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Parametric Instability with General Recycling

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INTRODUCTION

The original work on interferometer parametric instabilities (PI) [1] emphasized that parameters of Advanced LIGO (arm power level and TM internal mode Q values) were sufficient to allow instability growth. This took no account of the parametric feedback gain being modified by the recycling cavities (PRC or SRC). Subsequently the same authors [2] pointed out that, in an ideal interferometer (identical arm cavities and strictly degenerate PRC), the parametric feedback gain could be substantially enhanced via "power recycling", at least in a narrow instability frequency band corresponding to the PRC + arm double resonance width. However this work also pertained to only a strictly degenerate PRC as well as to identical arms. The next step in refinement [3] was to point out that this narrow PRC recycled enhancement is all but eliminated for reasonably expected arm [tolerance] differences and for sufficient PRC non-degeneracy.

Although the algorithm written, [3], to calculate recycling effects was fully general (including SRC, different arms, non-degeneracy, losses), the various possible regimes of interest were not fully studied and categorized. Subsequently, D. Ottoway has pointed out [4] that the SRC acts to modify instability gain in an important way, especially when substantial degeneracy for that cavity is considered. Here I fully exercise the algorithm [3] to elucidate some striking general features of the dual recycled interferometer (both PRC and SRC where the PRM and SRM have approximately the same reflectivity). The configuration has some revealing symmetries that predict (in an equivalent analytic model) important general features of parametric gain. These features are fully reflected in the numerical results.

1. Specification of the Model

The model considered here (as well as that of [4]) is strictly a single mode analysis. All frequencies are with respect to the carrier in an exactly double resonant configuration with no Michelson fringe or cavity offsets. Of course, in the actual interferometer, the only relevant PI modes would be higher transverse modes (HTM). Here, the simulation distinguishes transverse modes only by: 1. putting in a pre-determined arm loss by hand, and 2. adjusting frequency offset (from the carrier) to account for pre-determined arm cavity Gouy phases. We assume that PI is isolated to only *one* acoustic resonance in *one* TM (so that the incipient PI light originates as a localized "source" in one arm). The full set of parameters which specify the model are:

1. Δ = the frequency offset of the PI mode from that which would make it resonant in the exciting (source) arm. Δ=0 does not mean that this will be resonant in the "other" arm since the "other" arm may have a different Gouy phase (see next).

2. δ = the frequency offset of the PI mode, with respect to Δ, in the "other" non-exciting arm cavity.

3. Arg[ρ^2]= R.T. phase of PI mode in PRC. For the carrier this = Pi. If the PRC is exactly degenerate this = Pi for every PI mode. Frequency dependent transit time phase is assumed negligible (for this short cavity).

4. Arg[η^2]= R.T. phase of PI mode in SRC. For the carrier this = SRC GW sb detuning, and is the same for every PI mode if the SRC is degenerate. Frequency dependent transit time phase is assumed negligible.

4. r_p = amplitude reflectivity (positive, real) of the PRM ~ [nominally] .96954.

5. r_s = amplitude reflectivity (positive, real) of the SRM ~ [nominally] .97468.

6. r_{fp} = amplitude reflectivity (positive, real) of the ITMs ~ [nominally] .9975.

7. r_e = amplitude reflectivity (positive, real) of the ETM ~ [nominally] .999999.

8. L = RT power loss in each arm cavity (SRC and PRC loss assumed negligible, or fixed). Nominally 75ppm for the fundamental mode.

Since PI threshold ("gain">1) has been completely described [1,2] within this framework for a single (arm) cavity, throughout the following we describe all gain values ("enhancement" if >1) normalized to that for the same situation but with no PRC or SRC.

2. Review PRC enhancement

PI in an interferometer with *only* PR is worth separately reviewing since, 1. the published discussions only include this case, and 2. it has a particularly intuitive description (which makes clear its identity with the SR alone configuration).

Without loss of generality we consider PI excitation from a particular ETM acoustic mode (in the "exciting" arm). Incipiently, the field excited by the Doppler scattering from this acoustic perturbation may be considered resultant from an equivalent strength and frequency excitation beam mode "input" through the ETM. The steady state build up of this excitation field in the "exciting" arm is illustrated in Figure 1. More precisely the quantity of interest (gain proportional to the PI "R" value of Ref. [1,2]) is the real part of this steady state field impinging back on the ETM face.

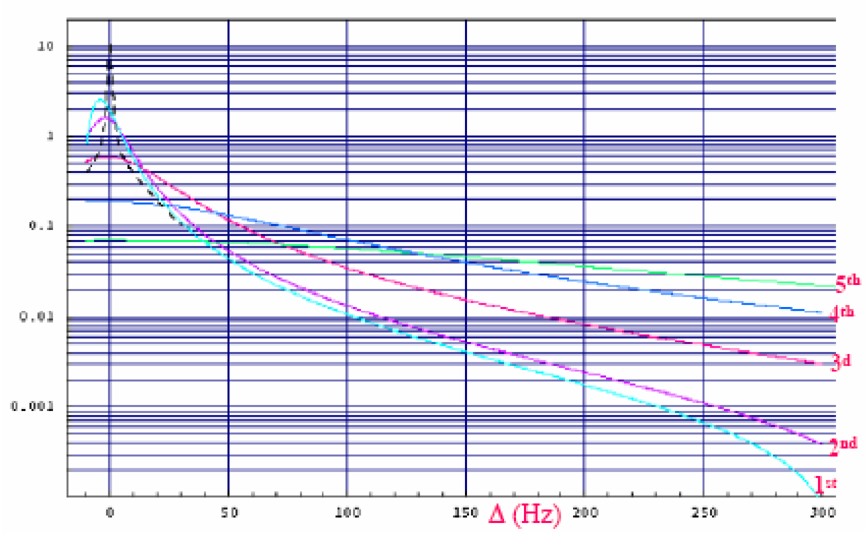


Figure 1. Build up of PI mode field in "exciting" arm. Field values are relative to values with no recycling. Dashed curve is for identical arms and carrier loss. Here ρ^2= -1.

This illustrates (dashed curve) that a very narrow enhancement is possible for an ideal interferometer (identical arms and degenerate PRC). However this enhancement can be killed in two ways. All the colored curves are for interferometers where the "other" arm has a slightly different Gouy phase (corresponding to only ~2m difference in mirror ROC). Maximum enhancement can only be approached for the two arms (for the particular PI HTM mode) being of order or closer simultaneously resonant than one PR double cavity line width.

The other mechanism for killing the enhancement is simply with sufficient arm loss for the PI HTM mode in question. For instance the numerals associated with the colored curves label HTM mode order, with concomitant steeply increasing diffractive arm RT loss (The loss here is parameterized in accord with the results in [5]). Only the first two mode orders have slight RC enhancement. These correspond to tilt modes and "donut" mismatch modes which are the only HTMs in the arms which have RT loss substantially < the ITM transmitivity. These also are special cases in that it is plausible that they would by easily controlled via WFS feedback and TCS Gouy phase fine tuning.

The fundamental (identical arm) enhancement can be simply explained. Instead of the very asymmetric PI excitation (at a single "exciting" ETM), consider a balanced, common mode excitation of *both* arms. By symmetry such an excitation is analogous to the frontal carrier excitation of the interferometer. There is a narrow double resonant enhancement in both arms, and no light is lost to the asymmetric port. Also consider a balanced but differential mode excitation of *both* arms. In this case all the the power (into the PRC) is lost to the asymmetric port. That is, none is recycled: no enhancement resulting. Then the actual PI excitation will be equivalent to ½ the superposition of this common and differential mode excitation. Therefore ½ the excitation strength is passively lost (to the asymmetric port) but ½ is recycled in the usual way we are familiar with for the carrier.

Of course, which "half" of the exciting field (common or differential) is recycled or lost depends only (strictly, if arms are identical) on whether there is a PRM or an

SRM present. Therefore we expect essentially the same results for PI enhancement for a SRC alone.

This ideal single RC PI gain enhancement is also diminished by detuning of the RC cavity phase (e.g by lifting the degeneracy, giving finite Gouy phase). This effect is illustrated in figure 2. The detuning effect is very broad and the gain peak shifts, both characteristic of detuning of double cavity resonance in general [6]. The nominal Adv. LIGO PRC (exact degenerate) corresponds to the "0.5" curve, whereas the [broadband] SRC would correspond to the "0.98" curve (i.e. PI gain *suppression* if this were the sole RC).

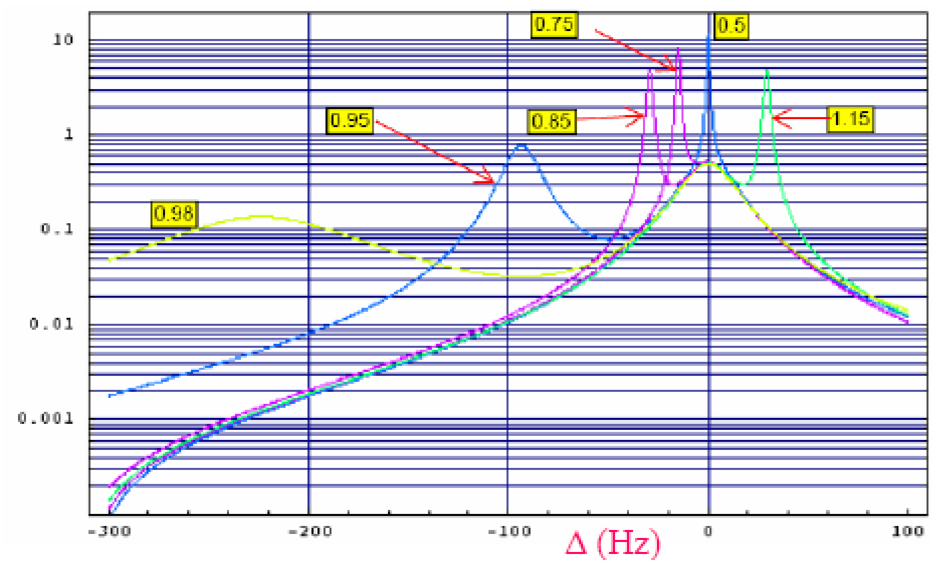


Figure 2. Relative PI gain for single RC (either PR or SR) as a function of Arg[ρ] (or η) values show in boxes in units of Pi. Arms identical. 0.5 peak curve is Identical to dashed curve in Fig. 1. Yellow is approx. the AdL SRC detuning.

Despite sharing many characteristics of the carrier double resonance, the PI excited field, even if Δ=0 (exact arm resonant), is *anti*-resonant in the RC (while the carrier is resonant). Nonetheless it may be shown that the ratio of arm to RC field strengths is the same as for the carrier (at least in the high finesse limit).

3. Full double recycled configuration.

If the PRC and SRC are identical ($r_s=r_p$, and $\rho=\eta$), then the narrow PI gain enhancement is doubled. If the two arms are identical (i.e. now a completely symmetrical interferometer) this is not so surprising, since *both* the common and differential excitations are recycled and equally contribute to the gain. This is illustrated in figure 3. What is surprising is that this result holds for any δ ("other" arm detuning)!

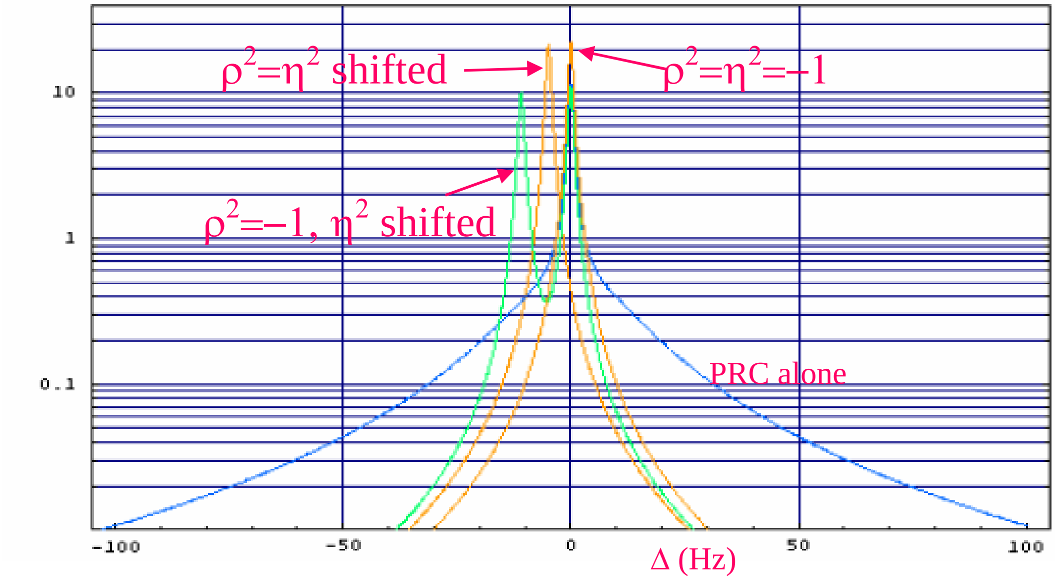


Figure 3 Both RCs acting together (except blue single cavity result, as in Fig. 1 for comparison). For equal PRC and SRC phase peak is ~2x PRC alone peak for *any* "other" arm cavity detuning. For SRC detuned from PRC the enhancement splits into two peaks (a "PRC enhancement", and a distinct "SRC enhancement"). For large detuning these peaks roll off like the double cavity envelope (Fig. 2).

This and other behavior of the general double RC gain are best described by considering an expression for the effective reflectivity of the "exciting" arm ITM (that is, the complex amplitude reflectivity of the remainder of the interferometer as seen from the PI excitation).

$$\mathcal{R} = r_{fp} - \frac{t_{fp} \cdot r_x}{1 - r_{fp} \cdot r_x}$$

This has been written to accentuate the analogy with the reflectivity of a simple two mirror cavity. Then, using the following "common/differential" definitions:

$$r_\Sigma \equiv \tfrac{1}{2}(r_p \rho^2 + r_s \eta^2)$$
$$r_\Delta \equiv \tfrac{1}{2}(r_p \rho^2 - r_s \eta^2)$$

we can express the "compound mirror" reflectivity:

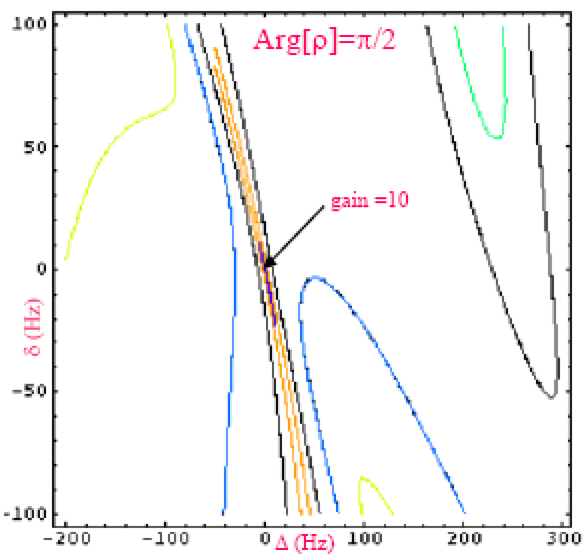
$$r_x = r_\Sigma + \frac{r_A \cdot r_\Delta^2}{1 - r_A \cdot r_\Sigma}$$

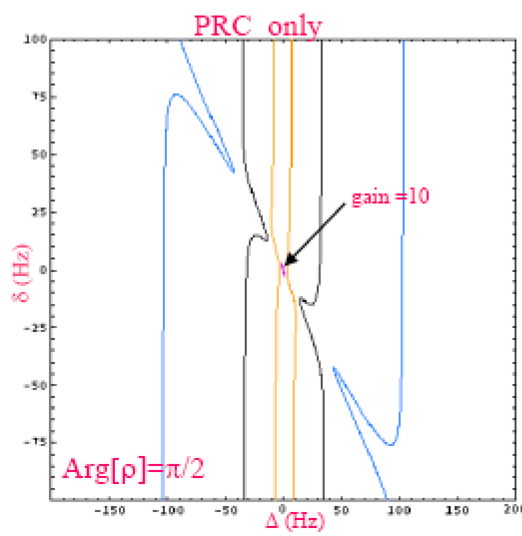
where r_A is the "other" arm reflectivity (as seen by the RC). For instance, if the PRC and SRC are "nearly" identical, $r_\Delta \approx 0$ and $r_x \approx r_\Sigma \approx -r_p$ nearly independent of the tuning of the other cavity, r_A. The proposed AdL r_p and r_s values are close enough that this condition holds effectively (the curves of Fig. 3 use these *unequal* values).

When the RC phases are different ($\rho^2 \neq \eta^2$) and at least with the arms identical, a very simple picture emerges. For each of the Σ and Δ modes there are almost independent gain peaks, each separately having height and detuning the same as for the single RC case (Fig. 2). For instance the Fig. 3 green curve ($\rho^2 = -1, \eta^2 = e^{i1.4\pi}$) consists of a Δ=0 PRC ≈ that in Fig 2, and a detuned SRC peak of the same height and position as that for the same tuned SRC alone. Whenever $\rho^2 = \eta^2$ these two RC gain peaks simply add (since the gain is ~field).

4. **Effects of difference in arm cavity Gouy phases.**

We have seen that various tunings of the PRC (via introduction of non-negligible Gouy phase) or SRC (via Gouy phase or signal recycling phase offset) merely move the position (in Δ) of the PI gain enhancement peak for small detuning from double resonance. In this regime the influence of unequal arm cavity Gouy phase ($\delta \neq 0$) is far less pronounced than for a single RC (Fig.1: killing of PI gain enhancement for small δ). We illustrate this now with η^2 fixed (at the nominal Adv. LIGO broadband signal tuning, Arg[η^2]=0.11) at a value far from the Fig. 2 regime. For such a large SRC detuning the Δ mode double resonance associated with it yields no PI gain enhancement (green ~peak contours in Fig. 4). Gain enhancement (within orange band in Fig. 4) is then exclusive to the PRC S mode. How this is ameliorated by "other"arm detuning (due to different arm Gouy phases) is described in the contour plots of Figure 4. Maximum achievable PI gain is constrained by finite PRC Gouy phase but only weakly unless the PRC is strongly focused.





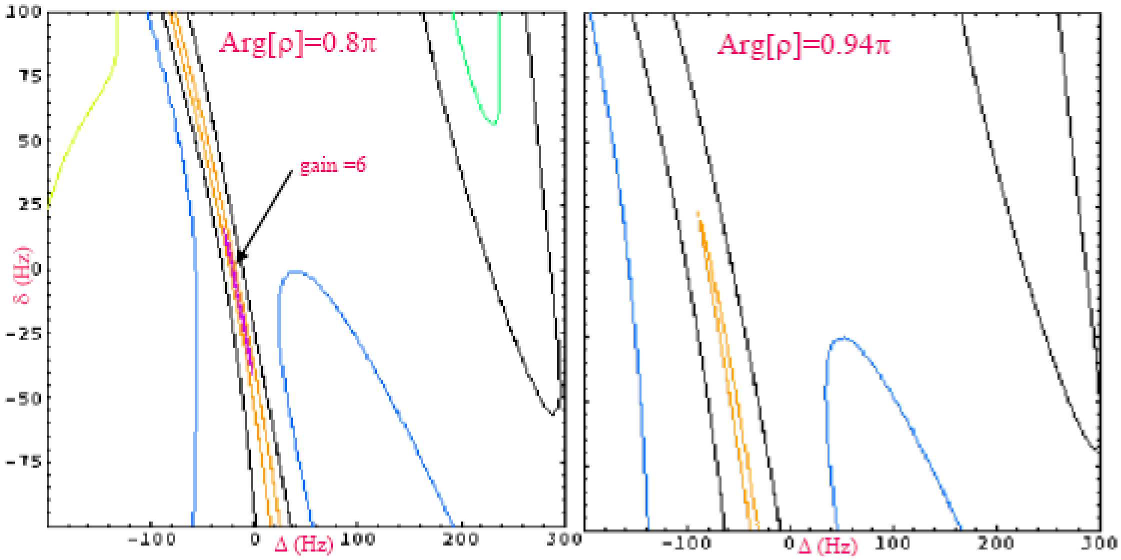


Figure 4. Contour plots of PI gain (relative to r_s=r_p=Δ=0) for Adv.LIGO SRC tuning (but Gouy phase =0). Plots differ in PRC Gouy phase, Arg[ρ]=π/2 being exactly degenerate. Orange is PI gain = 1 contour. Blue, g=.01. Black, g=.1. Yellow, g=.001. Green, g=.2

In each plot of Fig.4 the ~maximum enhancement band is contoured in red. This band is delineated by "other" arm detuning (δ). Compared to the PRC alone contour plot (a horizontal slice through which is the "1st" order curve of Fig. 1), a general characteristic of dual recycling is that this maximum enhancement band becomes much longer (in δ). It may be expected (from unavoidable TM optical fabrication tolerances) that $\delta \geq 20Hz$, which is still less than the enhancement band lengths encountered in dual recycling.

It must be emphasized that all the enhancement plots shown (except the higher order curves of Fig. 1) were for the least lossy, i.e. lowest order, HTM. Rapidly increasing diffraction loss with HTM order eliminates the possibility of PI gain enhancement for any PRC + SRC+ other arm δ tuning for all but the lowest few HTM orders. This is illustrated in Fig. 5. Diffraction loss simulation in Adv. LIGO like cavities predicts that only the first two HTM orders will have significant RC enhancement (Figs. 2-4 consider the first. The third order has maximal PI gain enhancement ~1 as show in Fig. 5). Even strong focusing introduced into the PRC (Fig. 5, right) only marginally eliminates PI gain enhancement (g<1).

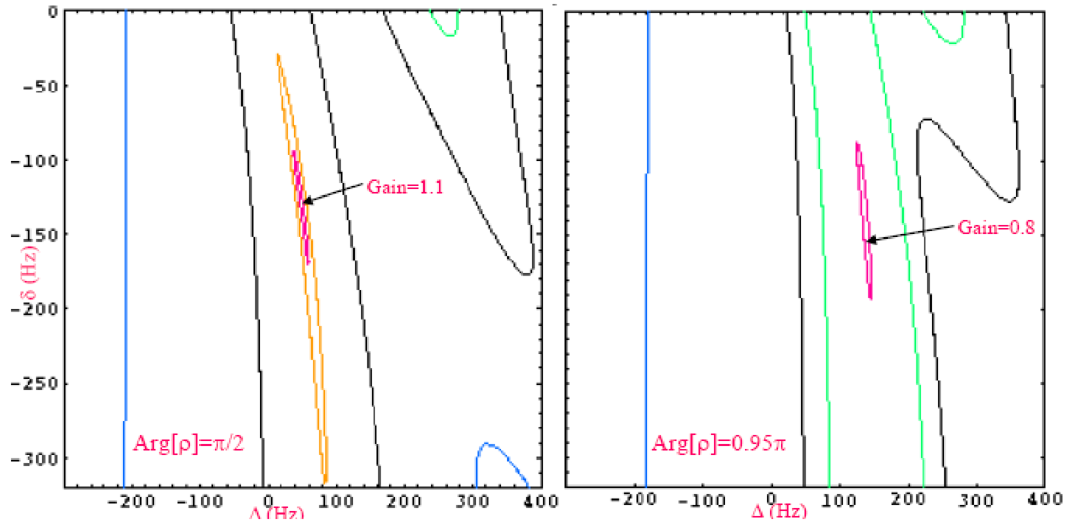


Figure 5. As in Fig.4, however now HTM order 3 (RT arm loss comparable to ITM transmission). Peak enhancement gain shown as red crest. Strong

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