Length control signals calibration for the 40m IFO

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Monica Varvella*for the 40m team

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Abstract

The calibration of the interferometer's Gravitational Waves channel is reported here to produce a measure of the gravitational wave strain incident on the detector. The procedure to measure the calibration value is reported in detail.

1 Introduction

The calibration of the interferometer's Gravitational Waves (GW) channel is here reported to produce a measure of the GW strain incident on the detector. Five steps are necessary to calibrate the data:

1) make a model of the interferometer response. This requires a conversion from ETMs motion ($\Delta(ETMX_{pos} - ETMY_{pos})$) to the Asymmetric Port (AP) with Q phase demodulation signal at 166 MHz, i.e. AP 166 MHz Q. In the future we will refer to this channel as ASQ;

2) take a measurement of the freely swinging Michelson;

3) calibrate the ITMs mirror actuator drive. This requires a conversion from ITMs motion to ITMs control signal and it can be done according to the following procedure:

   (a) lock the Michelson on the dark fringe (ASQ) with low unity gain frequency (UGF): in this way, the Q phase demod signal is at a zero crossing when the error signal is mostly linearly dependent on the Michelson length;

   (b) shake ITMs at some frequency higher than UGF where the servo gain is much less than 1 so that the servo is not able to cancel the signal at the AP using $ITMX\_LSC\_EXC$ and $ITMY\_LSC\_EXC$ and measure the transfer function at AS Q;

   (c) correct for the pendulum response;

*E-mail: varvellalal.in2p3.fr, *Laboratoire de l'Accelerateur Lineaire, Orsay, France.*

4) convert from the ITMs control signal to the ETMs control signals; this can be done in the following steps:
   (a) lock each arm;
   (b) shake ITMs ($SUS\_ITMX\_LSC\_EXC$ and $SUS\_ITMY\_LSC\_EXC$) in order to get a signal at AS Q ($ITMX_{pos}->ASQ$ transfer function);
   (c) shake ETMs ($SUS\_ETMX\_LSC\_EXC$ and $SUS\_ETMY\_LSC\_EXC$) recording the value of the excitation signal at AS Q to get the same expression obtained with ITMs;
   (d) get the ratio $T(ITMs\_EXC \rightarrow ASQ)/T(ETMs\_EXC \rightarrow ASQ)$.
5) lock the full interferometer to get the ETMs motion.

1.1 Calibration: a model for the interferometer response.

The model for the interferometer response requires a conversion from ETMs motion to the asymmetric port with Q phase demodulation signal at 166 MHz, i.e. the channel ASQ. The interferometer response $T(X_{ETM}->ASQ)$ is given by the following equation

$$T(X_{ETM}->ASQ) = \frac{\partial ASQ}{\partial X_{ETMX}} = \frac{\partial ASQ}{\partial E_{ETMX}} \frac{\partial E_{ETM}}{\partial X_{ETM}} \quad (1.1)$$

that is the product of the transfer function from ETM excitation to ASQ and the transfer function between the ETM excitation and ETM displacement. The first term can be splitted in the following products of transfer functions

$$\frac{\partial ASQ}{\partial E_{ETMX}} = \left(\frac{\partial ASQ}{\partial X_{ITM}}\right) \left(\frac{\partial X_{ITM}}{\partial E_{ITM}}\right) \left(\frac{\partial E_{ITM}}{\partial E_{ETM}}\right) \quad (1.2)$$

and it depends on the configuration used to measured, consequently it affects the interferometer response. The second term is constant and it will be calculated using the steps described in the following sections.

1.2 Calibration: freely swinging Michelson.

The Michelson interferometer is used as sensor to obtain an absolute calibration of the mirrors actuator drive: the laser wavelength is used as reference. By misaligning the end mirrors (ETMs) and both the recycling mirrors (PRC and SRC) a Michelson interferometer is obtained with two arm cavity input mirrors (ITMX and ITMY) and the beam-splitter (BS) as shown in Fig.1.

First of all, freely-swinging Michelson, composed of BS, ITMX, ITMY (full interferometer arms blocked, PRM and SRM misaligned) over several fringes at the AP demodulated at 166 MHz (i.e. ASQ) is compared to the AP DC signal ($LSC\_Dither\_AP\_DC$) as shown in Fig. 2.

The AS Q signal follows this behavior

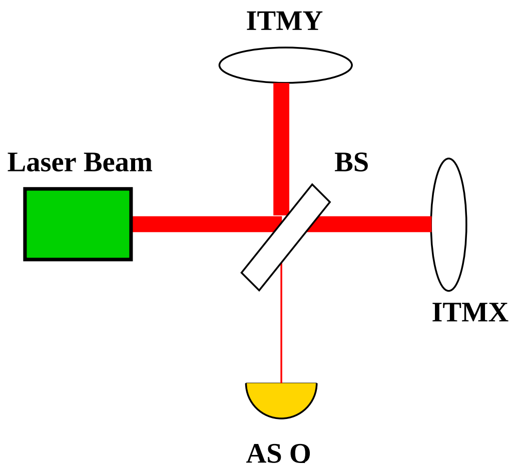


Figure 1: *Michelson.*

$$ASQ = A_p \; sin(2 \; k \; X_{ITM}) \tag{1.3}$$

where A_p is the peak height, X_{ITM} is the displacement of ITMX when ITMY is fixed, or the relative displacement $(X_{ITMX} - X_{ITMY})$ and $k = \frac{2\pi}{\lambda}$.

The slope of the ASQ signal at the zero crossing is

$$\frac{\partial ASQ}{\partial X_{ITM}}\Big|_{X=\lambda/2} = 2 \; k \; A_p = 4\pi \; \frac{A_p}{\lambda} \tag{1.4}$$

where $\lambda = 1.064 \; \mu m$ is the laser wavelength.

The amplitude A_p is measured in volts (counts), making sure it's a clear sweep through a fringe; from Fig. 2, $A_p = 8 \; V$. One gets

$$\frac{\partial ASQ}{\partial X_{ITM}} = 9.45 \times 10^7 \; V/m \; (ct/m). \tag{1.5}$$

1.3 Calibration: mirrors actuator drive. $ITMs\_EXC \rightarrow ASQ$

The displacement response of the mirror to an actuation signal can be obtained measuring the transfer function from the actuator (the drive of ITMX or ITMY) and the correspondent control signal ASQ. The validation is to measure the swept sine response of ASQ to the drive of a single mirror and ensure it faithfully follows the f^2 power law of a free mass. In order to do that, the Michelson has to be locked, on the dark fringe, so $X_{ITM} \sim 0$, the displacement is almost zero, but with relatively low gain.

The ASQ signal reads

$$ASQ = A_p \; sin(2 \; k \; X_{ITM}) \; \simeq \; A_p \; 2 \; k \; X_{ITM} \tag{1.6}$$

and it can be assumed linear under those conditions. Then ITMX has to be shaken and its displacement is the following

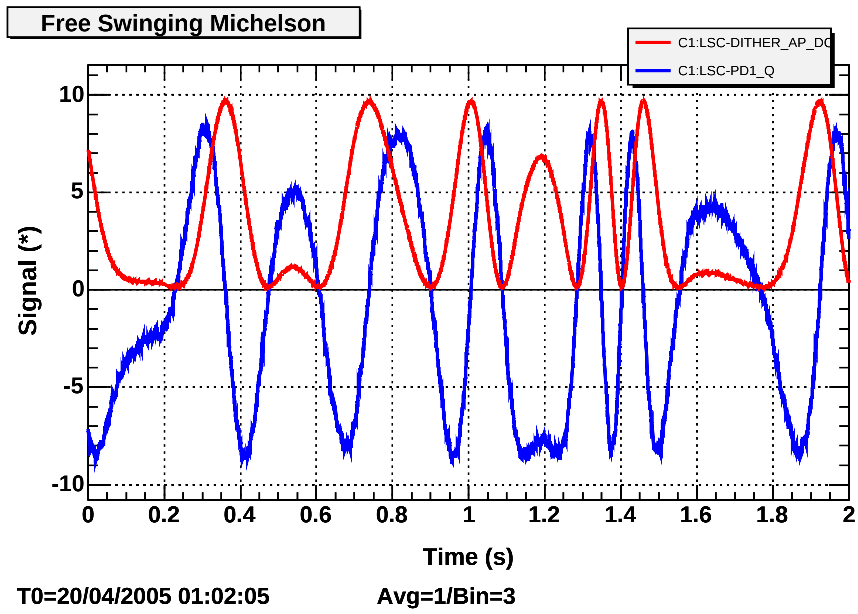


Figure 2: *Freely swinging Michelson: ASQ signal (LSC_PD1_Q) in blue and the AP DC signal ($LSC\_Dither\_AP\_DC$) in red.*

$$X_{ITM} = C_{ITMX}\ E_{ITMX}\ e^{i\omega_0 t}\ P(\omega_0) \tag{1.7}$$

where

- X_{ITM} is the displacement of ITMX once ITMY is fixed, in meters and it is much less than λ;
- C_{ITMX} is the near DC displacement to ITMX for a given excitation;
- E_{ITMX} is the excitation level in counts;
- ω_0 is equal to $2\pi f_0$ where f_0 is the shake frequency;
- $P(\omega_0)$ is the pendulum response, i.e. $\frac{1}{1+(f/f_p)^2}$ where f_p is the pendulum frequency.

Taking into account (1.7) the ASQ signal now reads

$$\begin{aligned} ASQ &= A_p\ 2\ k\ X_{ITM}\ T_{MICH}(\omega_0) \\ &\simeq A_p\ 2\ k\ C_{ITMX}\ E_{ITMX}\ P(\omega_0)\ T_{MICH}(\omega_0) \end{aligned} \tag{1.8}$$

where A_p is the same defined in Sec. 1.1 and $T_{MICH}(\omega_0) = \frac{1}{1+G_{MICH}(\omega_0)}$.

One has to choose f_0 such that $G_{MICH}(\omega_0) \ll 1$ and $f_0 \gg f_{MICH(UGF)}$. Under those conditions $T_{MICH}(\omega_0)$ 1.

Then the following transfer function can be measured

$$T_1\,(ASQ->E_{ITMX}) = \frac{\partial ASQ}{\partial E_{ITMX}} = A_p\ 2\ k\ C_{ITMX}\ P(\omega_0)\ T_{MICH}(\omega_0) \tag{1.9}$$

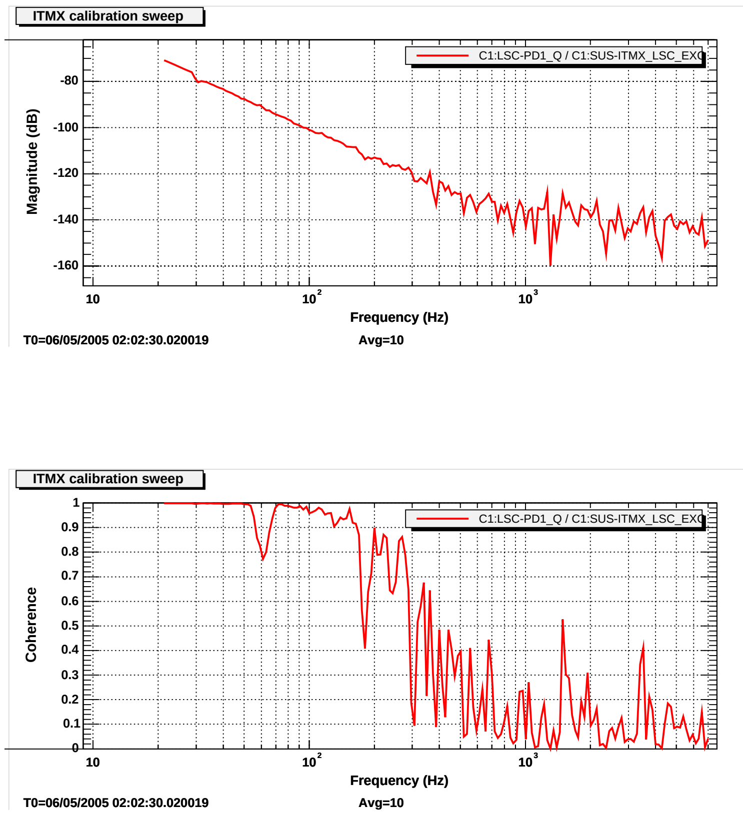


Figure 3: *Transfer function of ITMX versus ASQ when Michelson is locked near Michelson dark fringe.*

and verify, via a sweep, that the frequency dependence is well described by $P(\omega_0)\ T_{MICH}(\omega_0)$. This measurement gives C_{ITMX} in meters/counts of excitation, i.e.

$$C_{ITMX} = \frac{1}{2k\ A_p} \frac{1}{P(\omega_0)\ T_{MICH}(\omega_0)} \frac{\partial ASQ}{\partial E_{ITMX}} \tag{1.10}$$

where $P(\omega_0) = \frac{1}{1+(f/f_p)^2}$ and the frequency pole of the pendulum is 0.8 Hz (nominal value).

Using exactly the same procedure is possible to obtain C_{ITMY}.

Swept sine measurements of $\frac{ASQ}{SUS\_LSC\_EXC}$ have been done for both ITMs; Fig. 3 and Fig. 4 shows this measurements: one can see that the Michelson acts like a free pendulum at frequencies higher than 10 Hz while the feedback acts at lower frequencies. In the range between 20 Hz and 100 Hz the measurement is good enough for the calibration; one can use it to obtain the conversion factor between the ITMs motion and the ITMs control signal once the pendulum effect has been corrected. The measured pendulum frequency is 0.797 Hz for ITMX and 0.789 Hz for ITMY [3]. The value for C_{ITMX} and C_{ITMY} is close to $(1.6 \pm 0.1) \times 10^{-9}\ m\ /\ DAC\ ct$.

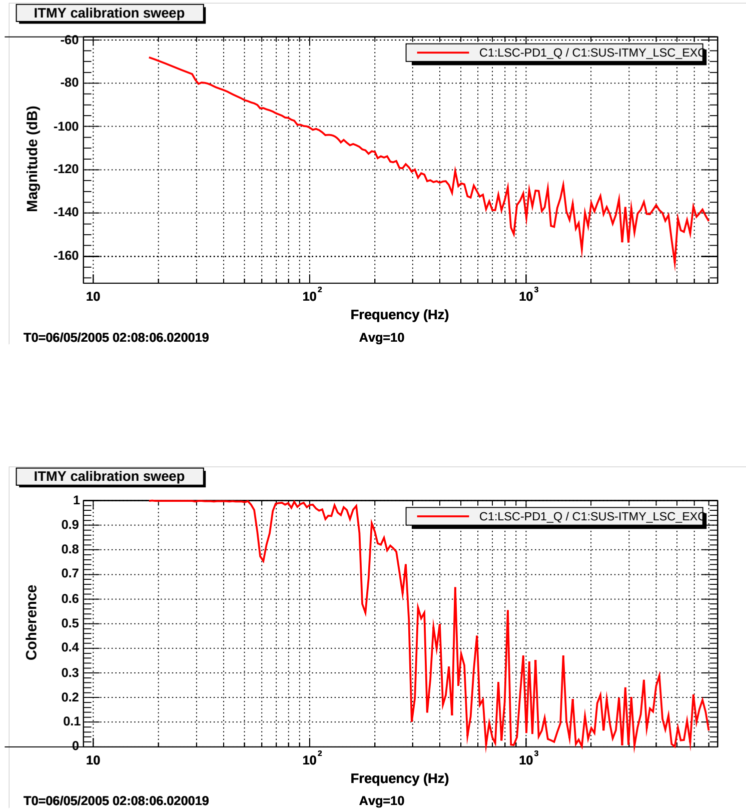


Figure 4: *Transfer function of ITMY versus ASQ when Michelson is locked.*

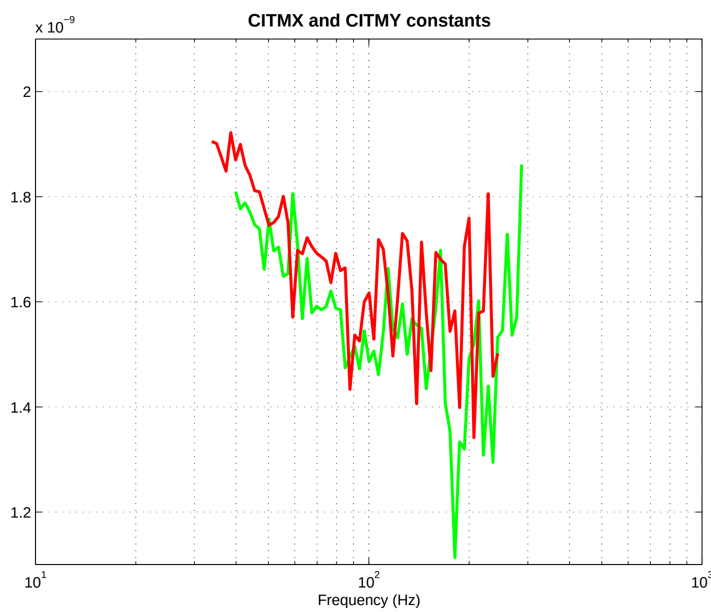


Figure 5: *CITMX and CITMY estimation from* $\left(\frac{ADCcounts}{DACcounts}P(\omega_0)\right)$ *calibration.*

1.4 Calibration: ITMS motion versus ETMs motion. $ITMs\_EXC \rightarrow ETMs\_EXC$

The third step is the bootstrap of ETM from ITM. Each arm needs to be locked separately as a simple Fabry-Perot cavity, then the swept sine response can be obtained shaking once the correspondent ITM mirror in order to get the signal at ASQ and then the correspondent ETM mirror using the same amplitude in order to get the same signal. The scheme of this calibration is shown in Fig. 6.

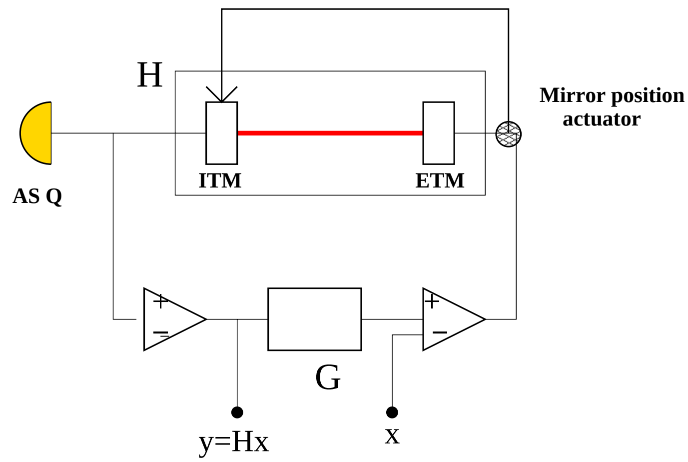


Figure 6: *Arm Plant scheme.*

It is necessary now to unblock one arm and lock it. The light on ASQ is different, so the amplitude is different and to distinguish it from A_p defined in Sec. 1.2, let's call it A'_p.

Let's shake ITMX with an excitation ($SUS\_ITMX\_LSC\_EXC$), the ASQ signal reads

$$\begin{aligned} ASQ &= A'_p\ 2\ k\ X_{ITM}\ T'_{MICH}(\omega_0) \\ &= A'_p\ C_{ITMX}\ E_{ITMX}\ P(\omega_0)\ T'_{MICH}(\omega_0) \end{aligned} \tag{1.11}$$

and measure the following transfer function

$$T_2 = \left(\frac{\partial ASQ}{\partial E_{ITMX}}\right) = A'_p\ C_{ITMX}\ P(\omega_0)\ T'_{MICH}(\omega_0)\ T_{ARM}(\omega_0) \tag{1.12}$$

Then, let's shake ETMX ($SUS\_ETMX\_LSC\_EXC$), the ASQ signal now reads

$$\begin{aligned} ASQ &= A'_p\ 2\ k\ X_{ETM}\ T'_{MICH}(\omega_0)\ T_{ARM}(\omega_0) \\ &= A'_p\ C_{ETMX}\ E_{ETMX}\ P(\omega_0)\ T'_{MICH}(\omega_0)\ T_{ARM}(\omega_0) \end{aligned} \tag{1.13}$$

where

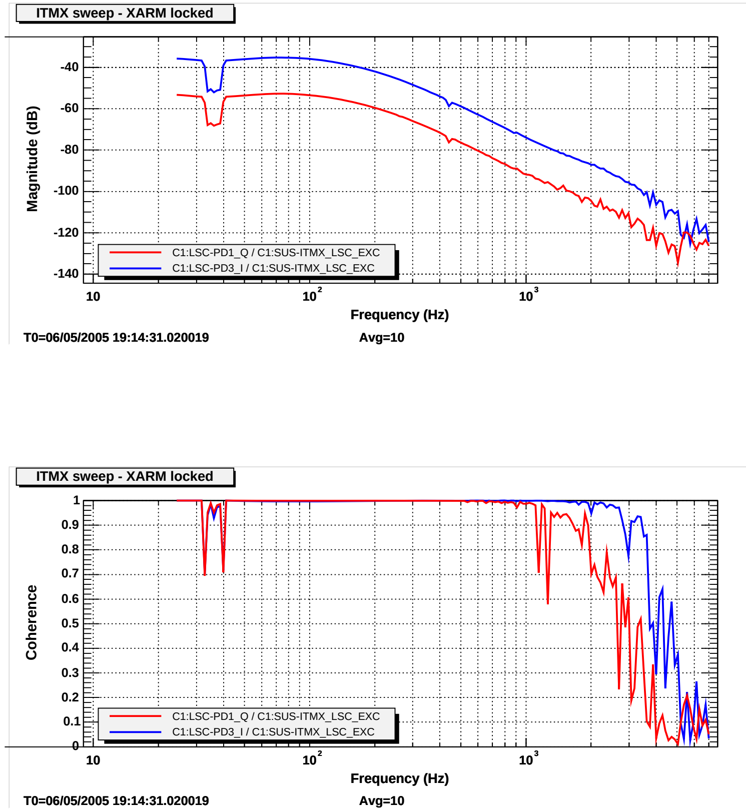


Figure 7: *Transfer function of ITMX versus ASQ and ITMX versus POX when Xarm is locked.*

$$T_{ARM}(\omega_0) \quad = \quad T_{servo}\ T_{cavity} = \frac{1 + (f_0/f_c)^2}{1 + G_{ARM}(f_0)} \tag{1.14}$$

and

$$f_c \quad = \quad \frac{c}{2\ L_{ARM}}\ \frac{1 - r_1\ r_2}{2\ \pi} = 1.55\ KHz \tag{1.15}$$

where f_c is the cavity pole frequency; shaking at less than f_c, $T_{cavity} \sim 1$.

The following transfer function can be measured

$$T_2 = \left(\frac{\partial ASQ}{\partial E_{ETMX}}\right) = A'_p\ C_{ETMX}\ P(\omega_0)\ T'_{MICH}(\omega_0)\ T_{ARM}(\omega_0) \tag{1.16}$$

and the ratio is

$$T_2\left(\frac{\partial ASQ}{\partial E_{ITMX}}\right) \Big/ T_2\left(\frac{\partial ASQ}{\partial E_{ETMX}}\right) = \frac{C_{ITMX}}{C_{ETMX}}\ \frac{1}{T_{armservo}} \tag{1.17}$$

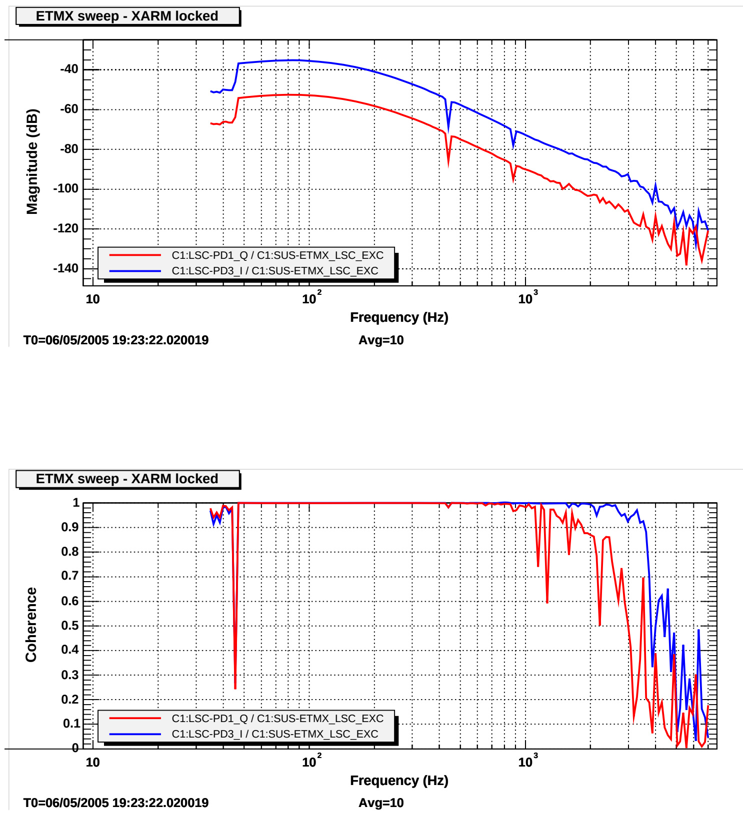


Figure 8: *Transfer function of ETMX versus ASQ and ETMX versus POX when Xarm is locked.*

Shaking with $f_0 \gg f_{armservo(UGF)}$, the $T_{armservo}$ is approximately 1 and C_{ITMX}/C_{ETMX} ratio can be measured.

The transfer functions measurements for ITMX and ETMX are shown in Fig. 7 and Fig. 8.

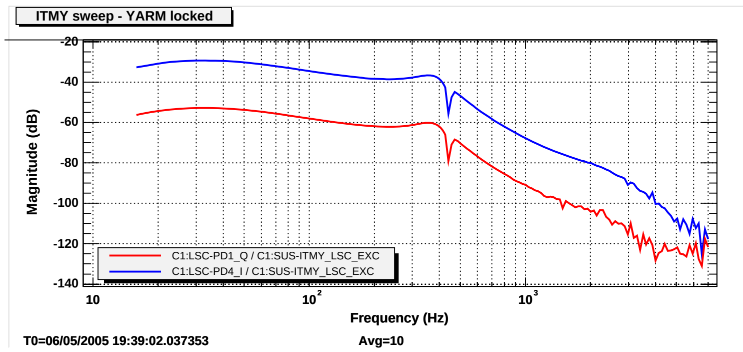
The same procedure can be repeated for ITMY and ETMY, locking the other arm. The transfer functions measurements for ITMY and ETMY are shown in Fig. 9 and Fig. 10.

The ratio of the transfer functions measurements ($ITM- > ASQ$,$ETM- > ASQ$) gives the conversion factor from ITMs control signal to ETMs control signal, ie the ratio of ETM to ITM drive.

The C_{ITMX}/C_{ETMX} ratio is close to 1 for both the arms as it can be seen in Fig. 12.

1.5 Calibration: ETMs motion. $ETMX\_EXC \rightarrow ASQ$

The fifth (and last) step requires the lock of the full interferometer; now the MICH, ARM and all the servos are different; the amplitude defined in Sec. 1.2 is now called A''_p and the ASQ signal reads



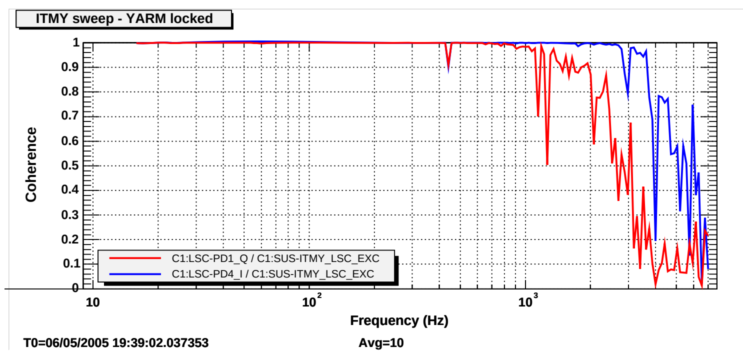


Figure 9: *Transfer function of ITMY versus ASQ and ITMY versus POY when Yarm is locked.*

$$ASQ = A_p''\ 2\ k\ X_{ETM}\ T_{fullIFO}(\omega_0) \tag{1.18}$$

where X_{ETM} is the $ETMX$ displacement and T_{full} is

$$T_{fullIFO}(\omega_0) = \frac{1 + (f/f_c)^2}{1 + G_{fullIFO}(f_0)} \tag{1.19}$$

with G_{full} is the servo, f_c is the cavity pole frequency and f_0 is the shake frequency.

Let's shake X_{ETM}, i.e. $ETMX$ displacement, one gets

$$X_{ETM} = C_{ETMX}\ E_{ETMX}\ e^{i\omega_0 t}\ P(\omega_0) \tag{1.20}$$

where all the quantities are known thanks to the previous measurements. Let's measure the transfer function

$$\frac{\partial ASQ}{\partial E_{ETMX}} = A_p''\ 2\ k\ C_{ETMX}\ T_{fullIFO}\ P(\omega_0) \tag{1.21}$$

where k, C_{ETMX}, $P(\omega_0)$ are known. So $A_p''\ T_{full}$ can be measured.

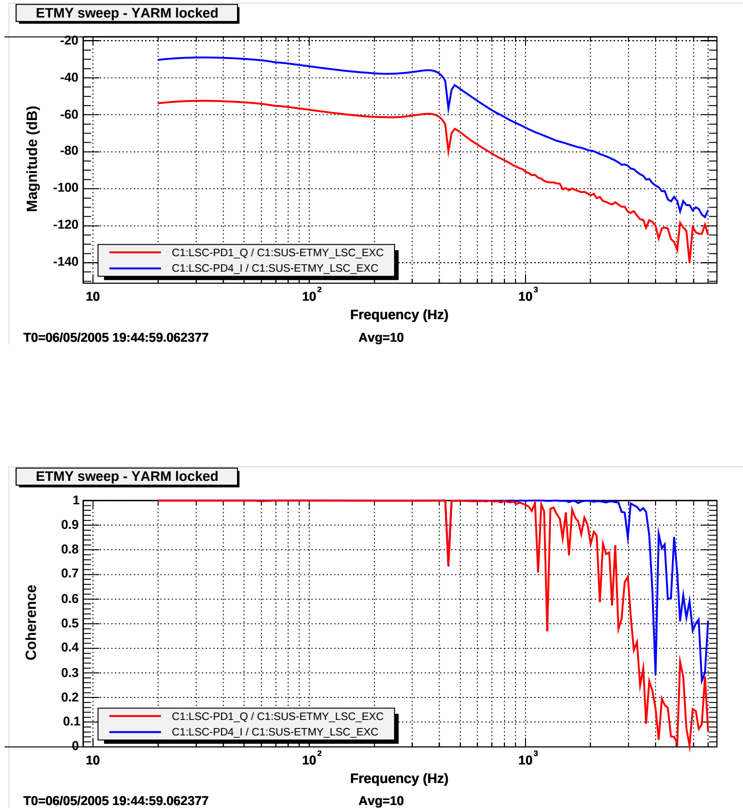


Figure 10: *Transfer function of ETMY versus ASQ and ETMY versus POY when Yarm is locked.*

The displacement X_{ETM} at ASQ can be calibrated taking into account the model for the interferometer response described in Sec. 1.1. From equation 1.1 one gets

$$
\begin{aligned}
T(X_{ETM} -> ASQ) &= \frac{\partial ASQ}{\partial X_{ETMX}} = A_p''\ 2\ k\ T_{fullIFO} = \\
&= \frac{\partial ASQ}{\partial E_{ETMX}} \frac{1}{C_{ETMX}\ P(\omega_0)}
\end{aligned}
\tag{1.22}
$$

where C_{ETMX} reads

$$
C_{ETMX} = C_{ITMX}\ T_3 \left(\frac{ASQ}{E_{ETMX}}\right) / T_3 \left(\frac{ASQ}{E_{ITMX}}\right) \frac{1}{T_{ARMservo}} \tag{1.23}
$$

and C_{ITMX} according to equation 1.10

$$
C_{ITMX} = T_1 \left(\frac{\partial ASQ}{\partial E_{ITMX}}\right) \frac{1}{A_p\ P(\omega_0)\ T_{MICH}(\omega_0)} \tag{1.24}
$$

Measuring $\frac{\partial ASQ}{\partial E_{ETMX}}$ from the configuration in which one arm, XARM in this case, is locked one gets

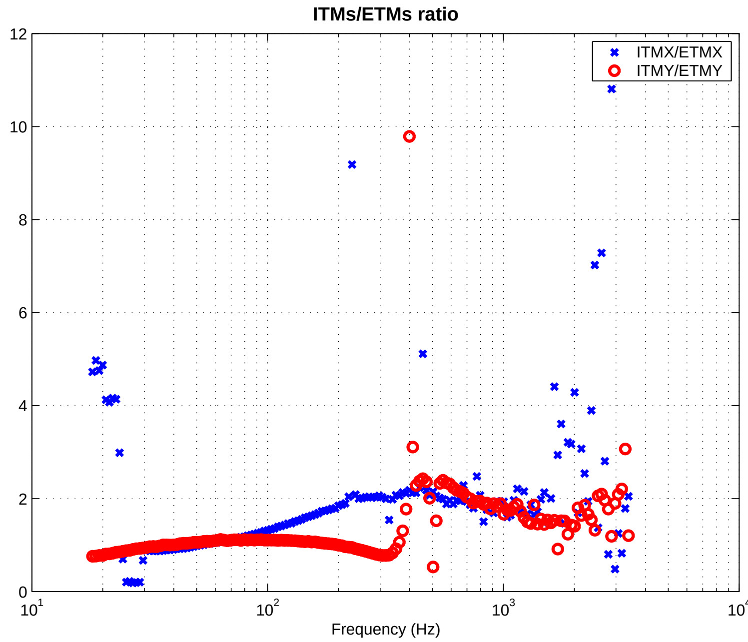


Figure 11: *Ratio of the sweeps ($ITM->ASQ$, $ETM->ASQ$) for each arm to get the ratio of ITM to ETM drive.*

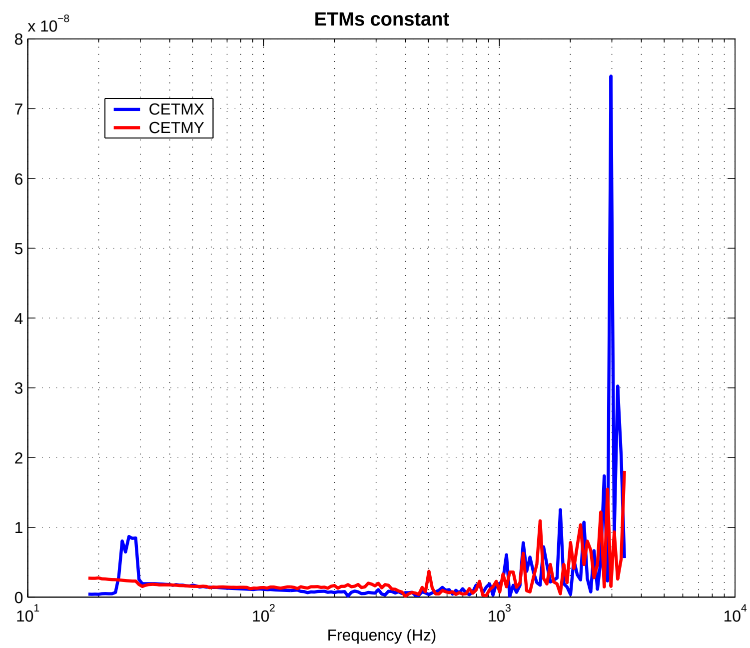


Figure 12: C_{ETMX} and C_{ETMY}.

$$T_X(X_{ETM}->ASQ) = \frac{\partial ASQ}{\partial X_{ETMX}} \simeq 2.1 \times 10^{10}\ ADC\ ct/\ m \quad (1.25)$$

The same procedure can be repeated for the Y_{ETM} displacement shaking

$ETMY$.

Measuring $\frac{\partial ASQ}{\partial E_{ETMY}}$ from the configuration in which one arm, YARM in this case, is locked one gets

$$T_Y(Y_{ETM} -> ASQ) \quad = \quad \frac{\partial ASQ}{\partial Y_{ETMY}} \simeq 1.6 \times 10^{10} \ ADC \ ct \ / \ m \quad (1.26)$$

2 Errors

The fractional uncertainty on the magnitude of the response function is $\frac{\Delta|T_X|}{|T_X|}$ and $\frac{\Delta|T_Y|}{|T_Y|}$. $\Delta|T_X|$ and $\Delta|T_X|$ has contribution from:

- the peak to peak measurement of ASQ obtained with the free swinging Michelson;
- the deviation from a single pendulum response of the ITM/ASQ transfer function;
- the uncertainty on a transfer function is given by [4]

$$\frac{\sqrt{1-\gamma^2(f)}}{|\gamma(f)|\sqrt{2\ N_d}} \quad (2.27)$$

  where $\gamma(f)$ is the coherence of the measurement and N_d is the number of averages. The three transfer function measurement used in the calibration, i.e. ITM/ASQ, ITM/ASI and ETM/ASI, all contribute an uncertainty calculated using the equation 2.27. This contributions are small since the coherence has been kept high during the measurements;
- the ratio ITM/ASI ETM/ASI transfer functions should be ideally flat

The different sources of uncertainty from transfer function measurements are combined in quadrature.

3 Conclusions and Discussion

The calibration of ETMs has been done according to the model described in Sec. 1.1. Table 1 summarizes the values for the constants C_{ITMX}, C_{ITMY}, C_{ETMX}, C_{ETMY}, A_p, whereas table 2 shows the DC values of the actuation funtions for the 40m IFO in $m/ADC\ ct$.

	$m/DAC\ ct$		$ADC\ ct/m$
C_{ITMX}	1.6×10^{-9}	A_p	9.45×10^{7}
C_{ITMY}	1.6×10^{-9}		
C_{ETMX}	1.6×10^{-9}		
C_{ETMY}	1.6×10^{-9}		

Table 1: Summary of constants values.

$ETMX$	$ETMY$ units	
4.6×10^{-11}	6.2×10^{-11}	$(m/ADC\ ct)$
2.1×10^{10}	1.6×10^{10}	$(ADC\ ct/m)$

Table 2: Summary of measurements of the DC values of the actuation functions for the 40m IFO.

Changing the configuration one can gets the transfer function in equation 1.1 from other optics, knowing already the constants C_{ITMX} and C_{ETMX} and $P(\omega_0)$.

References

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[4] "Random data analysis and measurement procedures", J.S. Bendat, A.G. Piersol Third edition ...