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A Note on Substrate Thermal Lensing in Mode Cleaner

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1 Introduction

1.1 Purpose

This document describes the thermal aberration in Mode Cleaner (MC) Mirrors due to coating absorption and substrate absorption. Associated thermal lensing is calculated and its effect on the mode matching into main IFO is inspected. A scenario is discussed where the thermal aberrations in the MC mirrors can help in reducing the lensing in Faraday Isolator (FI).

1.2 Scope

This document is prepared for the purpose of introducing thermal aberrations in MC and its impact on FI and overall mode matching. Typical readers of this include people involved in designing core optics, alignment sensing, and material scientists.

1.3 Definitions

Thermo-optic Coefficient: Quantitative measure of the change in refractive index with temperature, represented by dn/dT.

α_t: Thermal expansion coefficient.

1.4 Acronyms

TOC: Thermo-optic coefficient

ROC: Radius of curvature

HV Theory: Hello-Vinet Theory

1.4.1 LIGO Documents

1.4.2 Non-LIGO Documents

2 General description

Absorption in coatings and the substrates of the mode cleaner mirrors will lead to changes in the effective radii of curvatures, changing and distorting the spatial eigenmode of the mode cleaner. The heating in the coating will change the sagitta δs across the beam profile of the mode cleaner mirrors. This change can be approximated as follows:

$$\delta s = \alpha P_a / 4\pi\kappa$$

where α (=0.55 ×10^{-6} m^{-1}) is the thermal expansion coefficient, κ(= 1.381 Wm^{-1}K^{-1}) is the heat conductivity and *Pa* is the absorbed power. The Parameters for the MC cavity are as follows:

Table 1: Optical and Geometrical Parameter for the MC Cavity

Mode Cleaner Length	m	16.338
MC free spectral range	Hz	8,987,751
MC polarization		's'
MC1 radius of curvature	m	>10000
MC1 transmittance		0.0062
MC1 reflectance		0.9938
MC2 radius of curvature	m	26.769±0.025
MC2 transmittance		<0.00001
MC2 reflectance		0.9999
MC3 radius of curvature	m	>10000
MC3 reflectance		0.9938
MC3 transmittance		0.0062
Mirror absorption/scatter loss (each)	ppm	50
MC finesse		500
Cavity Pole Frequency	Hz	17,900
Cavity g factor		0.3897
MC waist	mm	2.102
Rayleigh range	m	12.44
Input Power	W	165
Stored MC Power	kW	26
Intensity at Flat mirrors	kW/cm^2	200
MC mirror mass	kg	2.92
MC mirror diameter	cm	15

MC mirror thickness	cm	7.5
MC1,3 HR center-center distance	cm	43.18
MC1,3 intracavity angle of incidence, θ	deg	44.625
MC2 angle of incidence	deg	0.76

Thermal lensing due to coating absorption on the high reflecting surface and bulk power absorption in the substrate can be modeled analytically through Hello-Vinet (HV) theory.[4,5] The HV theory provides an expression for the thermal aberrations in the substrate due to the two absorption mechanisms. Similarly the surface deformation due to the coating absorption in the HR side of the MC mirrors is calculated.

3 Thermal Lensing in MC Mirror Surface (Reflective Lens)

The surface of the MC mirrors may have a nominal coating absorption of 0.5 ppm. The circulated power in MC cavity is 26 kW so 13 mW power will be absorbed in the mode cleaner mirror coating. This absorbed power will create a surface deformation as shown in Fig. 1. Using H-V theory, the surface deformation is found to be of the order of 6.2 nm sagitta change for a beam of 3.4 mm. This corresponds to a thermal ROC of 10 km. As the thermal aberrations plotted in Fig. 1 as a blue line are not exactly quadratic, so some higher order losses are expected. Using overlap integral of a parabolic 10 km lens profile with the thermally aberrated mirror surface as shown in Fig. 1, shows that the higher order losses are expected to be less than 10 ppm. Hence the higher order losses are clearly not a problem in the MC cavity.

Similarly for MC_1 and MC_3 mirrors, the thermal aberrations are plotted in Fig. 2. Here the optimal ROC of the thermal aberrations is 4 km. The difference is due to the smaller beam size at these mirrors. Here again the higher order losses are still 10 ppm. The sagitta change for a beam waist of 2.05 mm is 0.52 nm.

The curved mirror ROC will change from 26.769 m to 26.8409 m due to the aforementioned 10 km lens. However, lumping the effect of 4 km thermal lens at the flat mirror into the curved mirror will see an effective change from 26.769 m to 26.9116 ROC for MC_2. Therefore, for modeling MC mirrors, the hot value of MC_2 mirror is 26.9116 m.

However, apart from these higher order losses, there are other sources of mode miss-match as compared to the nominal mode of the MC to which MMT is designed. The other sources of error could be:

1. Beam Waist change due to change of MC mirrors ROC due to manufacturing tolerances
2. Beam Waist change due to change of MC mirrors ROC due to thermal lensing

Note that the symmetrical nature of the MC ring cavity and the fact that the MC_1 and MC_3 have similar absorption properties and power, will ensure that the beam waist location remains at its nominal value. However, beam waist size will change depending upon primarily the ROC of MC_2 mirror. The nominal value of MC_2 ROC R_2 is 26.769 m and one way length is 16.338. This provides a beam waist of 2.1029 mm at a distance of 0.2159 m from MC_3 inside the cavity. The ROC of MC2 is similar to MMT_2 mirror so a similar tolerance of ±0.025m can be assigned. This corresponds to a ±27 km lens. Since the thermal ROC change is expected to be 10 km so the dominant loss mechanism is the thermal lensing. The assumed tolerance is a minor factor in the over all mode mismatch. Later it will be shown that even the surface thermal lensing in the curved mirror produces very little losses and hence can be neglected.

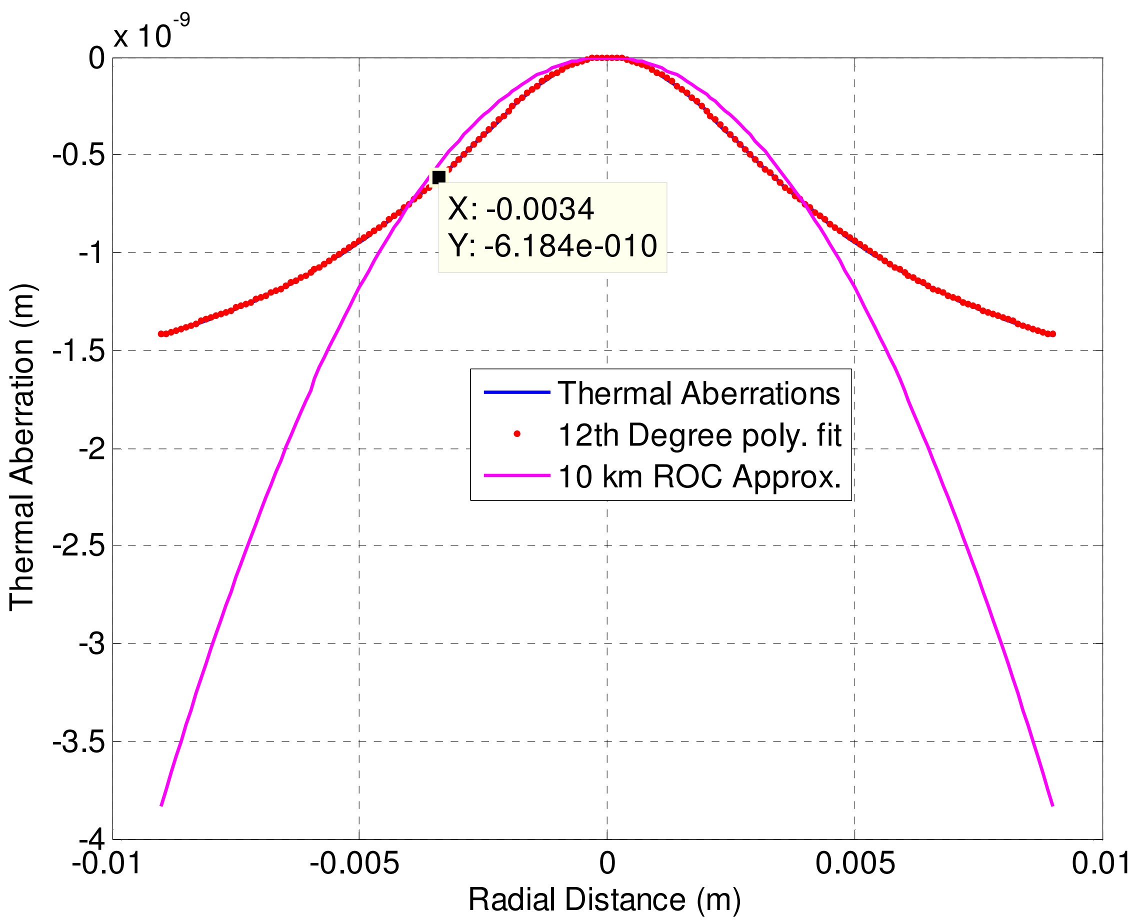

Fig. 1: Thermal lensing at surface for 0.5 ppm coating absorption at 26 kW and 10 ppm/cm bulk absorption for MC_2.

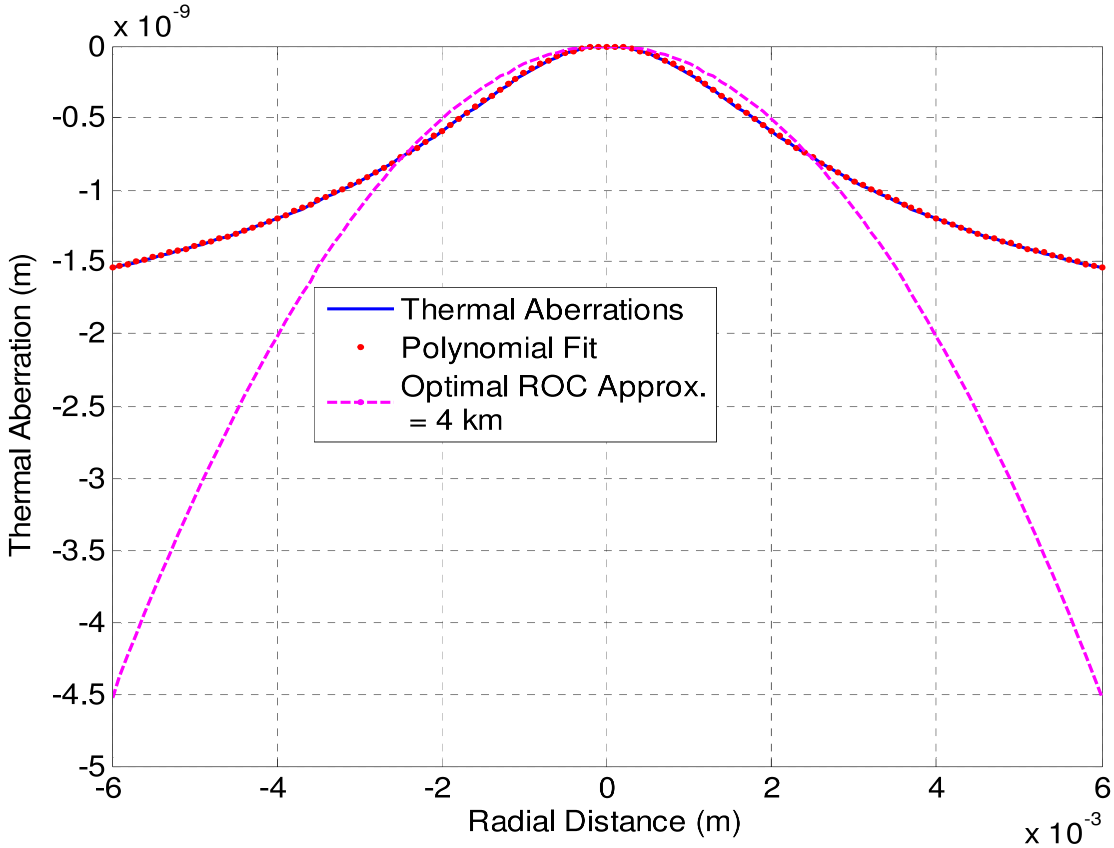

Fig. 2: Thermal lensing at surface for 0.5 ppm coating absorption at 26 kW for MC_1 and MC_3.

The beam size inside the MC may change due to the thermal lens at both MC_2 and $MC_3(MC_1)$. The effect of ROC change at MC_3 can be modeled by combining MC_2 and MC_3 mirror using thin lens approximation and defining an equivalent ROC at MC_2. Then the standard formula for the beam waist inside the cavity can be applied as: $\omega_0^2 = \frac{L\lambda}{\pi}\sqrt{\frac{g}{1-g}}$

where L is the length of the cavity as calculated for the equivalent MC_2 ROC R_2 and g is given by $(1-L/R_2)$. Using this combination, the beam size and the mode mismatch due to changing MC_2 ROC is plotted in Fig. 3. Note that the loss includes loss due to MC_2 and MC_3 surface thermal lens. Fig. 3 along with earlier calculations shows that a 20 ppm budget for mode-matching loss is sufficient for all ROC errors due to surface thermal lensing in the mode cleaner mirror.

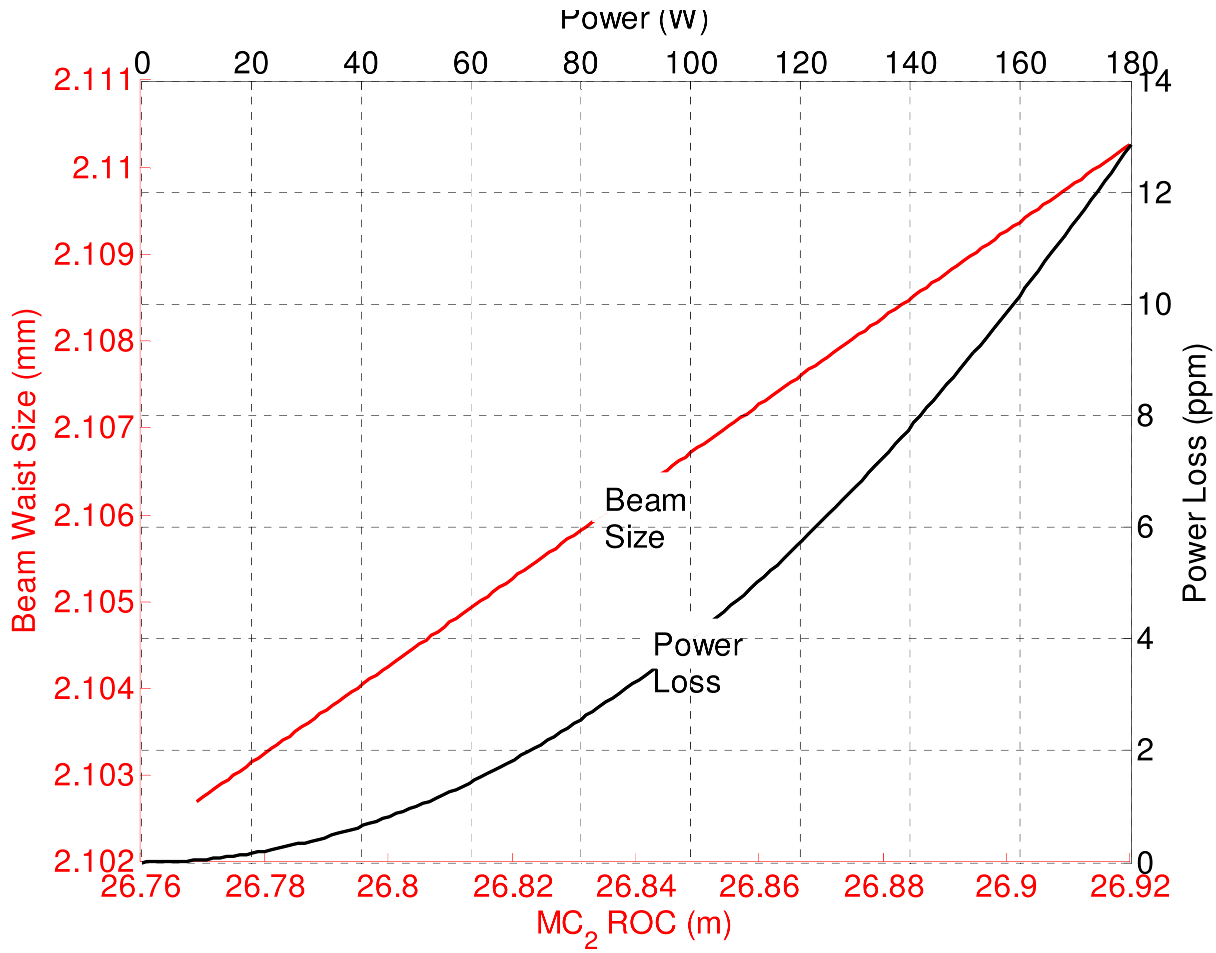

Fig. 3: Beam waist size variation due to ROC change of MC_2 plotted on left y-axis. The corresponding power loss in the mode matching is plotted on right y-axis. The top x-axis shows power input to the MC.

4 Thermal Lensing in MC Mirror Substrate (Transmissive Lens) Vertical Direction

The outgoing beam passing through MC_3 will experience a positive thermal lens inside the substrate. This lens also includes a small contribution from the surface thermal lens when the bam passes through it. The main contributing factor to the substrate thermal lens is coating absorption. At 0.5 ppm coating absorption, 13 mW heat is absorbed however the substrate absorption is 10 ppm/cm that translates into 12mW of heat in the substrate. The combined thermal lens due to coating and substrate absorption for MC_3 is shown in Fig. 4. The optimal spherical fit gives an estimate of about 135 km or 0.0148 diopter.

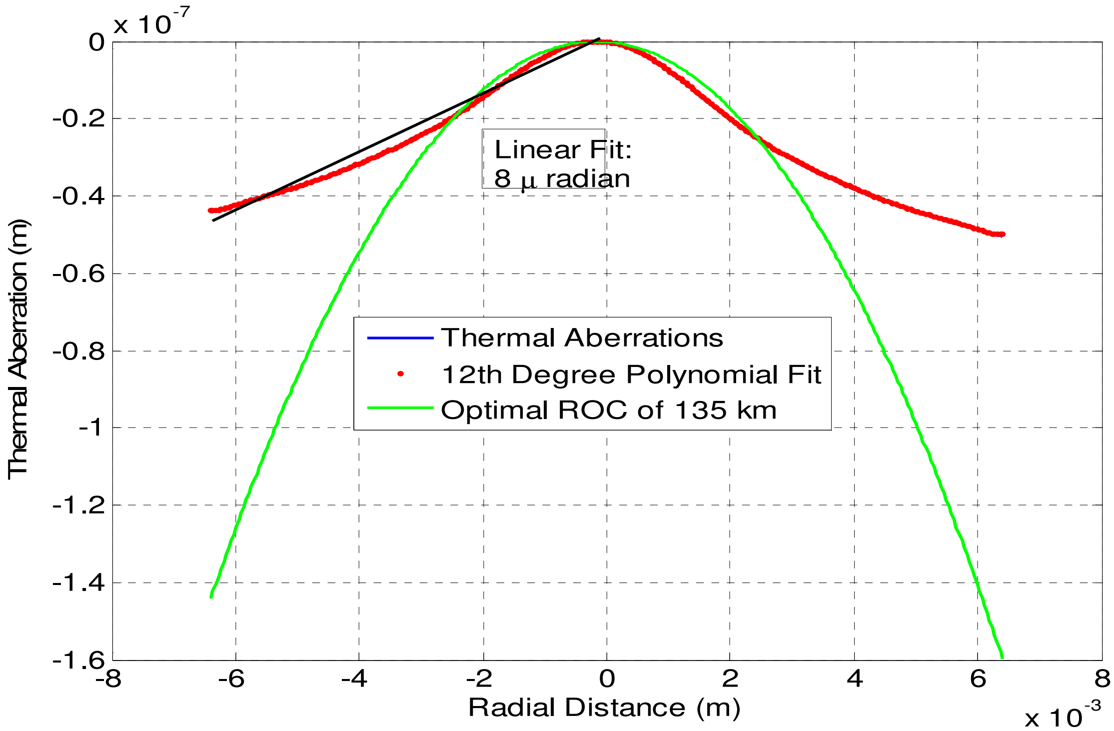

Fig. 4: Thermal lensing in ITM substrate for 0.5 ppm coating absorption at 26 kW and 10 ppm/cm bulk absorption for MC_2.

Note that the shape of the thermal aberrations is tilted as well as slightly off-centered. This is due to the fact that the coating thermal lens and the substrate thermal lens are not in-line. The effect due to the coating absorption produces a lens but because of the 45^0 incidence angle at MC_3, the beam experiences a slightly tilted and off-centered lens. The losses due to this are expected to be 0.03% calculated by taking the over-lap integral with an ideal Gaussian beam transmitted through the optimal spherical lens. Apart from this loss, there will be some power-dependent steering of the beam. This can be calculated to first order by the slope of the thermal aberration as shown in Fig. 4. here the slope of the line is 8 μrad at full power. This can be corrected by the alignment control.

The effect of tilted coating thermal lens is more clearly shown in Fig. 5 that shows the temperature profile of the substrate where we can see the off-centered profile.

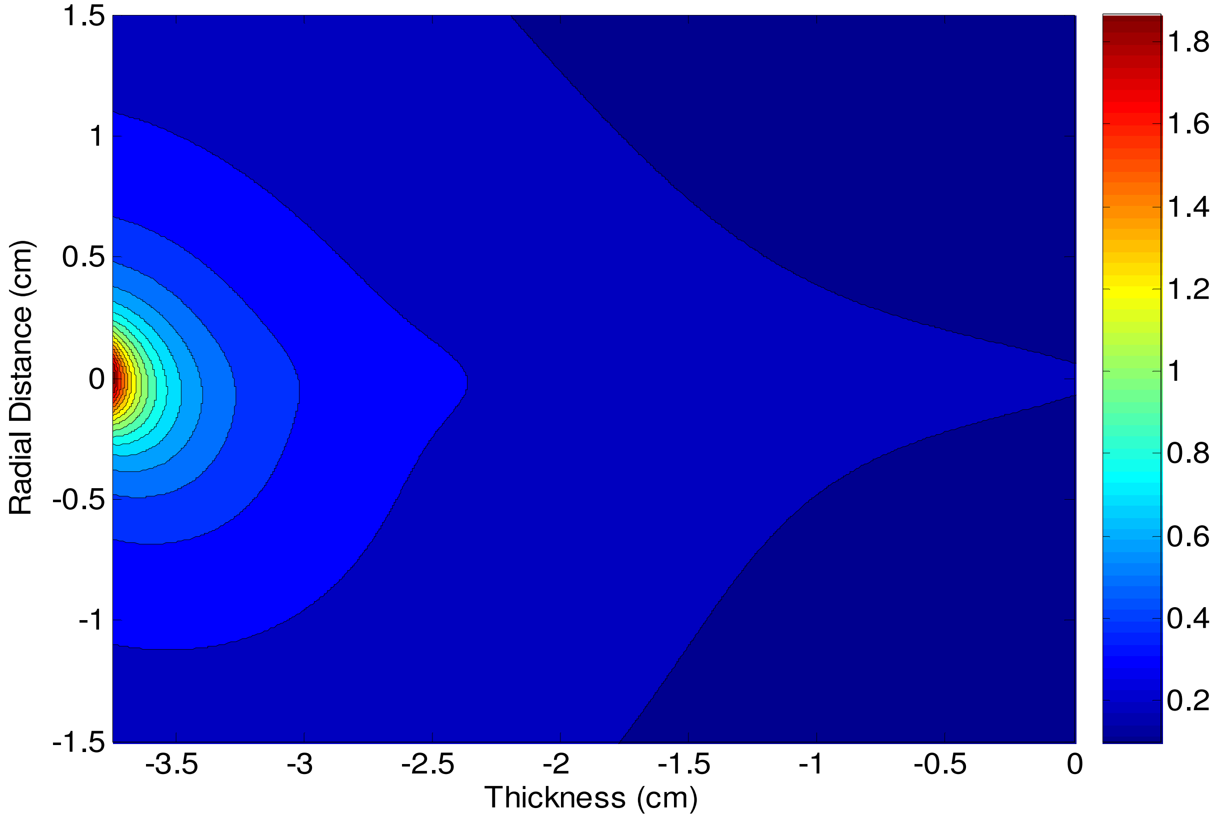

Fig. 5: Temperature profile across the substrate of MC_3. Here the lines are contours of constant temperature. X-axis represents the distance from the center of the substrate that is 7.5 cm thick. This shows the side towards the HR coating side of MC_3. Temperature variation on the AR coated side is negligible.

4.1 Thermal Lensing in MC Mirror Substrate (Along horizontal-direction)

So far the calculations above were for a symmetrical case. However, in the horizontal direction, the beam waist is larger by a factor of √2. This increase in beam waist reduces the thermal lensing in that direction because of lower intensity. In the case of surface thermal lensing, this effect is negligible because the over-all effect is quite small and results in losses in the order of ppm. In the case of substrate thermal lensing, this produces an astigmatic beam after it transmits through the substrate. The optimal ROC becomes 270 m for substrate thermal lensing. The higher order losses due to non-spherical losses can be calculated as a worst case scenario considering the vertical direction thermal lensing as a symmetrical case. The losses of 0.03% mentioned above thus are an over-estimate of the higher order losses. Another effect of the astigmatic lens is different tilting in x and y direction. 8 μrad tilt mentioned above is for the vertical direction. For horizontal direction, the expected tilt is 5.7 μrad.

The substrate thermal lens in the substrate reduces the mode matching as powers goes up. The reduction in mode matching can be calculated by evaluating the over-lap integral of the hot beam profile with the cold beam profile after the substrate lens. This calculation is once done for the x (horizontal) direction and then for the y-direction. The resultant mode miss-match is 0.4%. This is so far the worst lost in the mode cleaner due to thermal effects.

5 Strategy of Dealing with Thermal Lensing

The thermal lensing in MC will decrease the mode matching at full power if we design for cold case. Thermal lensing in the substrate will produce a positive astigmatic lens in the beam path that is power dependent. There are other power dependent thermal lenses in the IOO section. Most notably, the Faraday Isolator. Although there is a separate thermal compensation system for FI in the form of passive compensation via DKDP, some residual thermal lensing may be left. So based upon these, we propose to do the following:

- The MMT are designed for cold values of MC beam parameters
- FI thermal lensing is taken care of by DKDP. It will be ensured that the residual thermal lensing is negative.
- A ring heater will be installed around DKDP that may provide *in-situ* positive lensing.
- The substrate thermal lensing is astigmatic in nature and the DKDP ring heater will be designed in such a way that DKDP can provide an astigmatic lens.
- Since, the substrate thermal lensing is undoubtedly positive, a negative active compensation on DKDP can cancel its effect.
- Also we are assuming 0.5 ppm losses due to coating at the MC mirrors but it could be higher. As long as the residual thermal lensing in FI is within 0 to -100 m, thermal lensing in MC will improve the mode matching. The RH on DKDP can further improve the mode matching.

Fig. 6 shows the behavior of over-all mode matching due to residual FI lensing for various values of coating absorption at MC mirrors. Though different curves correspond to different values of coating absorption while assuming a 10 ppm/cam absorption of the substrate, the over-all thermal lensing in the MC could be due to any combination of the two. But this is most likely the range of thermal lensing that we may expect in MC. Fig. 6 also shows that it is better to keep the residual mode matching as negative because then the thermal lensing in FI improves the over-all mode matching. If the residual thermal lensing in FI is positive, MC thermal lensing makes it even worse. This can be ensured by acquiring a range of DKDP crystals of various thicknesses and then testing them in the lab before installation. The first DKDP that provides a transition from positive to negative residual thermal lensing should be selected.

Fig. 7 shows a zoomed plot of Fig. 6. It shows that clearly for an expected range of -30 m to -70 m, the thermal lensing in MC mirrors helps a lot to improve the thermal lensing.

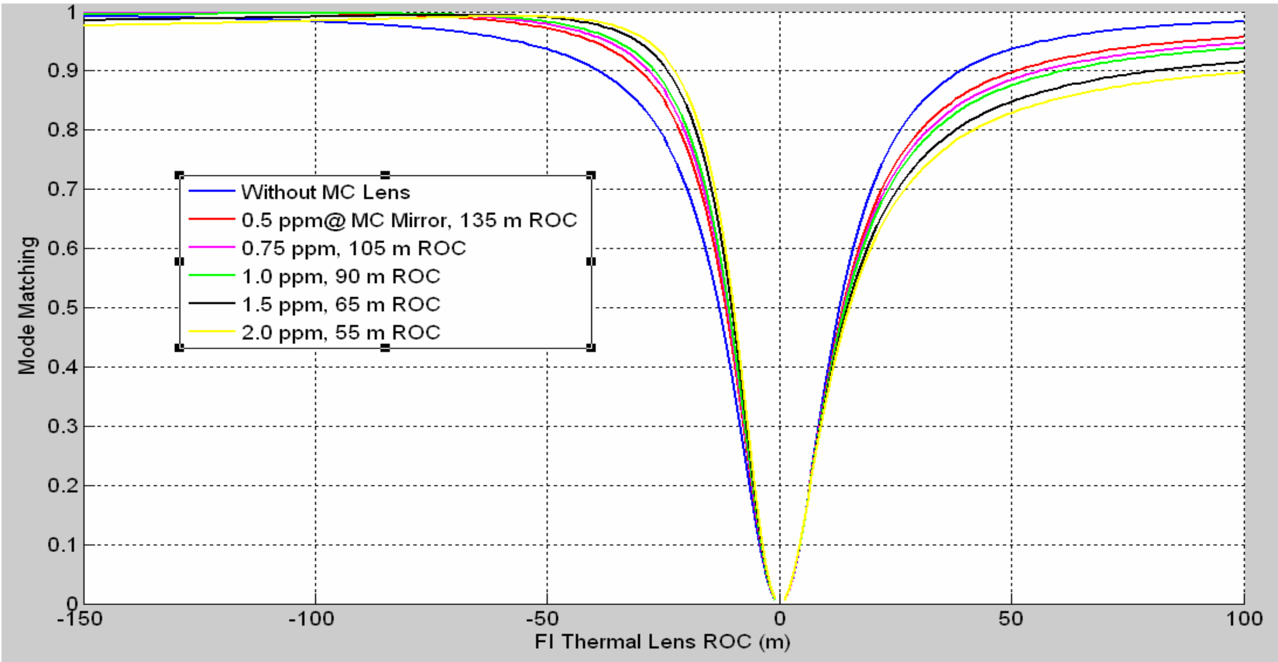

Fig. 6: Mode matching as a function of residual thermal lensing in FI for various values of lensing in MC mirrors.

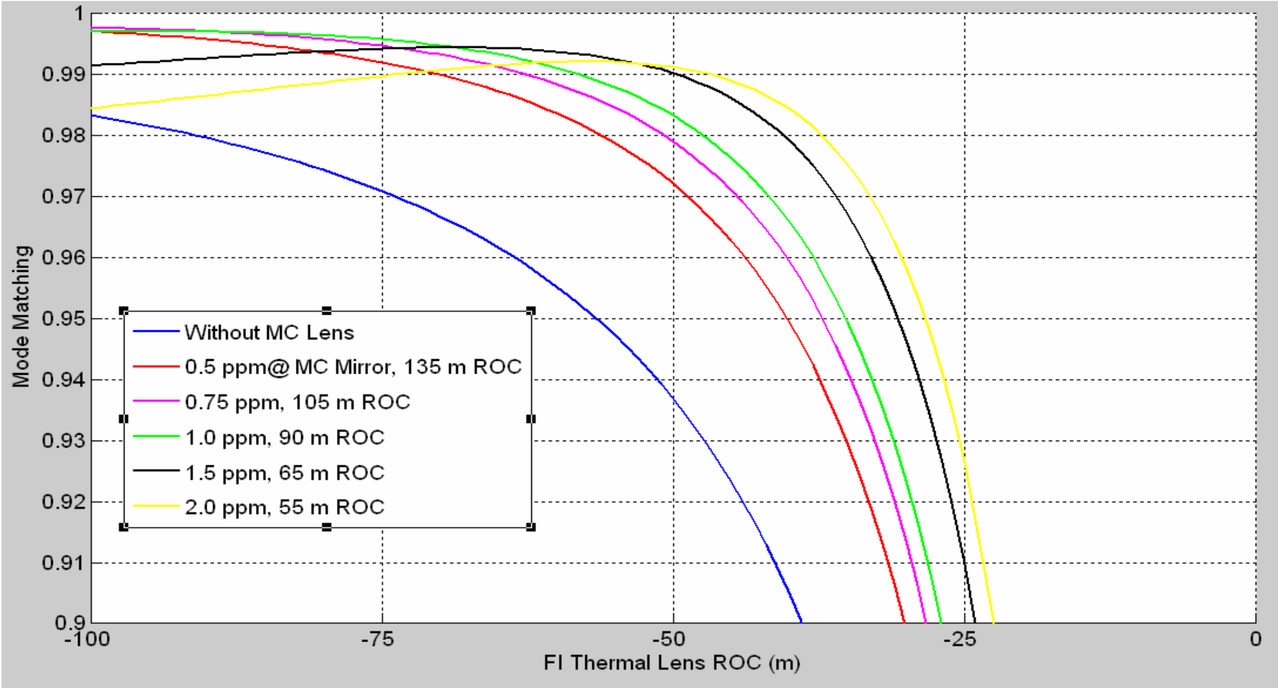

Fig. 7: Zoomed plot of Fig. 6.

The amount of thermal lensing required at DKDK is shown in Fig. 8. This is the optimal ROC that would be required to improve mode matching. Since, MC and FI are some distance apart, theoretically 100% mode matching can not be recovered but we can get very close to 99.9% mode matching. Also here we assume that the lens in FI is symmetrical but in reality we may need a little extra in the x-direction (or tangential direction) because MC lens supplies less positive thermal lensing in the tangential direction.

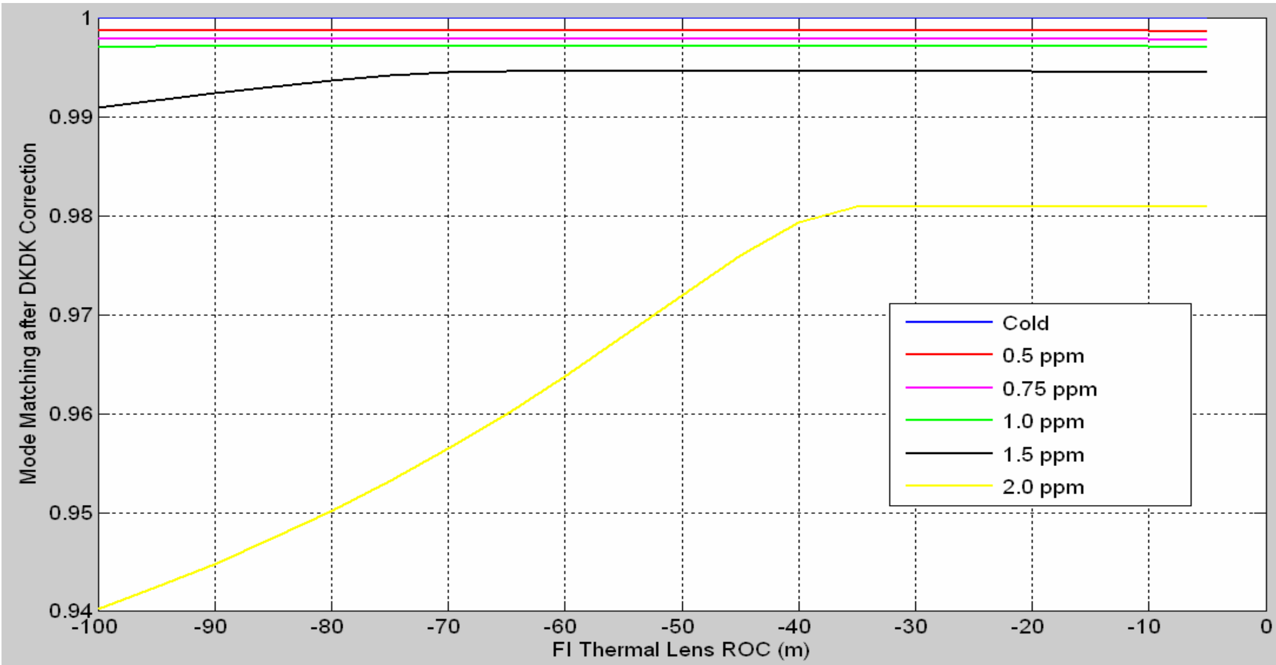

Fig.8: Mode matching after optimal thermal correction being applied at DKDP.

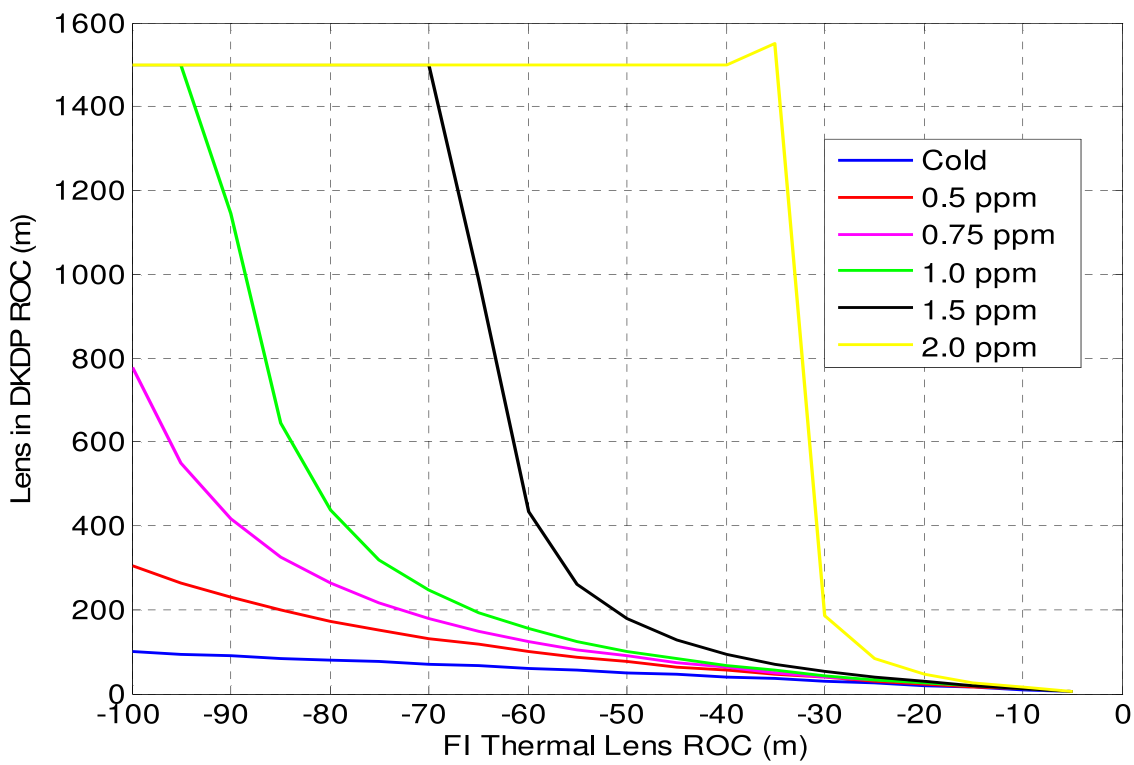

Fig.9: Optimal ROC correction required at DKDP from RH.

Here for the case of 1.5 and 2.0 ppm, the mode matching can not be recovered if the residual thermal lensing in FI is not stronger than -65 and -35 m respectively. Because in that case, MC positive lens already compensates the thermal lensing in the FI. Any further improvement will require negative lensing from DKDP through RH. However, this situation is very unlikely because MC lens coating absorption will be less than 1.0 ppm. However, if this situation arises, we can accommodate that by using a longer length of the DKDP crystal so that the intrinsic compensation provided by the DKDP crystal is negative. The corresponding ROC required at DKDP for mode matching improvement for various thermal lensing in MC is shown in Fig. 9. Thepoint at which the green, black and yellow curves become constant are the point where we need additional negative compensation from DKDP instead of a positive lens from DKDP RH.

6 Comparison of Analytical Model with Finite Element Analysis (COMSOL)

As a sanity check, the analytical model was compared with COMSOL based FEA model for thermal deformations. The substrate thermal lensing presents the more challenging situation because of high localized intensity of the beam inside the MC. The thermal models for the ITM substrate match well within 1% of each other. For the case of substrate thermal lensing in MC, the substrate thermal lens profile is shown in Fig. 10.

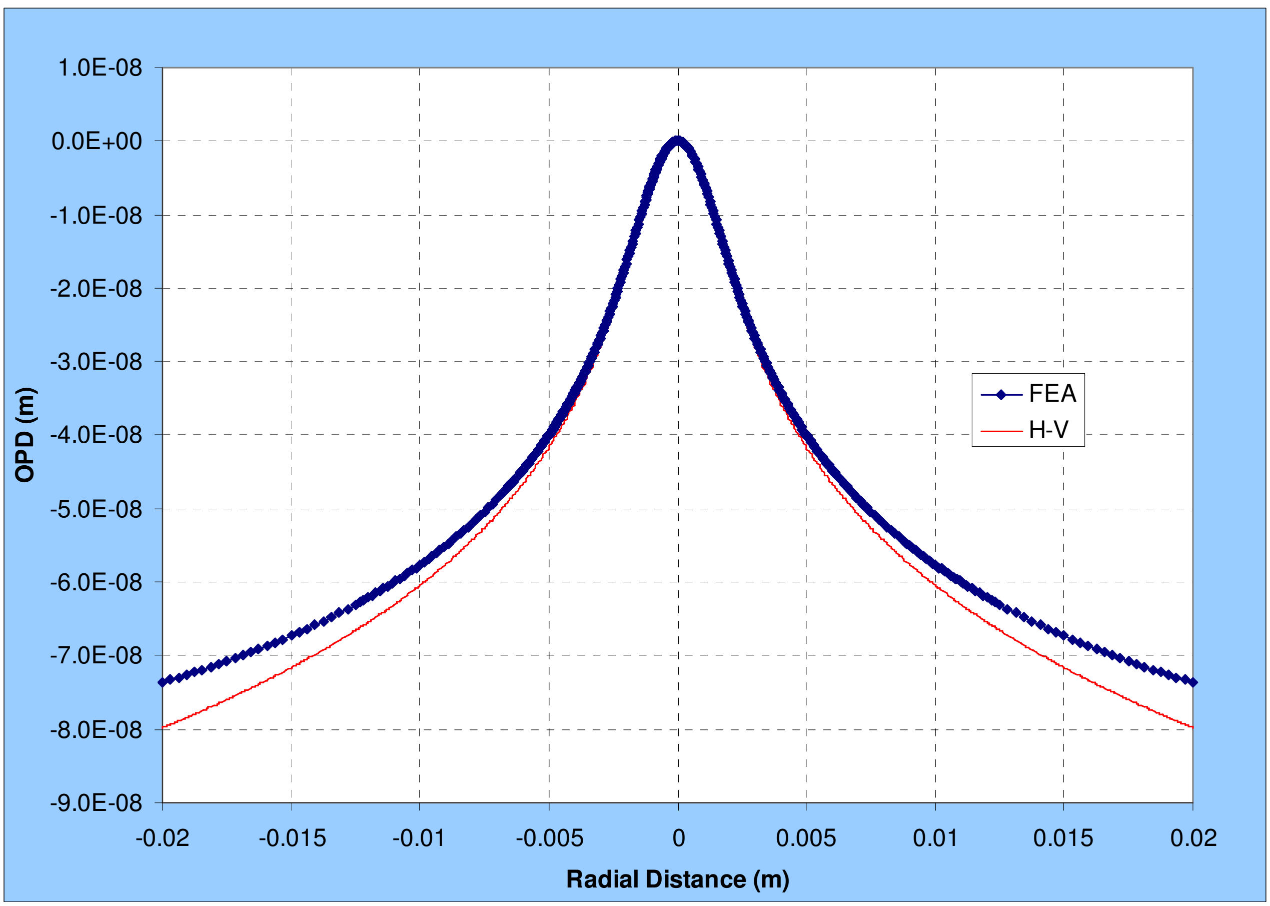

Fig. 10: Comparison of thermal OPD calculated through H-V Theory & FEA model for substrate thermal aberration in MC for 0.5 ppm coating absorption and 10 ppm/cm substrate absorption.

Here we have good agreement between the two models. The thermal lens associated with H-V theory is calculated to be 130 m while the FEA model predicts a thermal lens of 135 m. The two results are within 4% of each other. Analytical model is used throughout the IO document.

7 Conclusion

In conclusion, an alternative scheme for thermal lensing compensation in Advanced LIGO IOO is proposed. The scheme is based upon using DKDP as a passive negative compensation element to correct FI residual compensation and using a RH around DKDP to provide an active positive compensation element to correct any thermal lensing due to MC and FI+DKDP combination. This scheme will provide a power independent mode for the IOO MMT. This will also help maintaining an independent design for MC and FI for a stable and marginally stable cavity designs. Now the MMT mirrors only need to be compensated for their individual ROC tolerances or for as a back-up for thermal compensation in the arm cavity.

Appendix A Hot and Cold Values for MC Mirrors

		cold	hot
MC1 radius of curvature	m	>10000	-4000
MC2 radius of curvature	m	26.769	26.9116
MC3 radius of curvature	m	>10000	-4000
Cavity g factor		0.3897	0.3929
MC waist	mm	2.1028	2.1098
Rayleigh range	m	13.0558	13.1429