Reaction mass coupling at ETM/ITMs

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Introduction

A collection of the information needed to evaluate the tolerable stiffness of reaction chain wiring on the Advanced LIGO ITM/ETM suspensions. Refererences are given throughout.

Notes: all quantities are given in unscaled SI units (m,N, kg, etc.), and amplitude spectral density where appropriate.
Calculations are all for 10 Hz: motion of reaction and main suspensions should fall off with approimately the same power of frequency, and longitudinal suspension noise plays no role above about 10 Hz. It is assumed, without a comprehensive check, that longitudinal is the important degree of freedom.
TM = Test Mass, PM = Penultimate Mass, UIM = Upper Intermediate Mass, UM = mass at the top, SP = Suspension Point

In[356]:=
```
Off[General::Spell]; Off[General::Spell1]; (*some Mathematica stuff, ignore*)
```

Knowns

■ Coupling factors from PM, and UIM to TM at 10 Hz, and tolerable displacement noise at 10 Hz

These are needed to establish the tolerable displacement noise, and are taken from a recent Quad model (*MATLAB* extracted from *Mathematica*).

In[357]:=
```
cPMTM = 9.6 / 2.3 10^-3;
(*from ratio of SP to PM and SP to TM factors, read from MATLAB*)
cUIMTM = 9.6 / 1.4 10^-5;
(*from ratio of SP to UIM and SP to TM factors, read from MATLAB*)
```

In[359]:=
```
xTM = 10^-20; (*tolerable displacement noise at 10 Hz at TM, from SUS DRD*)
xPM = xTM / cPMTM; (*therefore at PM*)
xUIM = xTM / cUIMTM; (*and UIM*) (*... at UIM*)
```

■ "Stiffness" of suspensions stages at 10 Hz

Needed to determine the displacement due to unwanted coupled force from the actuator. This was obtained, directly, from the same suspension models.

In[362]:=

```
kcTMTM = 6 10^-6;
kcPMPM = 6 10^-6;  (*sic*)
kcUIMUIM = 1.3 10^-5;
```

■ Bias or *rms* force at the TM, PM, and UIM

```
biasTM = 5 10^-7;  (*Science mode ESD bias force see  T060006-00 and/or G060014-00,
 where this is the maximum force in "Mode E",
 the  mode that should be used in final science mode.  Note that Mode D,
 10 times stronger, may also be used to achieve close to the design performance,
 and so extra margin should be allocated*)
FrmsPM = 1.3 10^-3;
(*Extremely  - too?-  conservative "noisy mode" UIM actuator rms force,
 see http://www.its.caltech.edu/~rana/aLIGO/suselecreq.html,
 should be ~100 times smaller in final science mode,
 in which case requirements would be relaxed in proportion,
 but are in any case easily met*)
FrmsUIM = 1.2 10^-3; (*Extremely conservative noisy science mode UIM actuator
  rms force, see http://www.its.caltech.edu/~rana/aLIGO/suselecreq.html
   note angle biases are offloaded to UIM,
 otherwise these dominate by a factor ~300 *)
```

■ Actuator coupling at the TM, PM, and UIM

■ TM actuator (ESD) - ETM only (not fitted on ITM suspensions).

The electrostatic drive force, at the design operation point, has the following approximate dependence on the separation of the TM and reaction mass. This is based on semi-empiracle modelling for GEO.

In[365]:=

```
x = .;
FTM = biasTM x^(-3 / 2);
(*model - approximate; plausible: must be less than square law,
 more than linear; power law is a fit to an early
  measurement with only slightly different electrode geometry*)
dFTMx = D[FTM, x];  (*take gradient*)
x = 0.005; (*nominal operating point of ESD*)
```

■ PM/UIM actuators (EM)

The magnet designs have, in principle, zero coupling at the optimal operating poing, but this increases (approximately equally) for a displacement from that point in any direction. The figure given here was obtained with about 3 mm offset from the design point. This covers minor design errors, assembly errors and the operating range. The calculation was done using the magnet model by Barton. The UIM figure is applied to the PM, as an absolute worst case (the PM design is currently subject to change).

In[368]:=
```
dFUIMx = 500 ;  (*approximate, applies - conservatively - to PM*)
```

Calculations and Results

The allowed motion of the appropriate reaction mass is given (results are in m/Hz^(1/2)).

■ TM

```
coupleTM =  -dFTMx kcTMTM;  (*note that the bias force was already built in above*)
xrTM = N[ xTM  / coupleTM]
```

Out[369]=
3.92837×10^{-15}

■ PM

```
couplePM = FrmsPM dFUIMx kcPMPM; (*note that the bias was not built in above*)
xrPM = N[xPM  / couplePM]
```

Out[370]=
6.14316×10^{-13}

■ UIM

```
coupleUIM = FrmsUIM dFUIMx kcUIMUIM;
(*note again that the bias was not built in above*)
xrPM = N[xUIM  / coupleUIM]
```

Out[371]=
1.86966×10^{-11}

In[372]:=
```
(*but 300 times smaller if angle biases applied here*)
```

Conclusions

For the purposes of the following discussion the SP motion is taken to be 3 10^-13 m/Hz^(1/2), and the estmate of the required isolation is rounded up to the nearest integer multiple of 10 dB.
No isolation is needed for UIM or PM stages (at 10 Hz), from the point of view of meeting requirements at those stages. This is not true if angle biases are applied to the UIM, when 20dB isolation, from the SP to the UIM reaction mass, would be needed.
At least 40 dB isolation is required from SP to TM, and some margin should be added to allow for slightly increased ESD forces (the ESD could provide up to 10 times higher force while barely meeting science mode requirements). A reasonable, safe, goal would be to provide at least 60 dB of isolation from SP to reaction mass. One option would be to aim to provide something like 10, 15, 15 and 20dB for the 4 stages down the pendulum chain (there are fewer wires connecting to the lower stages).