Comparison of band-limited RMS between DARM_ERR and calibrated strain in S4

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This note summarises some results comparing the band-limited norm (BLN) or band-limited RMS of strain calculated from DARM_ERR (DE) with the calibrated strain data $h(t)$. The included plots show features of the relative difference between the BLN for each of the two estimates of strain over the whole of S4 for the H1, H2 and L1 interferometers.

For a given time interval (usually 60 seconds) we calculate the band-limited norm ρ_h of strain estimate $h(t)$ as

$$\rho_h \equiv ||h||_{\mathrm{BL}} = \sqrt{\sum_{k=k_1}^{k_2} |\tilde{h}_k|^2} \,. \tag{1}$$

where the frequency indices correspond to the frequencies over which the stochastic analysis is performed (50–500 Hz). The statistic to be analysed is the relative difference y of the BLNs for each type of strain estimate. Since we don't know which strain estimate is "best" the difference is taken relative to the average of the two BLNs, that is,

$$y = \frac{\rho_{\mathrm{DE}} - \rho_h}{\bar{\rho}} \,. \tag{2}$$

where

$$\bar{\rho} = \frac{1}{2}(\rho_{\mathrm{DE}} + \rho_h) \,. \tag{3}$$

Usually the definition of the BLN or RMS would include a scaling factor eg. $1/N$. This is omitted since we only ever look at the ratio of BLNs.

We also examine the relative distance as measured by the BLN:

$$\begin{aligned} z &= \frac{d(h^{\mathrm{DE}}, h)}{\bar{\rho}} \\ &= \frac{||h^{\mathrm{DE}} - h||_{\mathrm{BL}}}{\bar{\rho}} \,. \end{aligned} \tag{4}$$

as another measure of relative error between calibrated DARM_ERR and $h(t)$. (Note that a spectrally-weighted norm would be a better metric but code for this has not yet been included in the analysis).

The procedure to generate the frequency domain form of the input data is based on that used in `stochastic.m`:

1. Take a segment of data (60 seconds)
2. Downsample to 1024 Hz
3. High-pass filter above $f_1 = 50$ Hz
4. Apply a window function
5. Pad with zeroes
6. Take the DFT
7. Coarse grain down to the specified frequency spacing (0.25 Hz in this case)
8. Apply the response function ($R(f) \equiv 1$ is used for $h(t)$)
9. Notch out the same lines as used by the stochastic analysis. In addition, all calibration lines are notched out.

1 H1 Results

For H1 a total of 33419 segments were analysed. Comparing the raw BLNs, Figure 1 shows a histogram of ρ_{DE} in blue with the histogram for ρ_h overlaid in red. The bottom graph shows the CDFs. The distributions look very similar, although there is a noticeable bias (see below).

We performed a K-S test to see if the distributions of ρ_{DE} and ρ_h were different. Since we expect the values to be correlated, we obtained independent samples by only using even-numbered values for ρ_{DE} and odd-numbered values of ρ_h. As we would expect from the observed bias, the K-S test rejects the hypothesis that the distributions are the same at significance $\alpha = 0.05$. Figure 2 shows a scatter plot of ρ_{DE} versus ρ_h. As expected this shows a very high degree of correlation. The correlation coefficient is $r = 0.993778$.

The first plot in Figure 3 shows the relative difference between the BLNs for each segment as a function of GPS time at the start of the segment. The gaps are when H1 is out of science mode, and the horizontal dashed line indicates the mean value. The second graph shows a histogram of y with the best-fit normal curve overlaid, and the third shows the CDFs of y and the best-fit normal distribution.

1. The mean difference (bias) over the whole of S4 was $\mu = 0.0066$ ie. on average the relative error between ρ_{DE} and ρ_h is about 0.66%.
2. The standard deviation was $\sigma = 0.017$.

3. 31 or 0.09% of the differences were more than 5σ away from the mean.
4. 345 or 1.03% of the differences were more than 3σ away from the mean.
5. 95% of the differences were within $\pm 3.63 \times 10^{-2}$ of zero, indicated by the green shaded part of the histogram ie. typically differences were within $\pm 3.63\%$.
6. Although we have shown a normal distribution for comparison, a K-S test rejects the hypothesis that the distribution is normal at significance $\alpha = 0.05$.

The first graph in Figure 4 shows the relative BLN distance z between $h^{\mathrm{DE}}(t)$ and $h(t)$ for each segment as a function of GPS time. The second graph is a histogram of these values, which have mean $\mu = 0.1476$. Since this is clearly not normal we have not attempted to fit a normal to it.

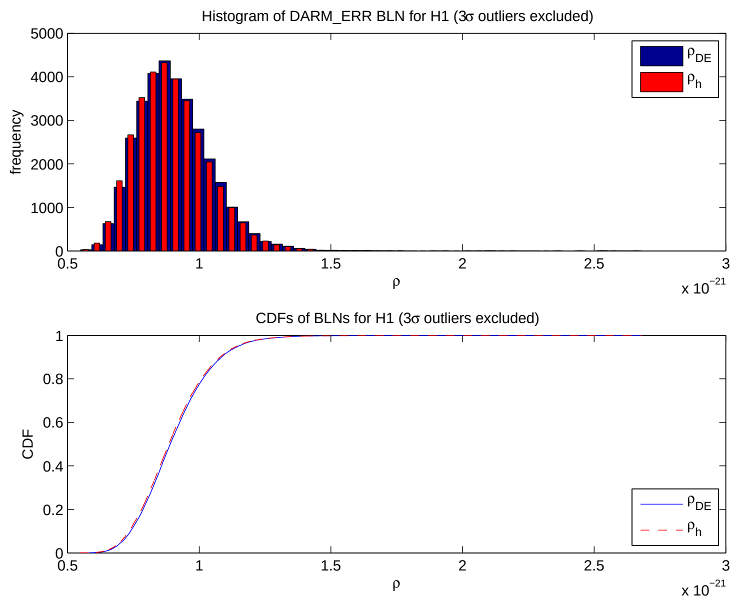

Figure 1: Distribution of ρ_{DE} and ρ_h for H1 (3σ outliers excluded)

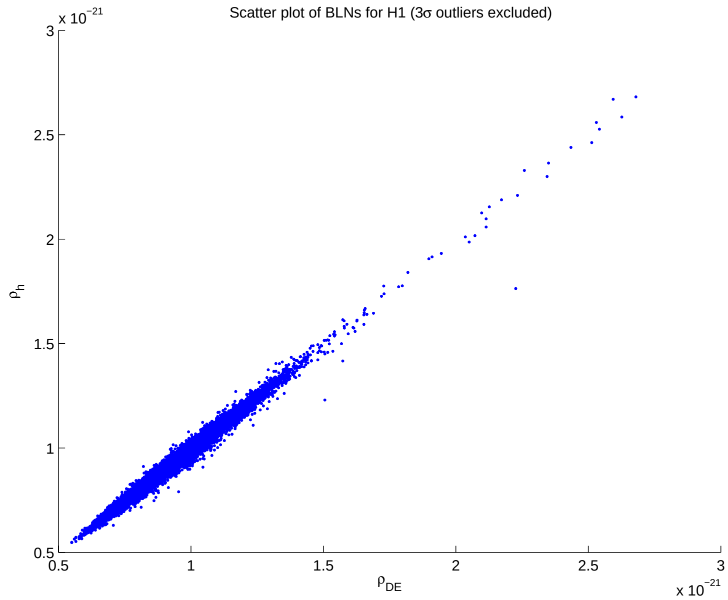

Figure 2: Scatter plot of ρ_{DE} and ρ_h for H1 (3σ outliers excluded)

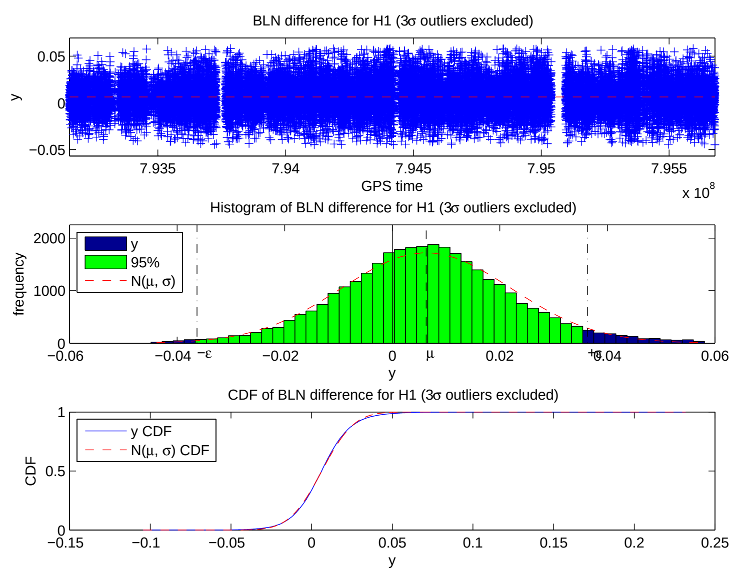

Figure 3: Relative difference of ρ_{DE} and ρ_h for H1 (3σ outliers excluded)

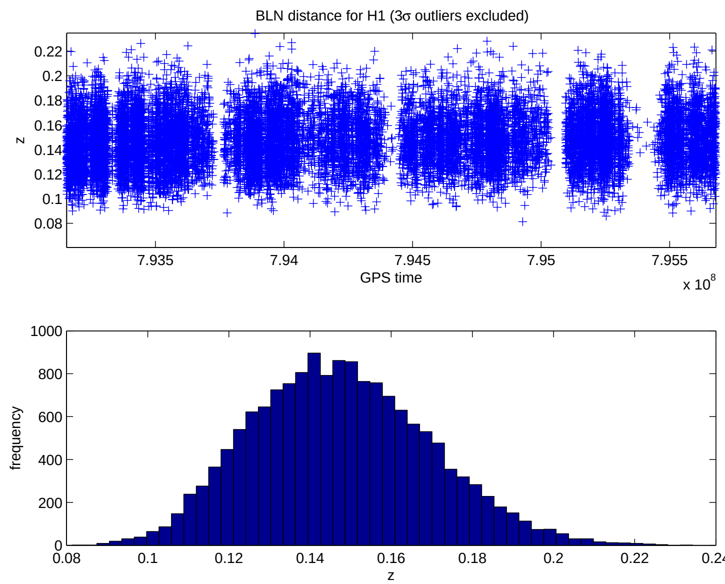

Figure 4: Relative BLN distance between $h^{\mathrm{DE}}(t)$ and $h(t)$ for H1 (3σ outliers excluded)

2 H2 Results

For H2 a total of 33139 segments were analysed. The distributions in Figure 5 look very similar. The bias is much smaller than for H1 and and a K-S test does not reject the hypothesis that the distributions are the same at significance $\alpha = 0.05$. The scatter plot Figure 6 shows a high degree of correlation, with correlation coefficient $r = 0.99037$.

Initially the values for the ρ_{DE} and ρ_h looked very different. It was found that the calibration line at 54.1 Hz was insufficiently notched out. After increasing the notch width from 0.25 Hz to 0.75 Hz the difference went away. The fact that such a large difference can occur may flag that we should be notching out calibration lines in the stochastic analysis, something which is currently not being done.

After re-running with the calibration lines more aggressively notched out (see Figure 7):

1. The mean difference (bias) over the whole of S4 was $\mu = 0.0011$ ie. the average relative error between ρ_{DE} and ρ_h is about 0.11%.

2. The standard deviation was $\sigma = 0.0224$.

3. 125 or 0.38% of the differences were more than 5σ away from the mean.

4. 406 or 1.23% of the differences were more than 3σ away from the mean.

5. 95% of the differences were within $\pm 3.96 \times 10^{-2}$ of zero ie. typically differences were within $\pm 3.96\%$.

6. A K-S test rejects the hypothesis that the distribution is normal at significance $\alpha = 0.05$.

For the distribution of relative BLN distance (see Figure 8) the mean is $\mu = 0.1183$.

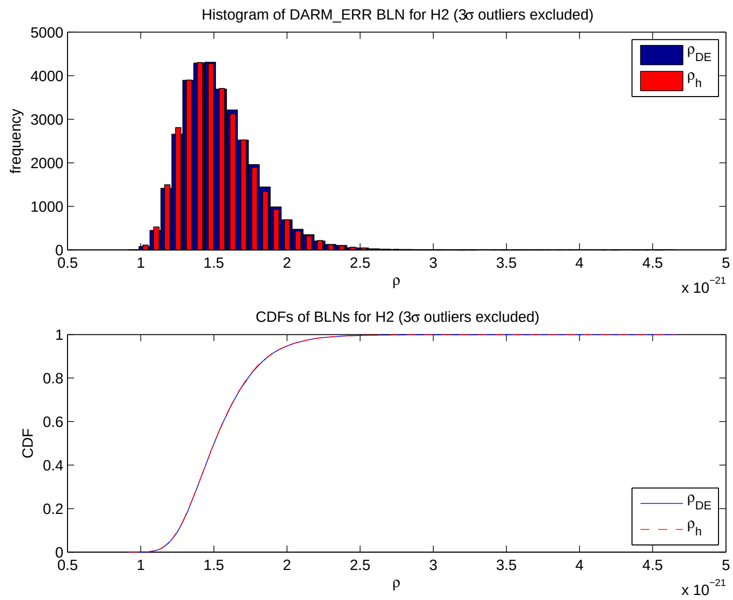

Figure 5: Distribution of ρ_{DE} and ρ_h for H2 (3σ outliers excluded)

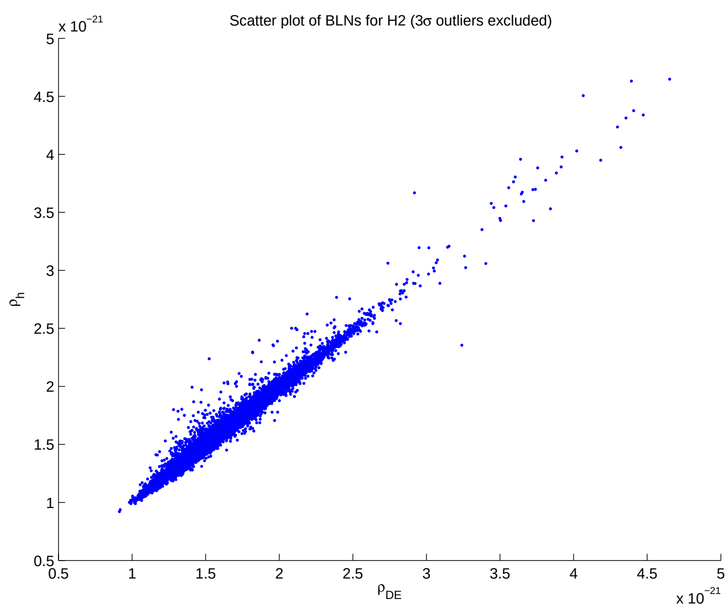

Figure 6: Scatter plot of ρ_{DE} and ρ_h for H2 (3σ outliers excluded)

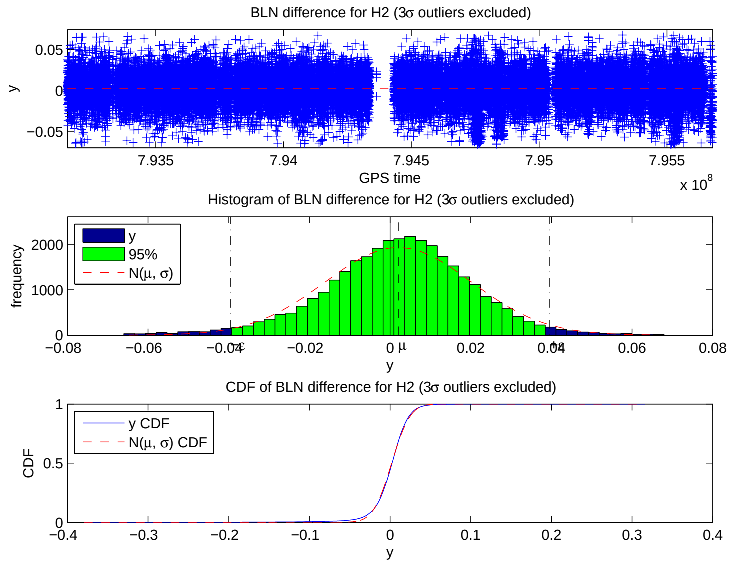

Figure 7: Relative difference of $\rho_{\rm DE}$ and ρ_h for H2 (3σ outliers excluded)

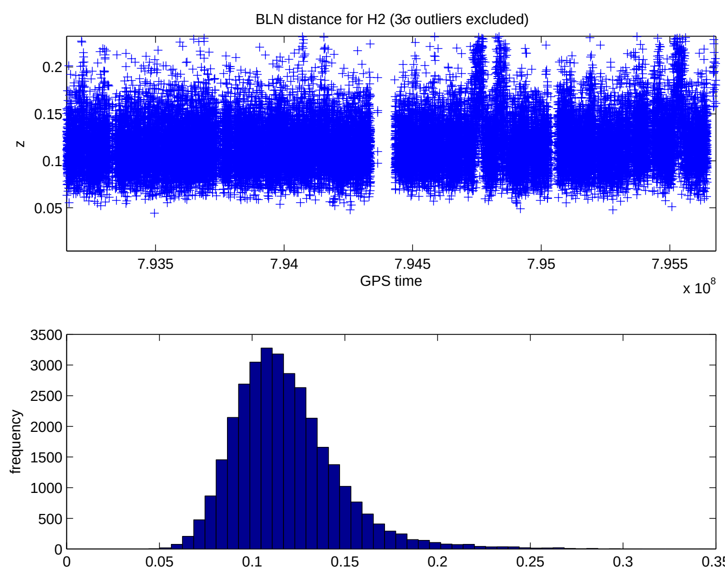

Figure 8: Relative BLN distance between $h^{\rm DE}(t)$ and $h(t)$ for H2 (3σ outliers excluded)

3 L1 Results

Initially a large number of outlying values were found in the GPS range 793340000–793410000 (approximately). Figure 13 shows the raw ρ_{DE} and ρ_h over GPS time in the first two graphs and their relative difference in the third. Since the large excursions occur in both DARM_ERR and $h(t)$ for L1 but not H1 or H2, it seems unlikely to be a bug in the analysis code. Until some further investigation is carried out to see where these excursions are coming from we have left these data points out of the L1 analysis.

The remaining data gives a total of 30379 segments. The distributions in Figure 9 again look very similar. The bias is much smaller than for H1, however a K-S test rejects the hypothesis that the distributions are the same at significance $\alpha = 0.05$. The scatter plot Figure 10 shows a high degree of correlation, with correlation coefficient $r = 0.999461$.

For the distribution of relative differences (see Figure 11):

1. The mean difference over the whole of S4 was $\mu = 0.000883$ ie. average relative error between ρ_{DE} and ρ_h is about 0.0883%.

2. The standard deviation was $\sigma = 0.0104$.

3. 14 or 0.05% of the differences were more than 5σ away from the mean.

4. 342 or 1.13% of the differences were more than 3σ away from the mean.

5. 95% of the differences were within $\pm 2.12 \times 10^{-2}$ of zero ie. typically differences were within $\pm$2.12%.

6. A K-S test rejects the hypothesis that the distribution is normal at significance $\alpha = 0.05$.

7. There appears to be a marked improvement in calibration for L1 about halfway through S4.

For the distribution of relative BLN distance (see Figure 12) the mean is $\mu = 0.0506$.

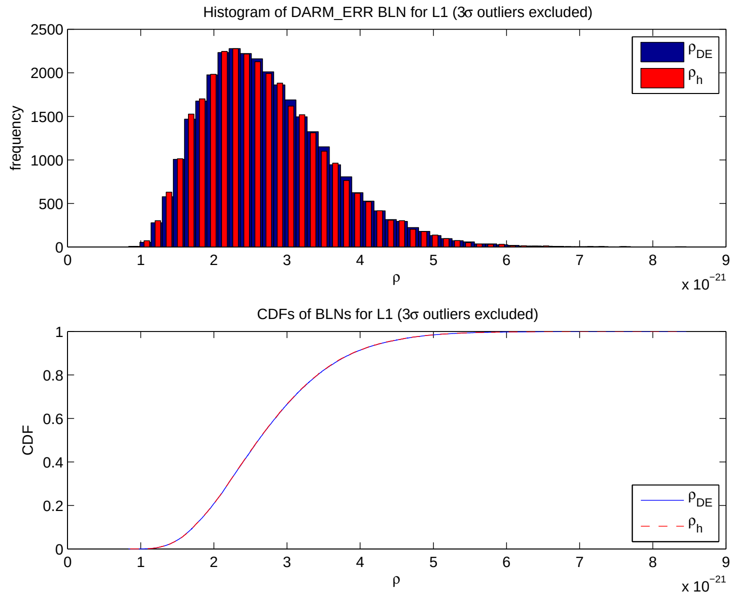

Figure 9: Distribution of $\rho_{\rm DE}$ and ρ_h for L1 (3σ outliers excluded)

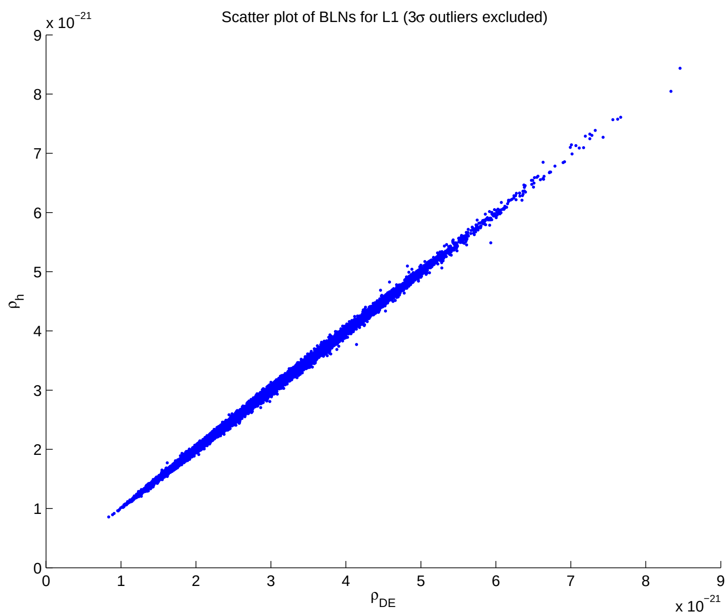

Figure 10: Scatter plot of $\rho_{\rm DE}$ and ρ_h for L1 (3σ outliers excluded)

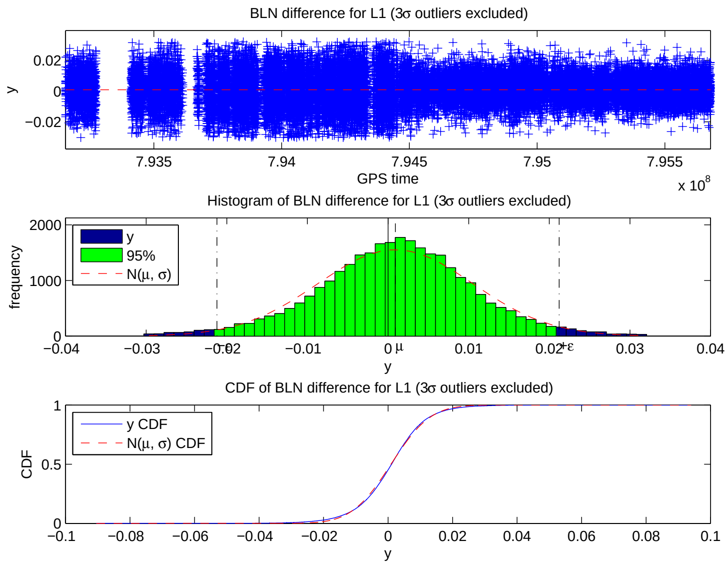

Figure 11: Relative difference of ρ_{DE} and ρ_h for L1 (3σ outliers excluded)

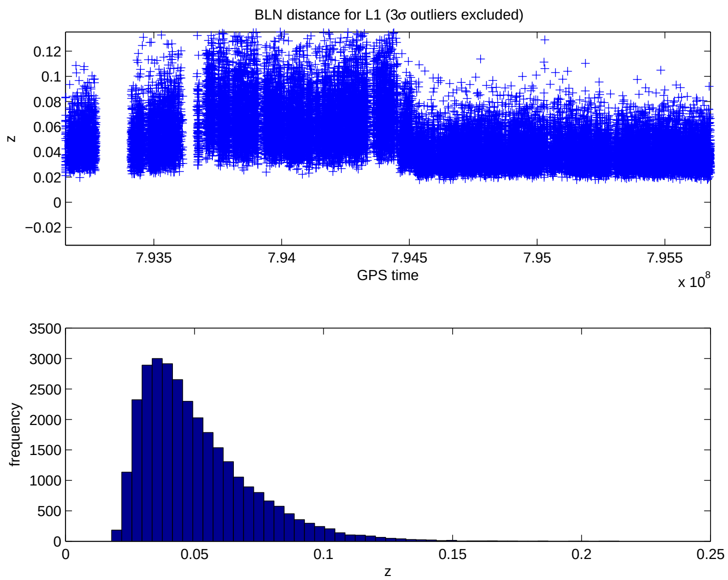

Figure 12: Relative BLN distance between $h^{\mathrm{DE}}(t)$ and $h(t)$ for L1 (3σ outliers excluded)

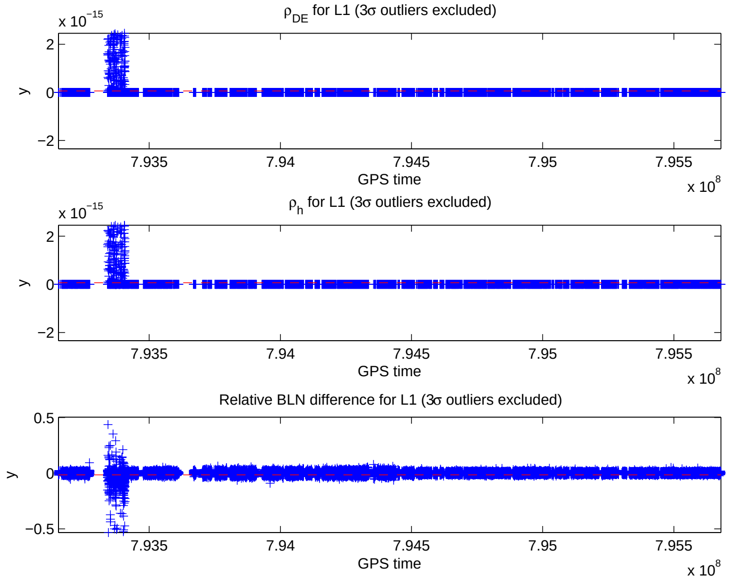

Figure 13: BLN of DARM_ERR and $h(t)$ over time for L1 (3σ outliers excluded)

4 Comparison of results from stochastic analysis

We have also compared results for the value of Ω_{GW} estimated using DARM_ERR and $h(t)$ for

- Hardware injections of stochastic signal
- Analysis of data between H1-L1, time-shifted to verify that a null result is obtained

NOTE: this comparison was performed using V1 calibration and has not been repeated for V4 calibration.

For the hardware injections we used the segment 794628335–794629295. The injected data corresponded to a signal with $\Omega_{\mathrm{GW}} = 0.01$. The results obtained using various channels were:

$$\begin{aligned} \text{AS\_Q:} \quad & \Omega_{\mathrm{GW}} = -8.991413 \times 10^{-3} \pm 1.016841 \times 10^{-3} \\ \text{DARM\_ERR:} \quad & \Omega_{\mathrm{GW}} = -9.242472 \times 10^{-3} \pm 1.048486 \times 10^{-3} \\ h(t): \quad & \Omega_{\mathrm{GW}} = -9.090732 \times 10^{-3} \pm 1.024118 \times 10^{-3} \end{aligned}$$

For the time-shifted analysis we used all of S4 science data for H1 and L1 only, with a 1-second time shift. The results obtained were:

$$\begin{aligned} \text{DARM\_ERR:} \quad & \Omega_{\mathrm{GW}} = 2.3910 \times 10^{-7} \pm 2.2317 \times 10^{-5} \\ h(t): \quad & \Omega_{\mathrm{GW}} = -2.8761 \times 10^{-6} \pm 1.8379 \times 10^{-5} \end{aligned}$$