LIGO Laboratory / LIGO Scientific Collaboration

LIGO-T070121-00-D *Advanced LIGO* 2 June 2007

Effect of Attachment Compliance on the Bending Frequencies of Optics Table Payload Structures

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Distribution of this document:
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1 Introduction

The payload structures (for example suspension structures) that are placed on the seismically isolated optics tables in Advanced LIGO must have a relatively high first resonance. The gain and phase perturbations to the seismic isolation system transfer functions due to the addition of the payload must not destabilize the active seismic isolation system control or cause a significant increase in the control law complexity, with a concomitant decrease in robustness. The desired minimum first resonance of payload structures[1,2] is 150 Hz.

The effect of the bolted attachment compliance, at the interface of the optics table and structure base, was considered[3] in establishing the required minimum number of bolted attachments and the diameter and spacing of the optics table's array of holes used for attaching payload. This consideration of the bolted attachment was based upon the frequency for a rigid (infinitely stiff) structure where all of the elastic deflection was in the bolts and dog clamps. The coupled dynamics analysis[2] of the quadruple pendulum and the BSC seismic isolation table considered the elastic interaction of the quadruple suspension structure and the seismic platform, but did not include the compliance of the bolted attachments.

A recent study[4], has examined the effect of the elastic compliance of a massive base to which the structure is attached, as is typical in stand-alone modal testing of the structure. This finite element study concluded that the compliance of massive steel blocks was insignificant. However this study did not include the effect of the compliance of the bolted/dogged connection.

In this report a simple beam model is proposed and then used to re-confirm the assertion that the compliance of a massive block (to which the structure is rigidly attached) is insignificant. This same model is then used to show that bolt/dog connections cause a significant reduction in the first elastic mode resonance.

2 Beam Model

Many of the large payload structures attached to the optics tables have high length-to-width and length-to-depth ratios and so are reasonably well approximated as a beam cantilevered from the table (Figure 1). The bending frequencies, f_i, of a cantilevered beam[5] are given by:

$$f_i = \frac{\lambda_i^2}{2\pi L}\sqrt{\frac{E_b I}{m}}$$

Where the eigenvalues, λ_i = 1.875, 4.694, 7.866, …, L is the beam length, I is the beam cross sectional moment of inertia, E_b is the elastic modulus of the beam and m is the lineal mass density of the beam. The effective beam parameters (Table 1) of an early representative quadruple

[1] D. Coyne, "Coupled Dynamics of Payload Structures on the Seismically Isolated Optics Table", G050427-01.

[2] D. Coyne, "Coupled Dynamics Analysis of the Seismic Isolation System (SEI) Stage-2 Structure and the Quadruple Suspension (SUS) Structure", T050014-00.

[3] D. Coyne, "Optics Table Hole Size and Spacing based on Suspension Attachment Frequencies", T050098-00.

[4] T. Hayler, J. Greenhalgh, "Effect of steel base blocks on frequency measurements on BS structure", T070117-01.

[5] R. Blevins, Formulas for Natural Frequency and Mode Shape, Krieger Publishing Co., 2001, pp.166-168.

pendulum structure[6] yield a first frequency of f_1 = 60 Hz, which is quite close to the first bending mode used in Ref. [4].

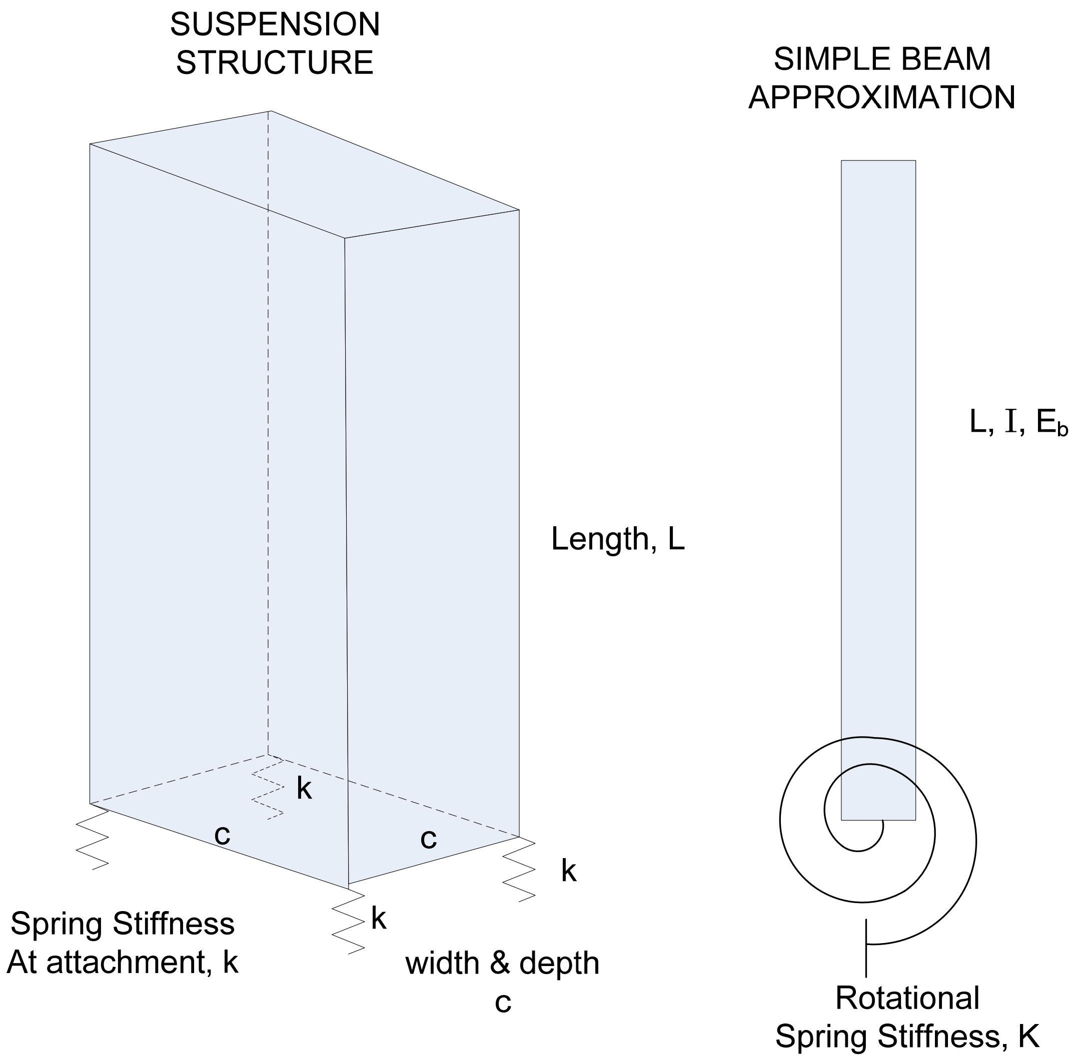

Figure 1: Simple beam model of the payload structure

Table 1: Effective beam parameters representative BSC chamber payload

Value	Symbol	Units	Description
69	Eb	GPa	Elastic modulus of 6061 aluminum
$1.55\ 10^{-4}$	I	m^4	Moment of inertia
2	L	m	Beam length
56.9	m	kg/m	Beam lineal density
0.5	c	m	Beam width and depth

[6] D. Coyne, “Frequency Analysis of the Quadruple Pendulum Structure”, T030044-03.

The attachment compliance can be represented as a discrete elastic rotational spring at the base of the beam. The eigenvalues for the bending frequencies with this boundary condition[7] are dependent upon the non-dimensional parameter $\frac{KL}{E_b I}$ and are given in Table 2, where K is the rotational stiffness at the end of the beam.

Table 2: Eigenvalues for the bending frequencies of a beam with one end spring-hinged and the other free (Ref. [7])

$\frac{KL}{E_b I}$	λ_i					
	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
0	0	3.926 591	7.068 581	10.210 089	13.351 763	16.493 357
0.01	0.415 932	3.927 805	7.069 291	10.210 666	13.352 143	16.493 664
0.1	0.735 782	3.938 466	7.075 616	10.215 046	13.355 499	16.496 383
1	1.247 917	4.031 139	7.134 132	10.256 621	13.387 756	16.522 725
10	1.722 742	4.399 523	7.451 057	10.521 785	13.614 188	16.719 626
100	1.856 787	4.649 726	7.782 671	10.897 588	14.014 861	17.133 516
∞	1.875 104	4.694 091	7.854 757	10.995 541	14.137 168	17.278 760

From the values in Table 2 one can calculate the reduction in first bending mode frequency as a function of the non-dimensional parameter that accounts for the compliance at the attachment. This variation is shown in Figure 2.

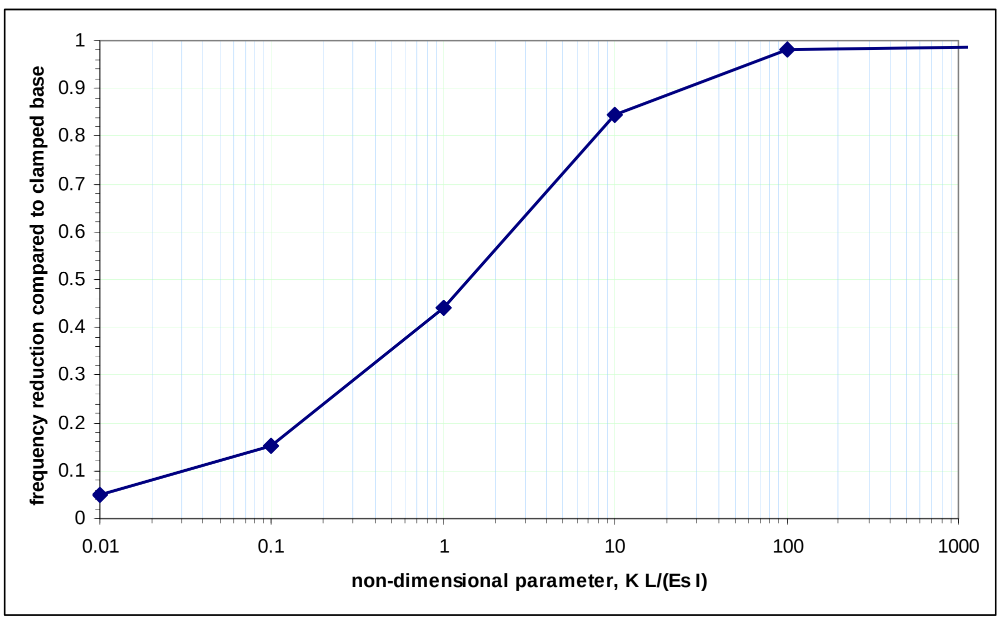

Figure 2: First bending frequency reduction with rotational compliance at the attachment

[7] K. Chun, "Free Vibration of a Beam with one end spring-hinged and the other free", J. of Applied Mechanics, vol. 39, pp. 1154-1155, 1972.

If (as in Ref. [4]) we assume that 4 discrete translational springs, each with spring constant k, represent the attachment at the 4 corners of the base of the structure, then the effective rotational stiffness is given by:

$$K = kc^2$$

where c is width (or depth) of the beam.

3 Attachment to an elastic half-space

To represent the situation where the payload structure is firmly mounted to a massive and very stiff support (as is typical during stand alone modal testing), consider the support as an elastic half space. The solution for the deformation due to a force at the boundary of a semi-infinite elastic body was given by J. Boussinesq originally and is recited in Timoshenko & Goodier[8]. The maximum deflection, w_{max}, due to a pressure, q, acting over a circular region of radius, a, is:

$$w_{\max} = \frac{2(1-\nu^2)qa}{E_s} = \frac{2(1-\nu^2)F}{\pi a E_s}$$

where ν is the Poisson's ratio, and E_s the elastic modulus, of the elastic half space material. Consequently the effective spring constant is:

$$k = \frac{F}{w_{\max}} = \frac{\pi a E_s}{2(1-\nu^2)}$$

and therefore

$$K = \frac{\pi a c^2 E_s}{2(1-\nu^2)}$$

Using E_s and ν values appropriate for steel (193 GPa and 0.3 respectively), and an attachment radius of a = 10 mm, the translation stiffness at each attachment is k = 3.3 GN/m, the total rotational stiffness is K = 0.83 GN/rad and the non-dimensional parameter $\frac{KL}{E_b I}$ = 156. From Figure 2, for this value we expect less than a 2% reduction in the bending frequency, which is roughly consistent with the observations in Ref.[4].

4 Attachment with a bolted dog clamp

From Ref. [3] the effective stiffness of the original bolted dog clamp (D050150-02) with a ¼-20 bolt is k = 2.98 10^6 N/m (or 1.7 10^4 lbf/in). This is 3 orders of magnitude more compliant than the deformation associated a steel half space. The non-dimensional parameter $\frac{KL}{E_b I}$ = 0.14 and the first bending frequency is reduced about 80%.

The revised dog clamp design reported in Ref. [3] attachment, with 3/8 bolting, has an effective stiffness of k = 9.75 10^6 N/m. In order to insure that the quasi-rigid body modes are over 150 Hz,

[8] S. Timoshenko, N. Goodier, Theory of Elasticity, McGraw Hill, 2nd ed., 1951, p. 368.

11 of these clamps are required. With just 4 clamps at the corners of the structure, $\frac{KL}{E_b I}$ = 0.46 and the first bending frequency is reduced about 65%.

Consider a clamping arrangement where 4 clamps are used on each side of the perimeter of the base of the structure, as depicted in Figure 3. In this case the rotational stiffness is

$$K = 2.4kc^2$$

or 2.4 times stiffer than for the case with 4 clamps at each corner. In this case $\frac{KL}{E_b I}$ = 1.1 and the first bending frequency is reduced about 50%.

5 Conclusion

All of the dog clamp/bolt attachment cases considered are for a worse positioning (most compliant) of the bolt in the dog clamp as indicated in Figure 4 (taken from Ref. [3]). Nonetheless this analysis indicates:

- potential for significant bending frequency reduction with inadequate bolting
- provides a formulation that should provide an approximate estimate of bending frequency reduction due to attachment compliance, and
- there is a need for improving the dog clamp/bolt stiffness in order to optimize the stiffness of this interface arrangement.

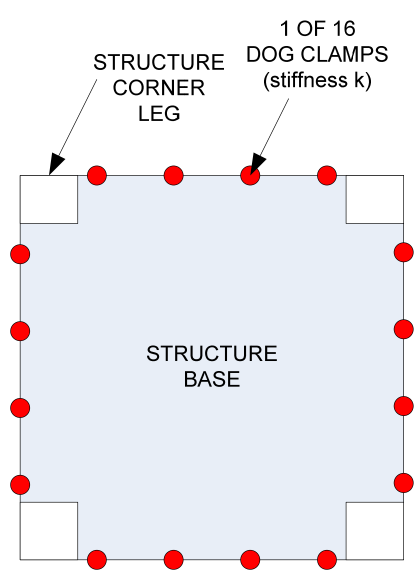

Figure 3: Geometry of dog clamps around the base of the structure

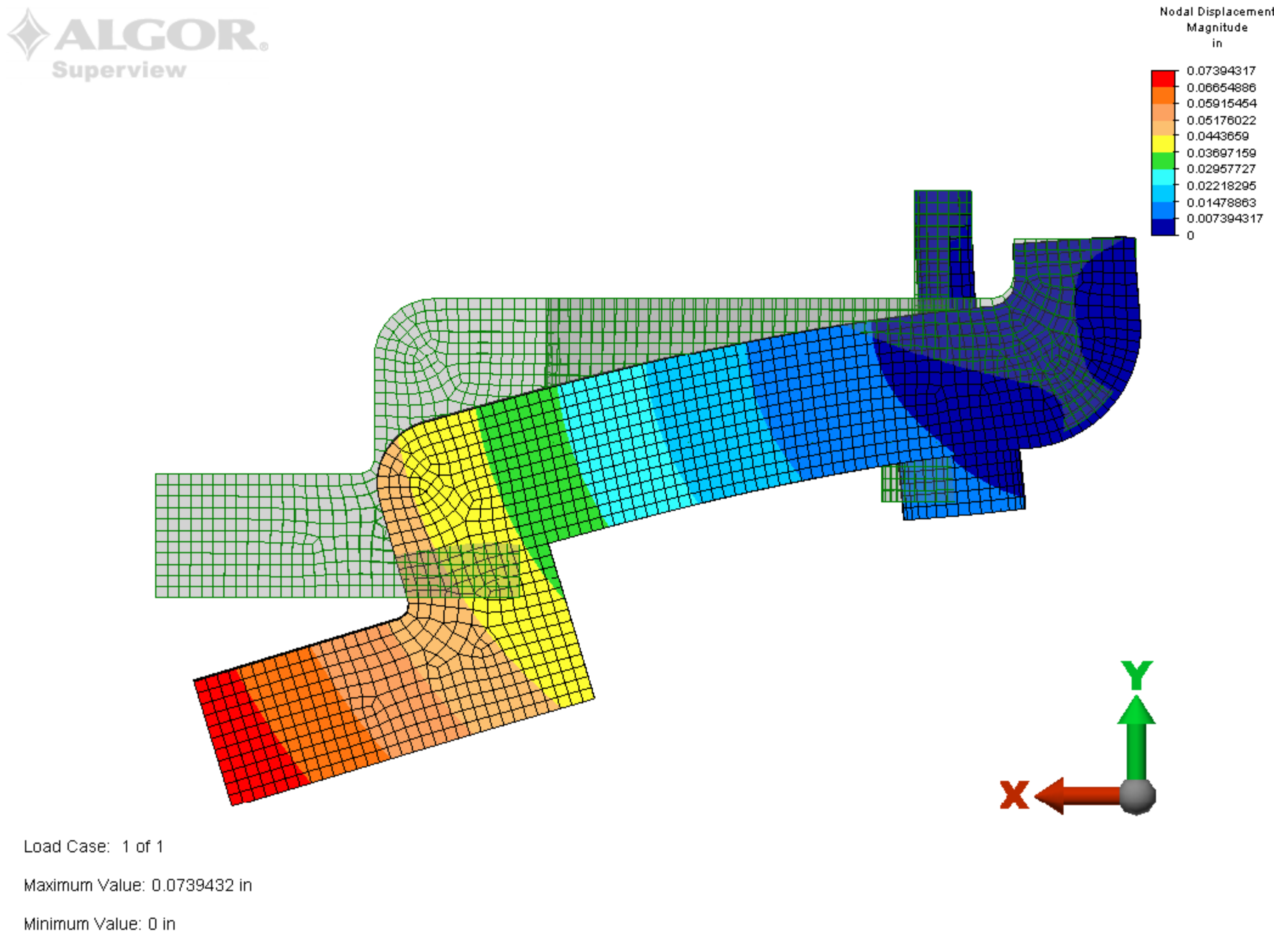

Figure 4: Finite element analysis of the stiffness of a dog clamp. Note the worst case positioning of the bolt.