If we want to change the asymmetry, we can move the x and y FP cavities in and out from the beam splitter. The maximum possible asymmetry is 344 mm. The minimum is 85 mm.

3.2 Folded FP Arm Cavity

The length of the FP arm cavity in the folded IFO is constrained similarly to that of the straight IFO, with the exception that the ITM/FM suspension fouls the hexagonal ring at its extreme locations. Table 2 gives the maximum, minimum, average, and recommended lengths for the folded FP cavity lengths.

Table 2. FP Cavity Lengths for Folded IFO

Max Length	3,994,721 mm
Min Length	3,994,206 mm
Average Length	3,994,463 mm
Recommended Length	3,994,450 mm

The y-arm folded interferometer FP Cavity is located by setting the center of the HR side of the ITM at (200 mm, 9842.04 mm) in global coordinates, with the beam in the cavity parallel to the global y-axis. The provisional Schnupp Asymmetry is 200 mm, which determines the location of the x-arm FP cavity and the BS. This sets the center of the HR side of the x-arm ITM at (9777.96 mm, -200 mm) in global coordinates, with the beam in the cavity parallel to the global x-axis. The center of the HR side of the BS is located at (9136.46 mm, 9173.10 mm).

Figure 5 shows this layout.

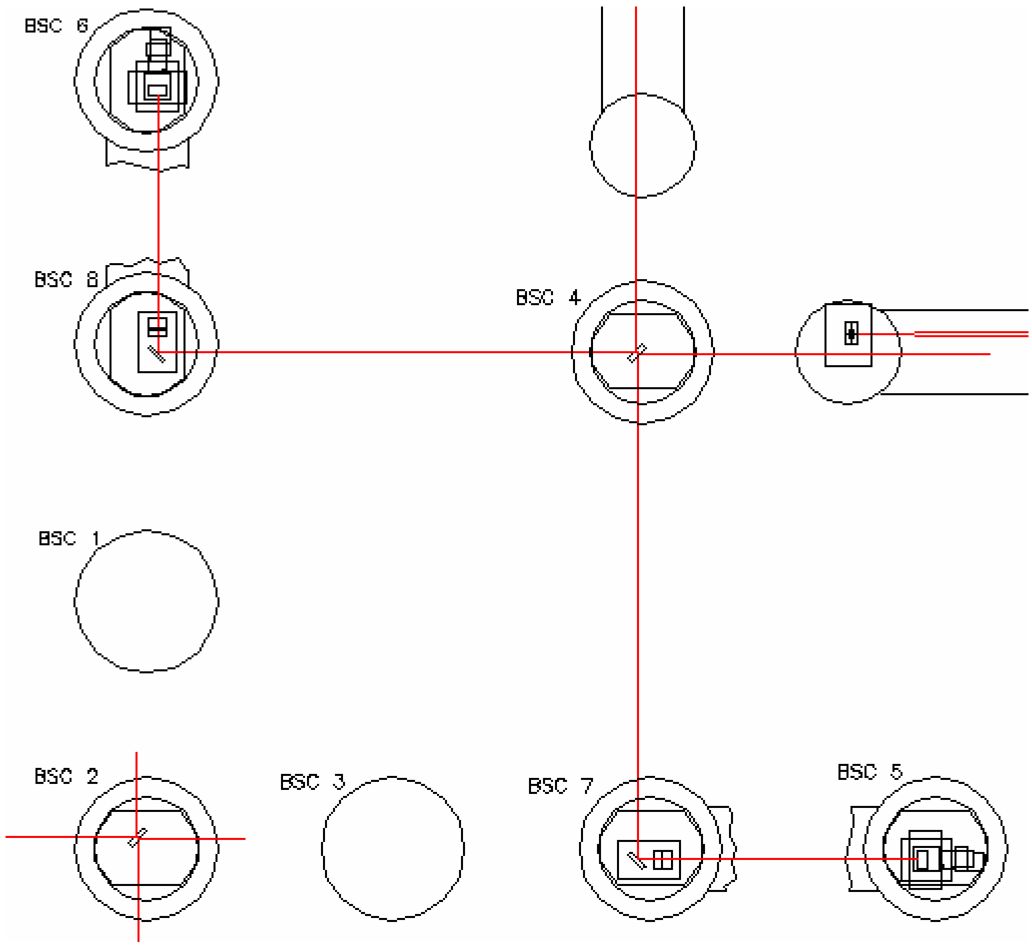


Figure 5. Folded IFO Layout

If we want to change the asymmetry, the maximum possible is 312 mm. The minimum is 23 mm. If we go to these extremes, it is possible that the BS suspension will be located outside of its range of motion. (The footprint of the BS suspension is not yet known in detail.)

4 Power Recycling Cavities

4.1 Marginally-Stable, Straight, Power Recycling Cavity

The y-arm of the PRC limits both the maximum and minimum lengths of the PRC. The maximum is limited by the interference between the ITMy frame and the BSC Support Rod. The minimum is limited by interference between ITMy clamp envelope and the Pickoff Mirror Suspension. The y-arm of the PRC is the short arm, so the average PRC length is determined by adding the Schnupp Asymmetry (A_S).

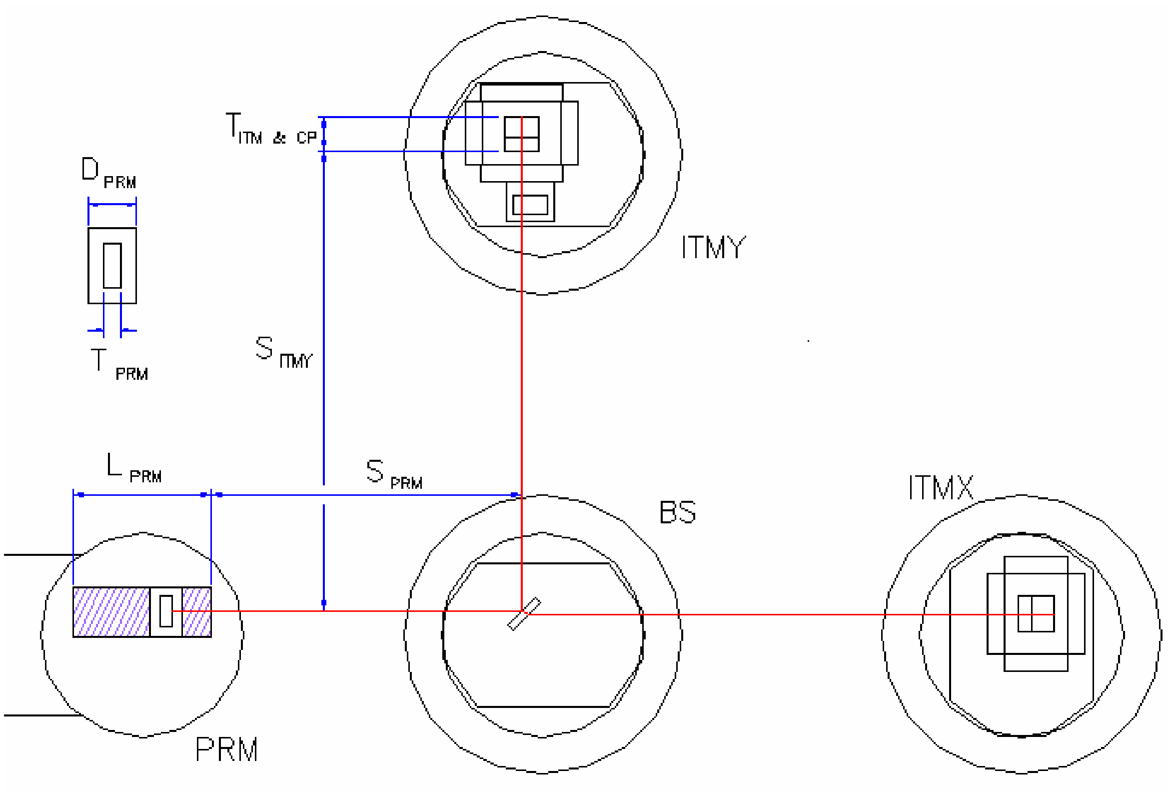


Figure 6. Marginally stable PRC, with symbols defined

With the definitions in Fig. 6, the dimensions for the Marginal PRC are:

L_{PRM} = 1320 mm
D_{PRM} = 300 mm
T_{PRM} = 100 mm
S_{PRM} = 2971 mm
S_{ITMY} = 4393 mm
$T_{ITM\ \&\ CP}$ = 330 mm
A_S = 200 mm
n = 1.44963

The maximum length it can have is

$$L_{MSPRC\ STRAIGHT\ MAX} = L_{PRM} - D_{PRM} + (D_{PRM} - T_{PRM})/2 + S_{PRM} + S_{ITMY} + n*T_{ITM\ \&\ CP} + A_S$$
$$= 9162 \text{ mm}$$

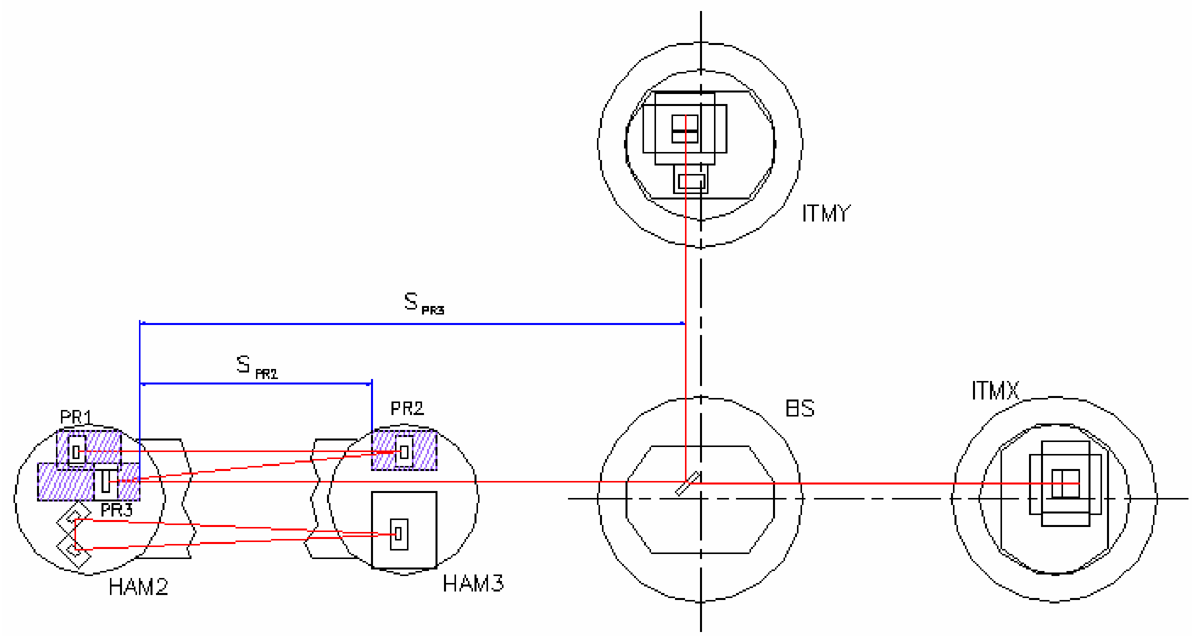
And the minimum is

$$L_{MSPRC\ STRAIGHT\ MIN} = (D_{PRM} - T_{PRM})/2 + S_{PRM} + S_{ITMY} + n*T_{ITM\ \&\ CP} + A_S$$
$$= 8142 \text{ mm}$$

The average length is

$$L_{MSPRC\ STRAIGHT\ AVE} = 8652 \text{ mm}$$

4.2 Stable, Straight, Power Recycling Cavity



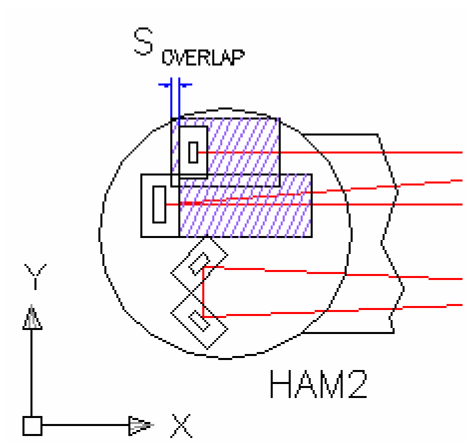


Figure 7. Stable PRC, with symbols defined

With the definitions in Fig. 7, the dimensions for the Stable PRC are:

S_{IMC} = 15,453 mm
$S_{OVERLAP}$ = 59 mm
S_{PR2} = 15,212 mm
S_{PR3} = 19,262 mm
S_{ANGLE} = 5 mm
L_{IMC} = 838 mm
S_{IMC} = 15,453 mm
D_{MCT} = 220 mm
T_{MCO} = 75 mm
L_{PRM} = 1320 mm
D_{PRM} = 300 mm
T_{PRM} = 100 mm
S_{PRM} = 2971 mm
S_{ITMY} = 4393 mm
$T_{ITM\,\&\,CP}$ = 330 mm
A_S = 200 mm
n = 1.44963

The term S_{ANGLE} accounts for the angled path between PR2 and PR3.

The maximum length it can have is

$$\begin{aligned} L_{SPRC\ STRAIGHT\ MAX} &= S_{IMC} + S_{PR2} + S_{PR3} - S_{OVERLAP} + S_{ANGLE} \\ &\quad + 3[\,L_{IMC} - D_{MCT} + (D_{MCT} - T_{MCO})/2] \\ &\quad + 2[\,L_{PRM} - D_{PRM} + (D_{PRM} - T_{PRM})/2] \\ &\quad + S_{ITMY} + n{*}T_{ITM\,\&\,CP} + A_S \\ &= 59{,}255 \text{ mm} \end{aligned}$$

And the minimum is

$$\begin{aligned} L_{SPRC\ STRAIGHT\ MIN} &= S_{IMC} + S_{PR2} + S_{PR3} + S_{OVERLAP} + S_{ANGLE} \\ &\quad + 1.5(D_{MCT} - T_{MCO}) + D_{PRM} - T_{PRM} \\ &\quad + S_{ITMY} + n{*}T_{ITM\,\&\,CP} + A_S \\ &= 55{,}480 \text{ mm} \end{aligned}$$

The average length is

$$L_{SPRC\ STRAIGHT\ AVE} = 57{,}367.5 \text{ mm}$$

While this is the median length for the stable PRC, layout considerations suggest that it is best to place PR3 toward the BS side of HAM2 to leave extra table space for the input optics. This would remove ~700 mm from the cavity length, giving a recommended cavity length of:

$$L_{SPRC\ STRAIGHT\ REC} = 56{,}700 \text{ mm}$$

4.3 Marginally-Stable Folded Power Recycling Cavity

A similar analysis gives for the marginal, folded interferometer the following:

$L_{MSPRC\ FOLDED\ MAX}$ = 14,337.5 mm

$L_{MSPRC\ FOLDED\ MIN}$ = 13,316.9 mm

$L_{MSPRC\ FOLDED\ AVE}$ = 13,827.2 mm

4.4 Stable Folded Power Recycling Cavity

The folded interfereometer allows the following for the power recycling cavitie:

$L_{SPRC\ FOLDED\ MAX}$ = 64,494.5 mm

$L_{SPRC\ FOLDED\ MIN}$ = 60,601.5 mm

$L_{SPRC\ FOLDED\ AVE}$ = 62,548 mm

While this is the median length for the stable PRC, layout considerations suggest that it is best to place PR3 toward the BS side of HAM8 to leave extra table space for the input optics. This would remove ~700mm from the cavity length, giving a recommended cavity length of:

$L_{SPRC\ FOLDED\ REC}$ = 61,850 mm

5 Mode Cleaner Cavity

The constraints on the IMC Cavity length are identical in all layouts.

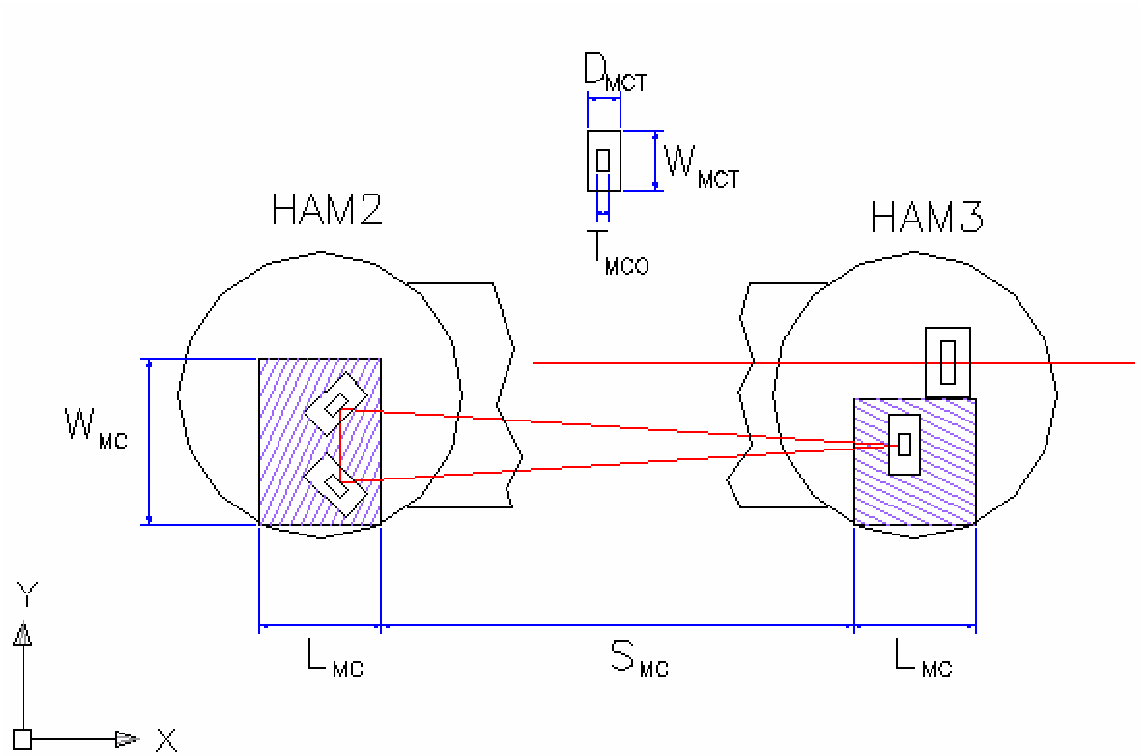


Figure 8. IMC Cavity Symbols

With the definitions in Fig. 8, the dimensions for the Marginal PRC are:

W_{IMC} = 1143 mm
L_{IMC} = 838 mm
S_{IMC} = 15,453 mm
D_{MCT} = 220 mm
W_{MCT} = 400 mm
T_{MCO} = 75 mm

$$L_{IMC\ MIN} = S_{IMC} + (D_{MCT} - T_{MCO})/2 + (2^{0.5} / 2)(W_{MCT} + D_{MCT} - T_{MCO})$$
$$= 15{,}910 \text{ mm}$$

$$L_{IMC\ MAX} = S_{IMC} + 2L_{IMC} + W_{IMC} / 2 - (D_{MCT} + T_{MCO})/2 - (2^{0.5} / 2)(W_{MCT} + D_{MCT} + T_{MCO})$$
$$= 17{,}061 \text{ mm}$$

$$L_{IMC\ AVE} = 16{,}485 \text{ mm}$$

Note: The MC triples are placed on ~3” risers to set the optics to the viewport height, in order to ease the removal of diagnostic beams. Now that all or almost all of the diagnostic beams will be collected in vacuum, there is no need to raise the MC triples. Lowering the MC triples to the table will alter the max and min IMC lengths, but not the average of the two.

6 Signal Recycling Cavity

The SRC length is from the ITM average to the SRC mirror, including the beam splitter refraction. The lengths are different for straight and folded interferometers.

6.1 Straight Interferometer, marginal SRC

Straight IFO SRC length 9626 mm max 8606 mm min

6.2 Folded Interferometer, marginal SRC

Folded IFO SRC length 14,238 mm max 13,218 mm min

7 Relationships Among Lengths

The values chosen for sideband frequencies must satisfy three equations; see section (1). The first gives the PRC frequency:

$$f_{prc} = (k + 1/2)c/2L_{prc} \qquad (1)$$

where (k = 0,1,2 ...) and L_{prc} is the power recycling cavity length. The second gives the input mode cleaner resonances:

$$f_{imc} = nc\,/2L_{imc} \qquad (2)$$

where n is an integer (1,2,3 ...) and L_{imc} the input mode cleaner length. The third gives the signal recyling cavity resonance:

$$f_{scc} = (p + \delta_\phi/2\pi)c/2L_{src} \qquad (3)$$

where p is an integer (1,2,3 ...), δ_ϕ is the signal recycling detuning (0.1 rad), and L_{src} is the SRC length.

There is a modest range of adjustments available for L_{prc} and L_{imc}, determined by the dimensions of the LIGO vacuum envelope, the size of the optical tables, and considerations of other articles that must reside on these tables. In turn, this determines a range of frequencies for both f_{prc} and f_{imc}. The union of these frequencies is available for the controls of the interferometer. The lengths are summarized in Table 3. (all dimension in m):

Table 3. Allowed range of lengths for LIGO cavities

Cavity	Configuration	Min	Max
IMC	All	15.910	16.461*
PRC	Straight, marginal	8.142	9.162
PRC	Straight, stable	55.480	59.255
SRC	Straight	8.606	9.626
AC	Straight	3994.455	3995.033
PRC	Folded, marginal	13.317	14.338
PRC	Folded, stable	60.602	64.495
SRC	Folded	13.321	14.238
AC	Folded	3994.206	3994.721

* The actual maximum length for the IMC is 0.6 m longer; however, this space is reserved for input injection optics. Using it for the mode cleaner would not affect any of the length issues (such as no 9 MHz frequency for the folded PRC) discussed below.

8 Modulation frequencies

The procedure we have used for fixing the modulation frequencies then begins by setting the arm cavity lengths. The length of the arm cavity for the straight (non-folded) interferometer is recommended to be 3994.750 m. The length for the folded interferometer is recommended to be 3994.450 m. Given these lengths, one can find a discrete set of frequencies spaced by 37.5 kHz that satisfy equations (7.3) and (7.4) and the nearly antiresonant condition for the sidebands with respect to the arm cavities. Finally, we can set the length of the SRC from equation (7.5).

The cases are discussed in the following sections.

8.1 Straight IFO, Marginal PRC

The length ranges for the IMC are 16.284-16.461 m. The length ranges for the PRC are 8.142-8.321 m. (This is the range for each where solutions exist.) A set of lengths (and corresponding frequencies) that meet the requirements is:

Table 4. Straight IFO, Marginal PRC

AC Length	m	3994.750
IMC Length	m	16.338
PRC Length	m	8.169
First frequency	MHz	9.1744
Second frequency	MHz	45.8722
SRC Length	m	9.855*
Alternative second frequency	MHz	64.2211
Alternative SRC length	m	9.373
High second frequency	MHz	183.4890
SRC length for high frequency	m	8.999

*This length is 229 mm *longer* than allowed by physical constraints in the SRC HAM.

One could use IMC and PRC lengths of 16.406 m and 8.203 m respectively also; this is the only other possibility if the low modulation frequency is to be near 9 MHz. The SRC is even f

8.2 Straight IFO, Stable PRC

The length ranges for the IMC are 15.910-16.461 m. The length ranges for the PRC are 55.685-57.614 m. A set of lengths (and corresponding frequencies) that meet the requirements are

Table 5. Straight IFO, Stable PRC

AC Length	m	3994.750
IMC Length	m	15.947
PRC Length	m	55.815
First frequency	MHz	9.3996
Second frequency	MHz	46.9979
SRC Length	m	9.619
Alternative second frequency	MHz	65.7971
Alternative SRC length	m	9.149
High second frequency	MHz	175.5922
SRC length for high frequency	m	9.246

8.3 Folded IFO, Marginal PRC

The length ranges for the IMC are 15.980-16.461 m. The length ranges for the PRC are 13.317-13.718 m. A set of lengths (and corresponding frequencies) that meet the requirements are

Table 6. Folded IFO, Marginal PRC

AC Length	m	3994.450
IMC Length	m	16.139
PRC Length	m	13.449
First frequency	MHz	27.8631
Second frequency	MHz	65.0140*
SRC Length	m	13.870
High second frequency	MHz	185.754
SRC length for high frequency	m	13.731

* 46.4386 MHz leads to a SRC length of 12.936 m, 255 mm too short.

The PRC is longer for the folded interferometer, making its first resonance be at 5.2-5.6 MHz and its second resonance be near 15.7-16.9 MHz. Neither frequency is consistent with the 9.26 MHz median input mode cleaner FSR; the first hit occurs for $n = 3$ and $k = 2$ in equations (7.3) and (7.4) above.

If the tube that links HAM2 and HAM 3 were increased in length by 1.8 m, then the input mode cleaner FSR is decreased to 8.33 MHz, and a frequency of 16.65 MHz could be used. (Note that this tube is scheduled for replacement in Advanced LIGO.)

8.4 Folded IFO, Stable PRC

The length ranges for the IMC are 16.160-17.061 m. The length ranges for the PRC are 60.602-63.979 m. A set of lengths (and corresponding frequencies) that meet the requirements are

Table 7. Folded IFO, Stable PRC

AC Length	m	3994.450
IMC Length	m	16.221
PRC Length	m	60.829
First frequency	MHz	18.4816
Second frequency	MHz	64.6857
SRC Length	m	13.941
High second frequency	MHz	175.575
SRC length for high frequency	m	8.551

If the tube joining HAM2 and HAM3 were increased in length by 1.8 m, a low frequency between 8.31 and 8.46 MHz could be employed.