Analysis of Stray Magnetic Fields from the Advanced LIGO Faraday Isolator T060025-00-D

Guido Mueller, Simon Stepuk, Volker Quetschke

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1 Introduction

The Faraday isolator contains strong permanent magnets in a quadrupole configuration. The magnetic fields from the isolator will create forces which will push or pull on the mirrors. Changes in the distances between the isolator and the mirrors will cause fluctuations in the forces and might reduce the stability of nearby optical components. In the first part of this note, we will give a brief review of magnetic forces, in the second part we will briefly discuss the stray magnetic field coming from the current advanced LIGO isolator. The third part will derive the requirements on the positional stability of the non-suspended isolator with respect to the mode cleaner mirrors.

2 Magnetic Forces and torques

Magnetic fields generate forces on permanent magnets like the magnets on the mode cleaner mirrors used to align the MC. These forces and torques are:

$$\vec{F} = \nabla(\vec{\mu} \cdot \vec{B}) \qquad \vec{\tau} = \vec{\mu} \times \vec{B} \tag{1}$$

where $\vec{\mu}$ is the magnetic dipole moment of the magnet. These forces and torques present only a problem when the magnetic field $\vec{B}$ changes in time. This is caused by the changes in the relative position of the isolator with respect to the mirror and subsequently the magnets.

3 Requirements

The Faraday isolator shares a HAM table with two mode cleaner mirrors, several steering mirrors, and the large mirror of the beam expanding telescope. The current baseline Advanced LIGO design assumes that the beam expanding telescope is not part of the recycling cavity although it is possible that the telescope moves into the recycling cavity. In that case, one small power recycling mirror and one large power recycling mirror

Mirror	Displacement	Coupling
Mode cleaner	$3\times10^{-17}\left(\frac{10\,\mathrm{Hz}}{f}\right)^2\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$	Laser frequency noise
Steering mirrors	$10^{-12}\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$	Pointing (estimated)
Recycling mirror	$3\times10^{-16}\frac{10\,\mathrm{Hz}}{f}\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$	Laser frequency noise

Table 1: Requirements on optical components near the Isolator. Source: LIGO T010097-0-D Auxiliary Suspended Optics Displacement Noise Requirements, P. Fritschel

will also be in the chamber. All other optical components will be to far away and will not be affected by the magnetic field of the isolator.

A second isolator will be placed at the dark port isolating the output mode cleaner from the main interferometer. The location and layout of these chambers are not well known yet. However the requirements on the output mode cleaner and on the signal recycling mirror, the most sensitive optical components in that port, are very similar if not less stringent then on the components in the input port. We will focus therefore only on the IO-chain.

The mass and the moment of inertia of an Advanced LIGO mode cleaner mirror (cylinder with radius $r = 7.5\,\mathrm{cm}$ and thickness $h = 10\,\mathrm{cm}$) are:

$$m = 3.89\,\mathrm{kg} \qquad I = m\frac{r^2}{4} + m\frac{h^2}{12} = 8.7\cdot10^{-3}\mathrm{kg\ m}^2 \tag{2}$$

The analyzed situation is shown in figure 1. The magnetic dipole moment of the LIGO I magnets on the mirrors is (see LIGO-T970149-00):

$$\vec{\mu} = 0.0107\mathrm{Am}^2 \tag{3}$$

The requirements on the stability of the mode cleaner and recycling cavity mirrors near the Isolator is shown in table 1.

A tilt by an angle ϕ of one of the mode cleaner mirrors will look like an apparent length change for a non-centered beam. The apparent length change is

$$\Delta l = \phi\Delta y \tag{4}$$

where Δy is the assumed offset of the beam position with respect to the center of mass of the mirror. Assuming that $\Delta y^{\mathrm{max}} = 1\,\mathrm{mm}$ we can derive the following requirement on the angular stability of the mode cleaner mirrors:

$$\phi^{\mathrm{max}}(f) = 3\times10^{-14}\left(\frac{10\,\mathrm{Hz}}{f}\right)^2\frac{\mathrm{rad}}{\sqrt{\mathrm{Hz}}} \tag{5}$$

In addition, the amplitude of the 10-mode into the interferometer has to stay below:

$$a_{10}^{max}(f) = \frac{7\cdot10^{-10}}{\sqrt{\mathrm{Hz}}}\sqrt{1+\left(\frac{230\,\mathrm{Hz}}{f}\right)^4}\frac{[10^{-8}\mathrm{rad}]}{\Delta\Theta_{\mathrm{ITM}}} \approx \frac{3.7\cdot10^{-7}}{\sqrt{\mathrm{Hz}}}\left(\frac{10\,\mathrm{Hz}}{f}\right)^2\frac{[10^{-8}\mathrm{rad}]}{\Delta\Theta_{\mathrm{ITM}}} \tag{6}$$

assuming a differential ITM tilt of $\Delta\Theta_{\mathrm{ITM}} = 10^{-8}$rad. This will put some requirements on the alignment of the optical components. The amplitude of the generated 10-mode can be estimated using the divergence angle α:

$$a_{10}(f) = \frac{\phi(f)}{\alpha} = \frac{\phi(f)}{1.7\cdot 10^{-4}\mathrm{rad}} < a_{10}^{\max}(f) \qquad \alpha = \frac{\lambda}{\pi\omega} \qquad \omega \approx 2\,\mathrm{mm} \tag{7}$$

The maximum allowed angular fluctuations are then:

$$\phi^{\max}(f) = 6.3\cdot 10^{-11}\,\frac{\mathrm{rad}}{\sqrt{\mathrm{Hz}}}\left(\frac{10\,\mathrm{Hz}}{f}\right)^{2}\frac{[10^{-8}\mathrm{rad}]}{\Delta\Theta_{\mathrm{ITM}}} \tag{8}$$

Obviously, the geometry effect (Eq. 5) dominates the requirements on angular tilt.

The expected displacement of the Faraday isolator on the optical table is given by the displacement of the HAM table (see page 7 E990303-03-D):

$$\Delta r(f) = \frac{10^{-11}\mathrm{m}}{\sqrt{\mathrm{Hz}}}\left(\frac{\mathrm{Hz}}{f}\right)^{3/2} \qquad \Delta r(10\,\mathrm{Hz}) = \frac{3.2\cdot 10^{-13}\mathrm{m}}{\sqrt{\mathrm{Hz}}} \tag{9}$$

Forces

We will use these values for the following calculations/estimations. The distance between the Faraday isolator and the MC mirrors is not yet known but it will be at least 10cm, likely more. The force $\vec{F} = \nabla(\vec{\mu}\cdot\vec{B})$ can have two effects: A common force at Fourier frequency f on all magnets will push or pull the mirror (mass m) and will change the length of the mode cleaner:

$$x_1(f) = \frac{1}{4\pi^2 f^2}\frac{F(f)}{m} = 6.5\cdot 10^{-5}\,\frac{\mathrm{m}}{\mathrm{N}}\left(\frac{10\,\mathrm{Hz}}{f}\right)^{2}\left(\frac{3.89\,\mathrm{kg}}{m}\right)F(f) \tag{10}$$

Differential forces on the MC mirror will also change the apparent length of the MC if the beam is not centered on the mirror. The torque on the mirror (radius r, thickness l) of a force applied at the magnet is roughly:

$$I\ddot{\phi} = \tau = Fr \tag{11}$$

Here we assume that the magnet is about a distance r away from the center of mass of the mirror. Note that this torque is generated by the two forces in equation 1, this is not the torque from equation 1. The angular rotation is then:

$$\phi(f) = \frac{1}{I}\frac{\tau(f)}{4\pi^2 f^2} = 2.2\cdot 10^{-5}\,\frac{\mathrm{rad}}{\mathrm{N}}\left(\frac{10\,\mathrm{Hz}}{f}\right)^{2}\left(\frac{8.7\cdot 10^{-3}\mathrm{kg\ m}^2}{I}\right)F(f) \tag{12}$$

This effect can be neglected compared to the first term (Eq. 10) and we can ignore all tilts associated with the force on the mirror (not the torque).

We choose a coordinate system in which $\vec{\mu} = \mu\hat{e}_x = 0.0107\hat{e}_x$ is parallel to the x-direction. The force is then:

$$F(f) = \nabla(\vec{\mu}\cdot\vec{B})(f) = \nabla(\mu\cdot B_x)(f) = \mu\nabla B_{||\mu}(f) \tag{13}$$

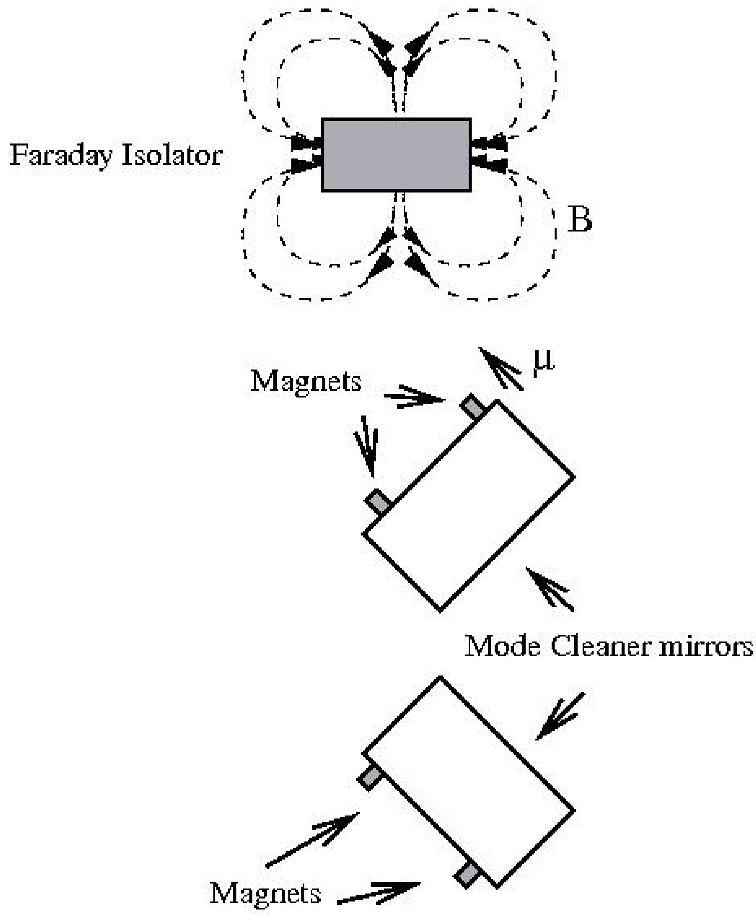

Figure 1: The drawing shows the two mode cleaner mirrors, the magnets, the isolator and the magnetic field lines. Note that this is just a sketch to describe the situation. The location of the isolator is not yet fixed nor is the shape of the conceptual magnetic field similar to the true magnetic field.

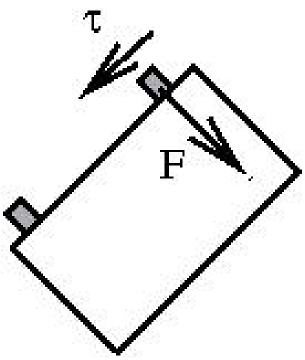

Figure 2: Forces created by the gradient in the magnetic field are assumed to act parallel to the dipole moment (worst case). Torques act perpendicular to the dipole moment.

where $B_{||\vec{\mu}}$ is the component of the magnetic field which is parallel to $\vec{\mu}$. The magnetic field generated by the permanent magnets is expected to be stable (this neglects thermal or Barkhausen noise) relative to the Faraday isolator. However, any motion of the FI with respect to the magnetic dipoles will change the field and subsequently also the gradient of the field at the dipoles. This will then change the force. In first order the change in the force is:

$$\Delta F(f) = \mu \frac{\partial}{\partial \vec{r}} \left(\nabla B_{||\mu} \right) \Delta \vec{r}(f) \leq \mu \nabla^2 B_{||\mu} \Delta r(f) \tag{14}$$

The displacement of the MC mirrors is then:

$$\Delta x_1(f) = 6.5 \cdot 10^{-5} \frac{\mathrm{m}}{\mathrm{N}} \left(\frac{[10\,\mathrm{Hz}]}{f} \right)^2 \cdot 0.0107\mathrm{Am}^2 \left(\frac{10\,\mathrm{Hz}}{f} \right)^{3/2} \frac{3.2 \cdot 10^{-13}\mathrm{m}}{\sqrt{\mathrm{Hz}}} \nabla^2 B_{||\mu} \tag{15}$$

or

$$\Delta x_1(f) = 2.2 \cdot 10^{-19} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}} \left(\frac{[10\,\mathrm{Hz}]}{f} \right)^2 \left(\frac{10\,\mathrm{Hz}}{f} \right)^{3/2} \frac{\mathrm{m}^2}{\mathrm{T}} \nabla^2 B_{||\mu} \tag{16}$$

$$< 3 \cdot 10^{-17} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}} \left(\frac{10\,\mathrm{Hz}}{f} \right)^2 = \Delta x_1^{\mathrm{max}}(f) \tag{17}$$

which gives:

$$\nabla^2 B_{||\mu}^{\mathrm{max}} < 1.35 \cdot 10^2 \left(\frac{f}{10\,\mathrm{Hz}} \right)^{3/2} \left(\frac{3.89\,\mathrm{kg}}{m} \right) \frac{\mathrm{T}}{\mathrm{m}^2} \tag{18}$$

$$= 1.35 \cdot 10^6 \left(\frac{f}{10\,\mathrm{Hz}} \right)^{3/2} \left(\frac{3.89\,\mathrm{kg}}{m} \right) \frac{\mathrm{Gauss}}{\mathrm{m}^2} \tag{19}$$

Torques

The torque

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{20}$$

is a torque on the dipole magnets. It will rotate the mirror by:

$$\phi = \frac{1}{4\pi^2 f^2} \frac{\tau}{I} \tag{21}$$

We again align our coordinate system with the magnetic dipole moment $\vec{\mu} = \mu \hat{e}_x$. The torque will then be:

$$\vec{\tau} = \hat{e}_z \mu B_y - \hat{e}_y \mu B_z \tag{22}$$

The rotation is then:

$$\phi = 3.1 \cdot 10^{-4} \frac{\mathrm{rad}}{\mathrm{T}} \left(\frac{8.3 \cdot 10^{-3}\mathrm{kg\ m}^2}{I} \right) \left(\frac{\mu}{0.0107\mathrm{Am}^2} \right) \left(\frac{[10\,\mathrm{Hz}]}{f} \right)^2 B_{\perp\mu} \tag{23}$$

where $B_{\perp\mu}$ is the component of the magnetic field which is perpendicular to μ. Any change in the position of the Faraday with respect to the magnets will cause a change in the angle:

$$\Delta\phi(f) = \frac{1}{4\pi^2 f^2}\frac{\mu}{I}\Delta r(f)\frac{\partial B_{\perp\mu}}{\partial r} = 10^{-16}\frac{\text{rad}}{\text{T}}\frac{,\text{m}}{\sqrt{\text{Hz}}}\left(\frac{\text{Hz}}{f}\right)^{3/2}\left(\frac{[10\,\text{Hz}]}{f}\right)^{2}\frac{\partial B_{\perp\mu}}{\partial r} \tag{24}$$

Comparing this with the maximum allowed angular fluctuations gives us another upper limit, this time on the magnetic field gradient:

$$\frac{\partial B_{\perp\mu}}{\partial r} < \frac{3\times10^{-14}\,\frac{\text{rad}}{\sqrt{\text{Hz}}}}{10^{-16}\,\frac{\text{rad m}}{\text{T}}\left(\frac{\text{Hz}}{f}\right)^{3/2}} = 300\frac{\text{T}}{\text{m}}\left(\frac{f}{\text{Hz}}\right)^{3/2} \tag{25}$$

$$= 3\cdot10^{6}\frac{\text{Gauss}}{\text{m}}\left(\frac{f}{\text{Hz}}\right)^{3/2}\left(\frac{8.3\cdot10^{-3}\text{kg m}^2}{I}\right)\left(\frac{\mu}{0.0107\text{Am}^2}\right) \tag{26}$$

Magnetic Fields

Axial

The field was measured for the Advanced LIGO Faraday rotator was measured and the result along the axial direction is shown in Fig. 3. The axial field outside the isolator can be approximated by the following formula:

$$B(x) = 0.25\,\text{G} + 0.076\frac{\text{Gx}^4}{(x-0.076\,\text{x})^4} \tag{27}$$

It falls with the fourth power of the distance. More critical than the field are the gradient and the curvature:

$$\frac{\partial B(x)}{\partial x} = -0.3\frac{\text{Gx}^4}{(x-0.076\,\text{x})^5} \qquad \frac{\partial B(x=12\,\text{cm})}{\partial x} \approx 1.8\cdot10^{6}\frac{\text{G}}{\text{m}} < \frac{\partial B^{\text{max}}_{\perp\mu}}{\partial r} \tag{28}$$

and

$$\frac{\partial^2 B(x)}{\partial^2 x} = 1.52\frac{\text{Gx}^4}{(x-0.076\,\text{x})^6} \qquad \frac{\partial^2 B(x=18\,\text{cm})}{\partial^2 x} \approx 1.2\cdot10^{6}\frac{\text{G}}{\text{m}^2} < \nabla^2 B^{\text{max}}_{||\mu} \tag{29}$$

So for the axial field, we find that a distance of 20 cm seems to be sufficient even when we assume worst case orientation.

Caveat

The magnetic field will be modelled a little bit better to ensure that our magnetic gradients are correct. So far, we have a few measurements along specific directions; the axial direction seems to be the worst, but should get a consistent model. However, the fast decline of the field strength and of the first two derivatives indicates that a distance of 30 cm away from the mode cleaner should always be fine.

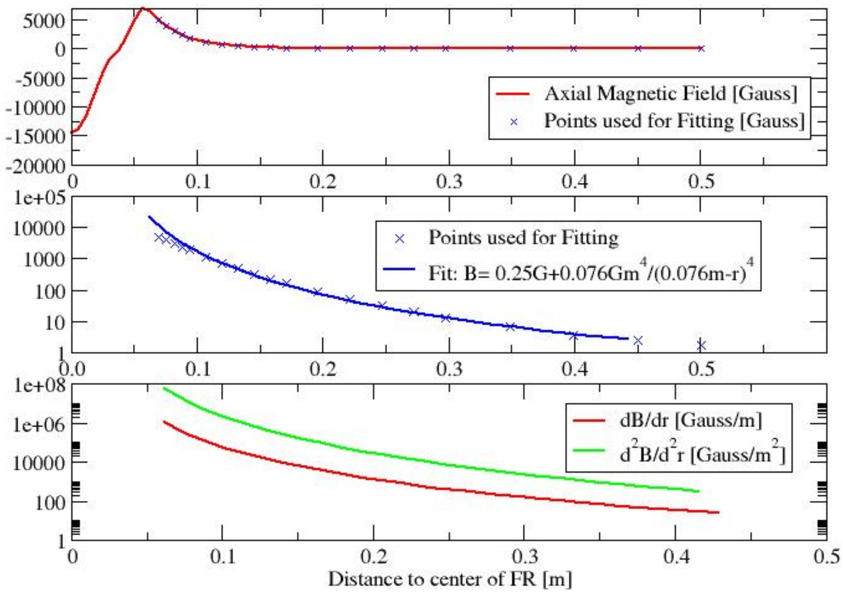

Figure 3: The axial magnetic field, a fit, and the first and second derivative.