

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY  
-LIGO-  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

<b>Technical Note</b>	<b>LIGO-T060214- 00- R</b>	09/07/06
<b>Test Mass Thermal Compensation Strategies</b>		
Phil Willems		

This is an internal working note  
of the LIGO Project.

**California Institute of Technology**  
**LIGO Project – MS 51-33**  
**Pasadena CA 91125**  
Phone (626) 395-2129  
Fax (626) 304-9834  
E-mail: [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)

**Massachusetts Institute of Technology**  
**LIGO Project – MS 20B-145**  
**Cambridge, MA 01239**  
Phone (617) 253-4824  
Fax (617) 253-7014  
E-mail: [info@ligo.mit.edu](mailto:info@ligo.mit.edu)

WWW: <http://www.ligo.caltech.edu>

## INTRODUCTION

The simultaneous thermal compensation of the input test mass (ITM) and compensation plate (CP) in Advanced LIGO has been shown to be problematic. If the test masses are not compensated directly, then the ROC of their HR surfaces is expected to change due to thermal expansion caused by absorption of arm cavity light power. This change will nominally cause the spot size at both test masses to change from 6.0 cm at low laser input power to ~5.3 cm at full laser input power. A fixed ring heater acting on the CP can only be optimized to compensate for one given spot size.

On the other hand, if the test masses are compensated to maintain the arm cavity mode spot size, then the resulting thermorefractive aberration in the ITM caused by its ring heater will cause the thermal lens to change from positive to negative. In this case the ring heater can provide no compensation at all.

This technical note discusses what can be achieved if the arm cavity mode is compensated by acting only on the end test mass (ETM).

## MODEL

A simple two-mirror cavity with spherical mirrors suffices to model the system, as the thermoelastic deformations are very well approximated by pure radii of curvature in terms of their effect on the cavity modes.

The spot sizes at the mirrors of a Fabry-Perot cavity are given by:

$$w_{itm} = \sqrt{\frac{L\lambda}{\pi}} \sqrt[4]{\frac{(1 - L/R_{etm})}{(1 - L/R_{itm})(1 - (1 - L/R_{itm})(1 - L/R_{etm}))}}$$
$$w_{etm} = \sqrt{\frac{L\lambda}{\pi}} \sqrt[4]{\frac{(1 - L/R_{itm})}{(1 - L/R_{etm})(1 - (1 - L/R_{itm})(1 - L/R_{etm}))}}$$

For the nominal parameters of Advanced LIGO ( $L=4000\text{m}$ ,  $\lambda=1064\text{nm}$ ,  $R_{itm}=R_{etm}=2076\text{m}$ ), the spot size on both mirrors is 6.0cm.

Assuming that Brownian noise in the coatings dominates the thermal noise on the test masses, the thermal noise amplitude of a test mass is inversely proportional to the spot size. Therefore, we can determine the relative thermal noise for the two test masses combined as being proportional to their root sum squared:

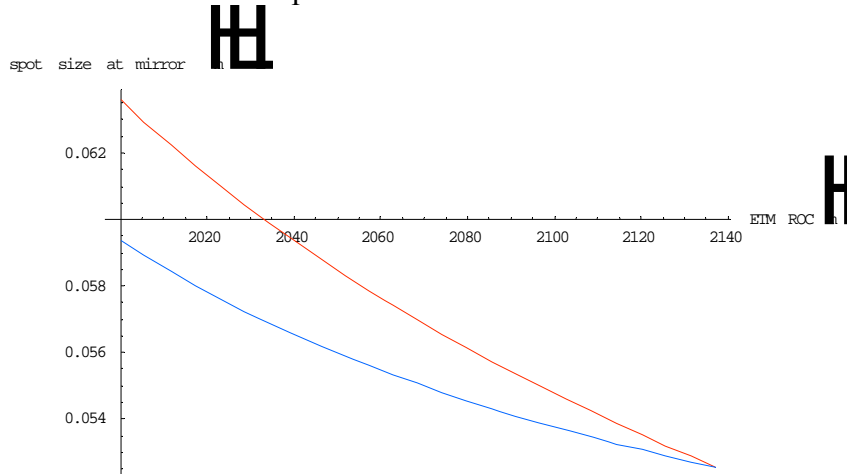
$$TN \propto \frac{\sqrt{w_{itm}^2 + w_{etm}^2}}{w_{itm}w_{etm}}$$

For 6cm spot sizes at both mirrors,  $TN=23.55$ .

At full power, the uncompensated test masses will distort to an effective ROC of 2137m. Since the coating contributes both the bulk of the absorbed power to the mass and the predominant part of the surface thermoelastic deformation, both masses have very nearly the same distorted ROC. The spot sizes then become 5.3cm, and the thermal

noise factor rises to 26.91, a 14% increase. This may be tolerable, but is worth preventing if possible.

Figure 1 shows the spots sizes at both mirrors when the ITM is allowed to thermally distort to a ROC of 2137 and the ROC of the ETM is tuned from 2137m to 2000m. The spot size at the ITM is restored to its nominal 6.0cm value when the ETM ROC is 2033m. The spot size at the ETM is 5.7cm for this ROC.



The thermal noise parameter for this configuration is 24.20, which is a 2.8% increase over the nominal thermal noise level. The increase in the test mass temperature will also contribute to a rise in the thermal noise level. The test mass ring heater design, applied equally to each test mass, would increase the temperature of each test mass by 5.4K, or 1.8%. Since the noise amplitude density is proportional to the square root of the temperature, this would cause an overall 0.9% increase in the thermal noise from both test masses combined. If all the compensation is applied to the ETM, then that mass will require approximately twice the applied heat, and so will rise 3.6% in temperature, but since the ITM temperature does not rise this again increases the thermal noise by 0.9%, to which we must add the 2.8% increase caused by spot size variation. So, the net thermal noise increase for the ETM-only compensation strategy is 3.7%, compared to 0.9% for the balanced compensation strategy.

For comparison, the level of coating thermal noise itself depends upon the mechanical loss of the coating materials, which are typically known to ~10-20%. This makes the thermal noise amplitude uncertain at the 5-10% level.

## IMPLICATIONS FOR TCS

This note shows that the spot size can be maintained at the ITM without direct actuation on the ITM itself. However, the wavefront ROC of the cavity mode will still be given by the thermally distorted ITM ROC, which is 2137m, or  $1.37 \times 10^{-5}$  diopters from the nominal 2076m ROC. However, this aberration is small compared to the ITM thermal lens in the substrate, which is  $\sim 10^{-4}$  diopters, so it requires only a small increase in the power needed from the TCS system. Compared to the complete change in the required TCS heating profile required if the ITM is directly compensated, it is a much more practical option.