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| Subject: | Indirect Length Coupling for the Beamsplitter with Horizontal Wedge |
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The baseline beamsplitter suspension design employs fused silica fiber suspensions and a vertical optical wedge. We have been considering a switch to a horizontal optical wedge for the beamsplitter in the hope that the reduced length coupling will permit a carbon steel wire suspension and thereby reduce complexity and cost.

To date when considering indirect coupling of suspended optic motion into optical cavity length (i.e. into the direction X, in figure 1), we have only considered vertical bounce modes (motion in direction Z)¹. As you recently pointed out the roll mode (rotation about X, R_x), and the side-to-side mode (motion in direction Y) can also cause a coupling into a cavity length change. In this memo, we outline the various coupling mechanisms and delineate those which are potentially important and those which are negligible.

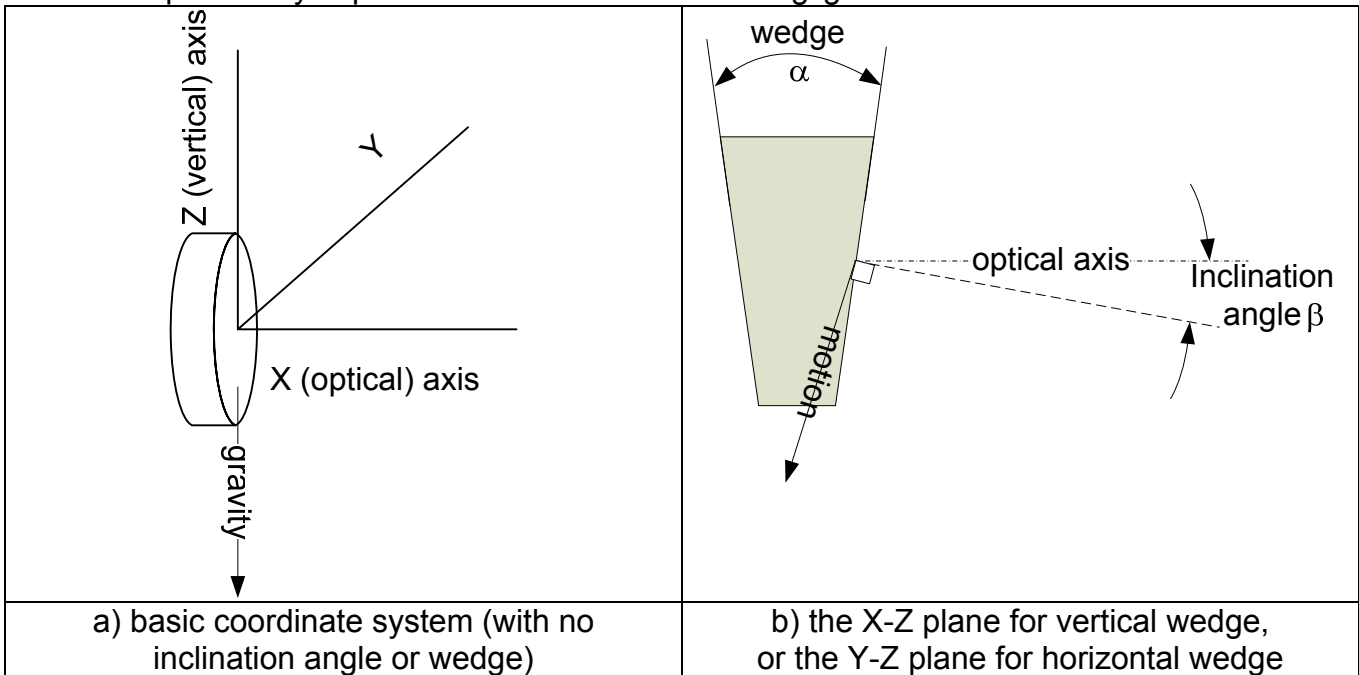


Figure 1: Coordinate System, wedge angle and inclination angle

¹ D. Coyne, "Optical Coupling of Vertical Displacement to Cavity Length", LIGO-[T040007-00](#), 21-Jan-2007.

The indirect length coupling mechanisms are summarized in table 1, where α is the wedge angle (and the subscript V indicates vertical and H indicates horizontal), β is the inclination angle of the optical axis to the axis of motion, δ is the optical beam decenter distance from the geometric center of the optic face, AR refers to the anti-reflection surface and HR to the high reflection surface (or the 50/50 surface in the case of the beamsplitter).

Table 1: Indirect Length Coupling Mechanisms/Parameters and Application to the Beamsplitter

| Motion | AR Face | HR Face | Frequency (Hz) | | Notes |
|-----------------------|------------|------------|----------------|-----------------------|---|
| | | | Steel Wire | FS Fiber ² | |
| Vertical, Z | α_V | β_V | 16.5 | ~11 | In band, of potential concern if the wedge is vertical; Nominally zero if the wedge is horizontal |
| Horizontal, Y | α_H | β_H | ~4 | ~4 | Negligible; controlled, low frequency. [The analysis of T040007 for vertical motion applies to this horizontal motion case as well.] |
| Pitch, R _Y | α_V | δ_Z | < 2 | < 2 | Negligible; controlled, low frequency |
| Yaw, R _Z | α_H | δ_Y | < 2 | < 2 | Negligible; controlled, low frequency |
| Roll, R _X | α^W | β^P | 23.5 | ~16 | Negligible; the decenter caused by the wedge and roll angle relative to the optic axis is too small relative to the beam size to cause significant coupling (see below) |

The vertical bounce mode coupling, which is of concern for the fused silica fiber suspension, although increased in frequency with a carbon steel wire suspension should have nominally zero length coupling with a horizontally wedged beamsplitter. (There is of course the precision is rotational alignment of the wedge when suspending the optic.)

The horizontal (Y) mode causes coupling to length via the horizontal wedge in the same manner that the vertical (Z) mode couples to length via the vertical wedge. However this mode has a low frequency (~ 4 Hz) and is well controlled and out of the sensing band. The pitch and yaw modes are likewise low in frequency, controlled and below the sensing band.

² N. Robertson, et. al., "Design of Beamsplitter Suspension for Advanced LIGO", LIGO-T040027-00, 9-Feb-2004, for frequencies for the beamsplitter suspension with fused silica ribbons. The steel wire suspension frequencies were recently calculated by Nora Robertson and will be published soon.

These modes also require a positional offset of the beam from the center of rotation of the mode (nominally the center of mass of the optic).

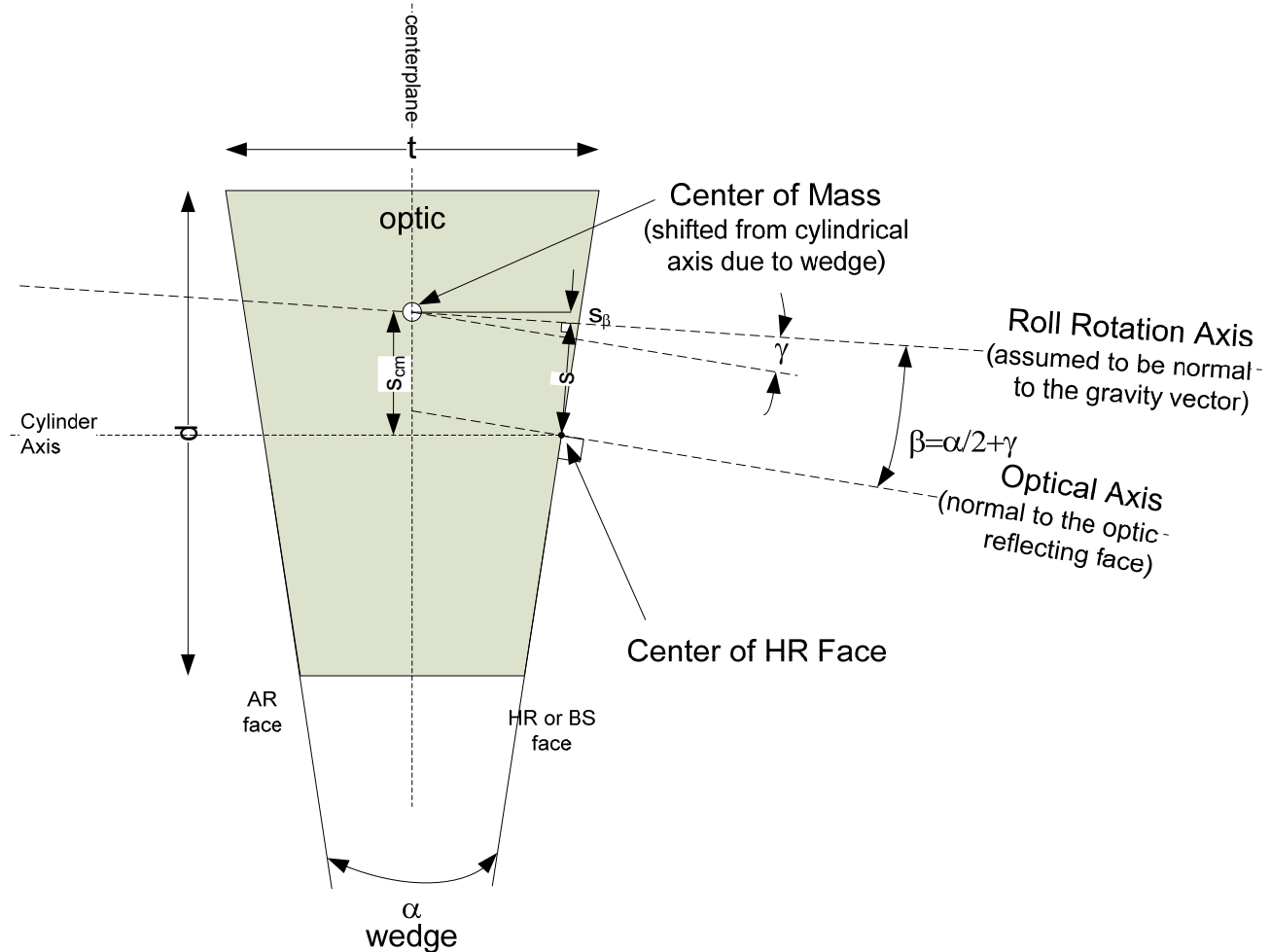


Figure 2. Wedged optic geometry.

The roll mode frequency is rather high, 16 Hz for fused silica fibers and 23.5 Hz for carbon steel wire. This is in the sensing band and so must be suppressed without injecting control noise. Just as for the pitch and yaw modes, the roll mode coupling requires an offset length. If roll mode coupling to sensed optical path length is a concern for a horizontally wedged beamsplitter, it may also be a concern for a vertically wedged beamsplitter.

Roll angle coupling is due to two offset lengths, denoted as S_β and S_{cm} in Figure 2. The shift in the center of mass position, S_{cm} , due to the symmetric wedge can be determined for the ungula as follows:

$$S_{cm} = \frac{\int_{-r}^{+r} \int_{-\sqrt{r^2-y^2}}^{+\sqrt{r^2-y^2}} y \left(\frac{\alpha y}{2} + \frac{t}{2} \right) dx dy}{\int_{-r}^{+r} \int_{-\sqrt{r^2-y^2}}^{+\sqrt{r^2-y^2}} \left(\frac{\alpha y}{2} + \frac{t}{2} \right) dx dy} = \frac{\alpha r^2}{4t}$$

The length offset due to the angle, β , between the optical axis and the roll axis, is approximately

$$s_{\beta} \approx \left(\frac{\alpha}{2} - \gamma \right) \frac{t}{2}$$

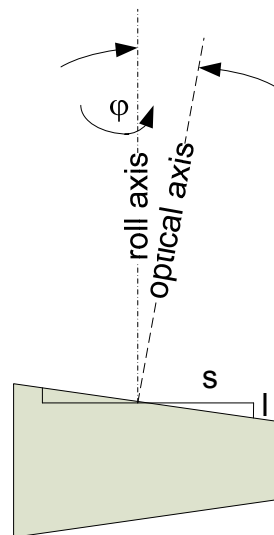
This is an approximate expression because the thickness is not precisely $t/2$ due to the wedge angle (but this is a small effect). Moreover, $S_{\beta} \ll S_{cm}$ for typical parameters of the beamsplitter.

The optical cavity length change, L , for an infinitesimally small beam, centered on the HR face, for a roll angle φ , is then

$$L = l \sin(\varphi - \pi / 2)$$

where

$$l = \frac{(s_{cm} - s_{\beta})}{\cos(\beta)}$$



With a small angle approximation

$$\frac{L}{\varphi} \approx s_{cm} - s_{\beta} \approx \frac{\alpha r^2}{4t}$$

The beamsplitter radius, $r = 185$ mm, thickness, $t = 64$ mm, inclination of the HR face to the local horizontal³, $\beta_v \approx 3.6$ mrad, and the wedge angle, $\alpha \approx 1$ deg (17 mrad). The shift in the center of mass position due to the wedge angle, $s_{cm} = 2.3$ mm. Consequently the reflected length to roll coupling for the beamsplitter is only $\sim 2.3 \times 10^{-3}$ m/rad, for an infinitesimal beam. For a finite sized beam some regions of the surface will move further away, as others move closer, and so the length coupling will be attenuated. The beam diameter for advanced LIGO at the beamsplitter is ~ 60 mm, so the 2.3 mm shift to the roll axis is only a slight perturbation and the coupling should be attenuated by surface averaging, perhaps on the order of $(2/30)^2$, or a factor of ~ 0.004 (a proper analysis is pending). If this is correct then the roll to length coupling might be expected to be $\sim 2.3 \times 10^{-3} \times 0.004 = 9 \times 10^{-6}$.

³ For the horizontal wedge cases wherein the the PRM is raised above the HAM3 table height and the beam does not come down to the nominal HAM optical beam height of -167 mm (in LIGO global coordinates) until HAM 2, the slope is 3.6 mrad. If the beam is forced to come to -167 mm in HAM 3, then the slope is 10 mrad.

The beamsplitter suspension roll transfer function (Figure 3) peak at the highest mode is approx 1.9 (assuming the phi of wire is 2×10^{-4} , giving Q of 5×10^3). Frequency is 23.3 Hz. Assuming that the residual roll motion of the ISI table is 2×10^{-13} rad/rt Hz (This is a conservative estimate based on meeting the 10 Hz displacement noise target across the 1m span of the table. By 23 Hz the noise level on the table should be less than this value.) Then the residual roll noise is $1.9 \times 2 \times 10^{-13} = 3.8 \times 10^{-13}$ rad/rt Hz.

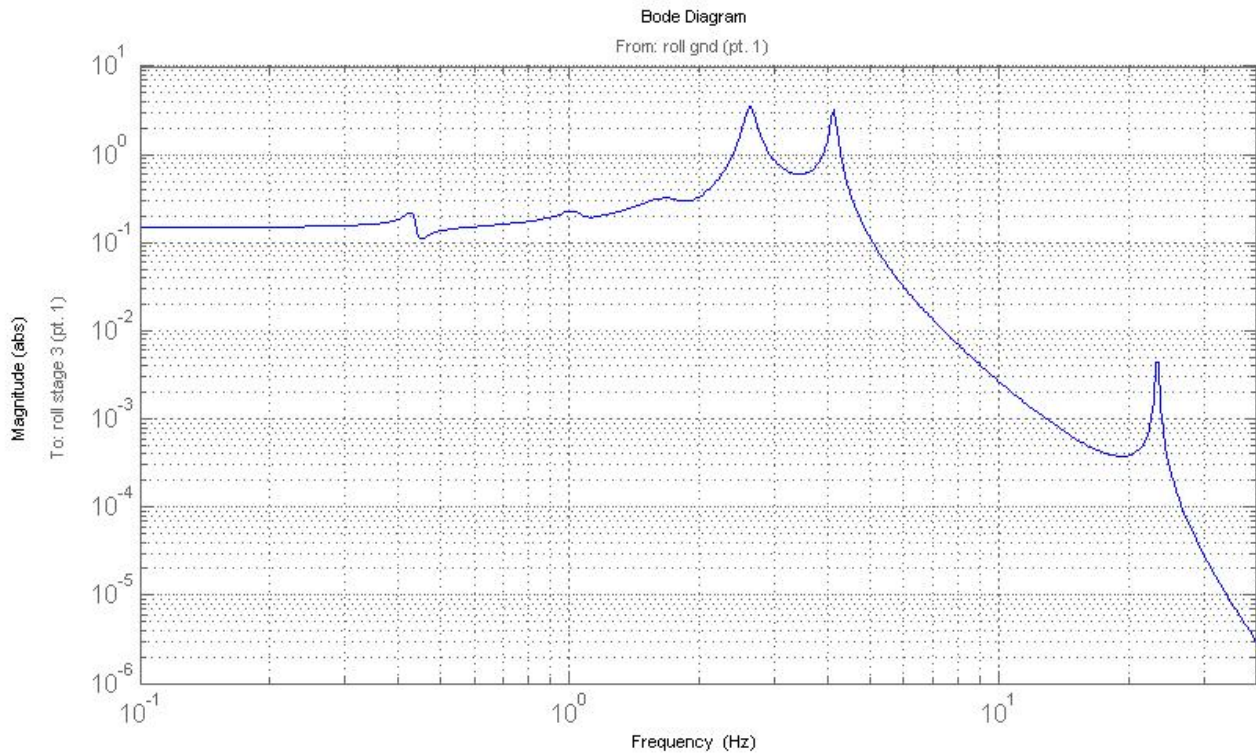


Figure 3. Roll Transfer Function

With the estimated roll to length coupling for a 6 cm beam, 9×10^{-6} , the longitudinal motion due to the roll peak is $3.8 \times 10^{-13} \times 9 \times 10^{-6} = 3 \times 10^{-18}$ m/rt Hz.

For comparison the sensitivity in h for Adv. LIGO at ~ 20 Hz looks to be approx 5×10^{-23} per rt Hz. Multiplying that by 2000 gives 10^{-19} m/rt Hz for motion of each test mass. To compare Michelson (beamsplitter) displacement to test mass displacement one needs to reduce the former by the cavity gain factor, about 800. Thus our BS roll peak would correspond to $3 \times 10^{-18} / 800 = 4 \times 10^{-21}$, or well below the TM motion.

This is preliminary and requires a more careful analysis of the roll coupling (we don't believe it yet.) Note also that the above analysis is for the reflected path and not for the difference in reflected and refracted paths (which is of course what is of most interest). Need to compare this analysis to measured coupling in the Initial LIGO interferometer (TBD).

