Thermal noises calculations: Gaussian vs Mesa beams

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Introduction

Gravitational Wave (GW) detectors monitor the possible presence of a GW by carefully measuring, by interferometric means, the distance between the surfaces of suspended and seismic attenuated mirrors. An error is introduced by the fact that a perfect detection of a GW would require the measurement of a test mass centre of mass, while only the test mass surface position is measurable. The incessant fluctuating redistribution of thermal energy inside the mirrors produces a fluctuating change of test mass's shape and a change of the position of its reflecting surface, which is undistinguishable from a GW induced motion of the test mass's centre of mass. Mirror thermal noise is expected to be the limiting factor of the sensitivity of the next generation of GW detector interferometers [4] in the frequency range between ~50Hz and ~200Hz.
Thermal noise takes origin from the dissipation mechanisms that redistribute energy inside the mirror structure.

There are various types of internal mirror thermal noise, each one associated with a specific dissipation mechanism. The relative importance of one with respect to the other is determined by the mechanical and thermodynamical properties of the test mass materials. Of course we are dealing here only with thermal noise of mirrors in complete thermo-dynamical equilibrium, disregarding any effect that may come from macroscopic thermal differences and heat flows. Brownian thermal noise [7] is due to intrinsic losses in the material and is associated with all forms of dissipation that are describable by an imaginary part of the Young's modulus. Thermo-elastic noise [9] is created by the stochastic flow of heat within each test mass, producing fluctuations of temperature; due to the thermal expansion coefficient, the test-mass material expands in the hot spots and contract in the cold spots, creating fluctuating bumps and valleys on the mirror faces.

It has been pointed out [11] that it is critically important how the losses are distributed inside the test masses. Losses far from the beam spot contribute less to the total thermal noise, whereas losses near the spot, for example in the dielectric coating directly reflecting the beam, contribute more. Coating thermal noise [12] due to internal losses is expected to be the dominant contribution to the thermal noise for mirrors with a SiO_2/Ta_2O_5 coating on a fused silica substrate, whereas the thermo-elastic noise of the substrate is the dominant contribution for sapphire mirrors at room temperature.

The local surface fluctuations produced by thermal noise are averaged by the intensity distribution of the laser beam spot over the mirror surface. Reading the entire mirror surface with uniform sensitivity would minimize the thermal noise.

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The standard design of interferometers uses light beams with a Gaussian distribution of power, which are eigenfunctions of cavities with spherical mirrors, a well-developed and understood technology. Many authors [7],[9],[12],[13],[15], analyzed the influence of the beam radius w (radius at which the power drops down by a factor $1/e^2$) on the different types of thermal noise for Gaussian beams. They found the following scaling rules for the noise spectral densities when the mirror is considered as an infinite half-space, $S_X \propto 1/w^n$ where

- n=2 for coating Brownian and thermo-elastic noise,
- n=1 for substrate Brownian and
- n=3 for substrate thermo-elastic noise.

The larger is the beam radius r_0, the better is the averaging of the fluctuations and thus lower will be the noise. However the beam size is constrained by the allowable diffraction losses requirements, which cannot exceed a few *ppm*. Taking into account the diffraction loss constraints, a Gaussian beam effectively averages out the thermal fluctuations only over a few percent of the mirror surface.

A significant reduction in mirror's thermal noise can be achieved by using modified optics that reshape the beam from a conventional Gaussian profile into a Flat-Top ("mesa-beam") profile. A large-radius, flat-topped beam with steeply dropping edges (necessary to satisfy the diffraction loss constraint) will lead to a better sampling of the fluctuating surface, lower noise in the determination of the mirror surface position and better sensitivity for GW detectors. The calculation of substrate thermoelastic noise reduction using Mesa beam has been done in [3] for sapphire test mass. More recently Vinet [5] calculated the substrate Brownian thermal noise reduction using Mesa beam for Virgo mirror size.

Calculation of the coating thermal noise, which is expected to be the most significant contribution to the thermal noise budget for the test masses of the next generation of GW interferometers, has never been published for non Gaussian beams and finite cylindrical test masses.

In this paper we present a comparative study of the various sources of thermal noise in different mirror and beam configurations, considering both Gaussian and Mesa beam profiles, addressing the problem of thermal noise reduction, through mirror aspect-ratio and beam size optimization. Some of these results have been already presented in [8] and [14].

We analyzed fused silica and sapphire mirror substrates with the conventional Ta2O5/SiO2 (λ/4) coating.

We fixed the mirror mass at 40 Kg, constrained by the Advanced LIGO suspension system design.

We fixed the diffraction loss constraint at 1ppm (10^{-6}) for both the Gaussian and the Mesa beam, calculating the diffraction losses with the so called clipping approximation; in this approximation the losses are computed by the amount of light that falls outside the mirror and the beam profile is assumed to retain its shape even though the diffraction from the edge of the mirror.

The parameters used in this evaluation are the following:

Parameters : (c.g.s. units)	**Fused Silica:**	**Sapphire:**	**Coating**	
			Ta2O5	**SiO2**
Density ρ (g/cm^3)	2.2	4	6.85	2.2
Young modulus Y (erg/cm^3)	7.2 10^{11}	4 10^{12}	1.4 10^{12}	7.2 10^{11}
Poisson ratio σ	0.17	0.29	0.23	0.17
Loss angle ϕ	5 10^{-9}	3 10^{-9}	10^{-4} (total)	
Lin. therm. expansion α (K^{-1})	5.5 10^{-7}	5 10^{-6}	3.6 10^{-6}	5.1 10^{-7}
Specific heat per unit mass (const. vol.) C (erg/(g K))	6.7 10^6	7.9 10^6	3.06$10^6$	6.7 10^6
Thermal conductivity κ (erg/(cm s K))	1.4 10^5	4 10^6	1.4 10^5	1.4 10^5
Total thickness (cm)	variable	variable	$19\,\lambda/4n_1$	$19\,\lambda/4n_2$

We will use the following expression for the normalized power distribution of the Gaussian beam over the mirror surface

$p_G(r) = \dfrac{2e^{-2r^2/w^2}}{\pi w^2}$ and the expression in Eq.(1) for the non-normalized field distribution of the Mesa beam at the mirror location.

$$u_{MB}(r) = \int_0^D e^{-\frac{(r^2+r'^2)(1-i)}{2w_0^2}} I_0\left(\frac{rr'(1-i)}{w_0^2}\right) r'dr' \qquad (1)$$

Where I_0 is the modified Bessel function of zero order and $w_0 = \sqrt{\dfrac{L}{k}}$.

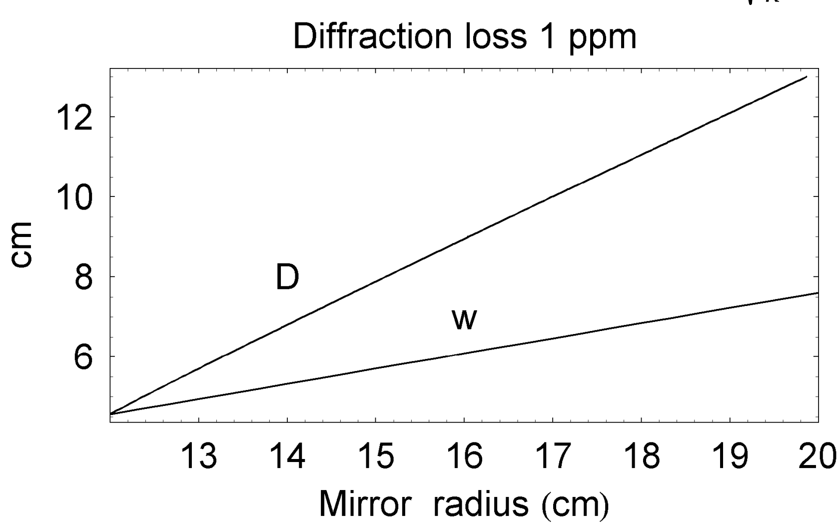

Fig. 1 Gaussian beam radius w and Mesa beam integration disc radius D as a function of the mirror's radius in order to satisfy the diffraction loss constraint.

Thermal noise calculation

We used the Levin approach to the Fluctuation Dissipation Theorem [11] in order to calculate the power spectral density of the test mass displacement. The generalized coordinate $X(t)$ is given by the average, weighted by the beam spot power distribution $f(\vec{r})$, of the normal displacement $u_z(\vec{r},t)$ of the test mass surface.

$$X(t) = \int_{Mirror} d^2\vec{r}\ u_z(\vec{r},t)\ f(\vec{r})$$

The spectral density of the displacement noise due to thermal fluctuation in the test mass is given by

$$S_X(\omega) = \frac{8 k_B T}{\omega^2} \frac{W_{diss}}{F_0^2}, \qquad (2)$$

where W_{diss} is the average (over the period $\frac{2\pi}{\omega}$) of the dissipated energy in response of the applied oscillatory pressure $P(\vec{r},t) = F_0\ f(\vec{r}) \cos(\omega t)$.

In this way the thermal noise evaluation reduces to the calculation of W_{diss} in accordance to the specific mechanism and localization of dissipation, i.e. Brownian or thermo-elastic and substrate or coating. The main ingredient for these calculations is the model proposed by BFV [7] and corrected by Liu and Thorne [10] of the approximate solution of the elasticity equations for a cylindrical test mass subjected to the oscillatory pressure with the same spatial profile as the beam power distribution. The deformation of a cylindrical test mass is calculated using the displacement vector given in Eq. (30) of [10] with the coefficients $p_m = \frac{2}{a^2 J_0^2(\zeta_m)} \int_0^a f(r) J_0(\zeta_m r/a)\, r dr$ calculated according to the specific cylindrically symmetric power distribution; a is the radius of the cylindrical mirror, ζ_m is the m th zero of $J_1(x)$, J_1 and J_0 are the first and zeroth order Bessel functions. The fundamental approximation underneath these calculation is the quasi-static approximation for the calculation of the displacement fields according to the oscillatory pressure P which is a good approximation for oscillatory period larger than the time required for sound to travel across the test mass ($\tau_{sound} \approx \frac{H}{c_s} \approx 30\mu s$, where H and c_s are the mirror thickness and sound velocity in the substrate respectively).

The geometry we consider for the reflective surface of the mirrors consists of a thin film of thickness d on a substrate whose thermo-mechanical properties are different from those of the film.

To simplify the analysis, we assumed that the multilayer coating can be approximated as a uniform layer with appropriately averaged properties.

In the following paragraphs we will give some details on how the different contributions to thermal noise are calculated.

Substrate Brownian thermal noise:

The conventional thermal noise of the substrate is given by Eq. (2) with the time averaged dissipation W_{diss} given by

$W_{diss} = 2\omega\phi_s\langle U\rangle$,

where ϕ_s is the loss angle of the substrate, $\langle..\rangle$ denotes the time average over the oscillatory period and U is the elastic energy stored in the test mass, $U = \int_{test\ mass} \frac{1}{2}\varepsilon_{ij}\sigma_{ij}\,dV$, ε_{ij} and σ_{ij} are the component of the strain and stress tensor respectively calculated from the displacement vector and the constitutive relations (elastic moduli tensor) for an homogeneous and isotropic material

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\phi\phi} = \frac{u_r}{r}, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \quad \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right), \qquad (3)$$

$$\sigma_{ii} = \lambda\varepsilon + 2\mu\varepsilon_{ii}, \quad \sigma_{rz} = 2\mu\varepsilon_{rz}, \quad \varepsilon = \varepsilon_{rr} + \varepsilon_{\phi\phi} + \varepsilon_{zz}$$

where λ, μ are the Lamé coefficients of the substrate.

Substrate thermo-elastic noise

In this case the thermo-elastic dissipation is given by

$$W_{diss} = \left\langle \int_{test\ mass} \frac{\kappa}{T}\left(\vec{\nabla}\delta T\right)^2 dV \right\rangle \qquad (4)$$

Where κ is the substrate thermal conductivity and δT is the temperature perturbation induced by the elastic deformation due to the oscillatory pressure and is given by

$$\delta T = -\frac{\alpha Y T}{C\rho(1-2\sigma)}\varepsilon \qquad (5)$$

Where α is the linear thermal expansion coefficient, Y and σ are the Young modulus and Poisson ratio respectively, C is the specific heat per unit mass at constant volume and ρ is the density of the substrate. The equation (5) follows from the adiabatic approximation of the general thermal conductivity equation. The adiabatic approximation is discussed in details in [10] where it is shown that if the time scale for diffusive heat flow is much longer than the pressure oscillating period (or in other way that the beam

radius $w >> r_t$ where $r_t = \sqrt{\frac{\kappa}{\rho C \omega}}$ is the characteristic length for diffusive heat transfer), we can approximate the oscillations of stress, strain and temperature as adiabatic, neglecting the heat flow term in the thermal conductivity equation.
Using equations (3) for the calculation of the expansion ε, substituting in (5) we can calculate the thermo-elastic dissipated energy in (4) and then the spectral density of the displacement noise given in (2).

Coating Brownian thermal noise

In this case the averaged energy dissipated by the intrinsic losses in the coating is given by

$$W_{diss} = 2\omega \phi_c < U_c > \qquad (6)$$

Where ϕ_c is the loss angle of the coating and U_c is the portion of elastic energy stored in the coating (in this calculation we assume an isotropic and homogeneous coating with averaged elastic coefficient Young modulus and Poisson ratio). In the thin film approximation we assume that the energy stored in the coating is given by $U_c \approx \delta U_c\, d$
Where d is the thickness of the coating and $\delta U_c = \int_S \frac{1}{2} \varepsilon_{ij}^c \sigma_{ij}^c \, dS$ is the energy density stored at the surface, integrated over the surface.
Following [12], the stresses and strains in the coating can be calculated in terms of the stresses and strains at the surface of the substrate because of the boundary condition between coating and substrate: the coating must have the same tangential strains as the surface of the substrate and the coating experiences the same perpendicular pressure as the surface of the substrate. Since the only exerted force is normal to the plane $z = 0$ we must have $\sigma_{rz}^c = 0$ ($\sigma_{rz}(r, z=0) = 0$ is a boundary condition for the elastic problem of the substrate)

$$\varepsilon^c{}_{rr} = \varepsilon_{rr}(z=0) \qquad \varepsilon^c{}_{\phi\phi} = \varepsilon_{\phi\phi}(z=0) \qquad \sigma^c{}_{zz} = \sigma_{zz}(z=0) \qquad (7)$$

$$\sigma^c{}_{ii} = \lambda_c \varepsilon^c + 2\mu_c \varepsilon^c{}_{ii}, \quad \sigma^c{}_{rz} = 2\mu_c \varepsilon^c{}_{rz}, \quad \varepsilon^c = \varepsilon^c{}_{rr} + \varepsilon^c{}_{\phi\phi} + \varepsilon^c{}_{zz}$$

In this way we can calculate all the fields necessary for the calculation of the elastic energy stored in the coating, using the expressions already found for the substrate.

Coating Thermo-elastic noise

In the thermo-elastic problem of the coating is important to note that the coating thickness, the diffusive heat transfer length and the beam radius satisfy the following relation

$$d << r_t << w$$

This relation justify the approximation of the multilayer film as a uniform film with averaged properties and when computing the oscillating temperature distribution we can

consider the temperature variation as adiabatic in the transversal direction and that only the thermal diffusion orthogonal to the surface of the mirror need be considered.
The fundamental equation we need is the one dimensional thermal conductivity equation driven by a thermo-elastic source term.

$$\left(\frac{\partial}{\partial t} - K_\beta \frac{\partial^2}{\partial z^2}\right)\delta T_\beta = -\left(\frac{Y\alpha T}{(1-2\sigma)C\rho}\frac{\partial \varepsilon}{\partial t}\right)_\beta = -\mathrm{B}_\beta \qquad (8)$$

Where ε_β is the expansion at the mirror surface ($z = 0$) associated with the zeroth-order elastic fields calculated for the previous sections and $\beta = s, c$ indicates quantities evaluated in the substrate and the coating respectively. For a multilayer coating this equation determines an averaged temperature field and the coating quantities are averaged in the following way (following [15], d_a, d_b are the thickness of the two materials)

$$(X)_{avg} \equiv \frac{d_a}{d_a + d_b} X_a + \frac{d_b}{d_a + d_b} X_b$$

$$K_c = \frac{\kappa_c}{(C\rho)_{avg}}, \quad \kappa_c^{-1} = (\kappa^{-1})_{avg}, \quad \mathrm{B}_c = \frac{(C\mathrm{B})_{avg}}{C_{avg}}$$

Assuming a time dependence of the form $e^{i\omega t}$ for the oscillatory thermal and elastic fields, equations (8) can be cast in this form

$$(i\omega - K_\beta)\delta T_\beta = -i\omega \mathrm{B}_\beta \qquad (9)$$

with the boundary conditions of zero heat flux at the surfaces of the test mass and continuity of temperature and heat flux at the boundary between coating and substrate

$$\left.\frac{\partial \delta T_c}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial \delta T_s}{\partial z}\right|_{z=H} = 0, \quad \delta T_c = \delta T_s\big|_{z=d}, \quad \left.K_c \frac{\partial \delta T_c}{\partial z} = K_s \frac{\partial \delta T_s}{\partial z}\right|_{z=d} \qquad (10)$$

The general solution of equation (9) is given by

$$\delta T_\beta = -B_\beta + C1_\beta e^{\gamma_\beta z} + C2_\beta e^{-\gamma_\beta z}, \quad \gamma_\beta = (1+i)\sqrt{\omega / 2K_\beta} \qquad (11)$$

The boundary conditions (10) determine the four arbitrary constants.
The averaged dissipated power for the coating thermo-elastic noise is given by

$$W_{diss} = \left\langle \int_{V_s} \frac{\kappa_s}{T}\left(\frac{\partial \delta T_s}{\partial z}\right)^2 dV_s \right\rangle + \left\langle \int_{V_c} \frac{\kappa_c}{T}\left(\frac{\partial \delta T_c}{\partial z}\right)^2 dV_c \right\rangle \qquad (12)$$

Inserting (12) in (2) we have the spectral density of displacement noise due to coating thermo-elastic fluctuations.

Results:

All the results are calculated at the frequency of 100 Hz , for a Fabry-Perot cavity length of L = 399901 cm and laser wavelength λ = 1064 e-7 cm.

For finite size test mass we performed the following calculations: we changed the mirror radius from 12 cm to 21 cm (with 1 cm per step) and in the same time we increased the Gaussian beam radius w and the Mesa beam integration disc radius D to satisfy the 1 ppm constraint for diffraction losses. The thickness of the mirror is reduced correspondingly to satisfy the total mass constraint. In this way all the geometric parameters in the problem are functions of the mirror radius. For example the Gaussian beam radius w can be expressed as $w(a) = \sqrt{-2a^2 / Log(10^{-6})}$ (this is the expression used for the calculation of the noises for semi-infinite mirror which are represented in the figures below as function of mirror radius)

Fused Silica substrate

Figure (2) shows the different contributions to the mirror thermal noise evaluated for infinite and cylindrical shaped mirror using Gaussian beam.
The infinite mirror thermal noises (dashed lines) are actually functions of the beam radius which correspond to 1 ppm diffraction loss in the finite mirror case.
Depending on the mirror radius and the specific noise considered, the finite size correction can be as large as several 10 %.
It is interesting to note that all the thermal noise contributions present a minimum in the finite cylindrical model which would represent the best choice for the mirror and beam dimensions. The descent part of the noise curves follows the basic idea that increasing the beam radius the noise will get lower and the rising part of the curves can be explained heuristically by the fact that all the noises contributions are related somehow to the elastic deformation of the test mass under a surface pressure and this effect is bigger in gong-shaped mirrors than in bar-shaped ones.

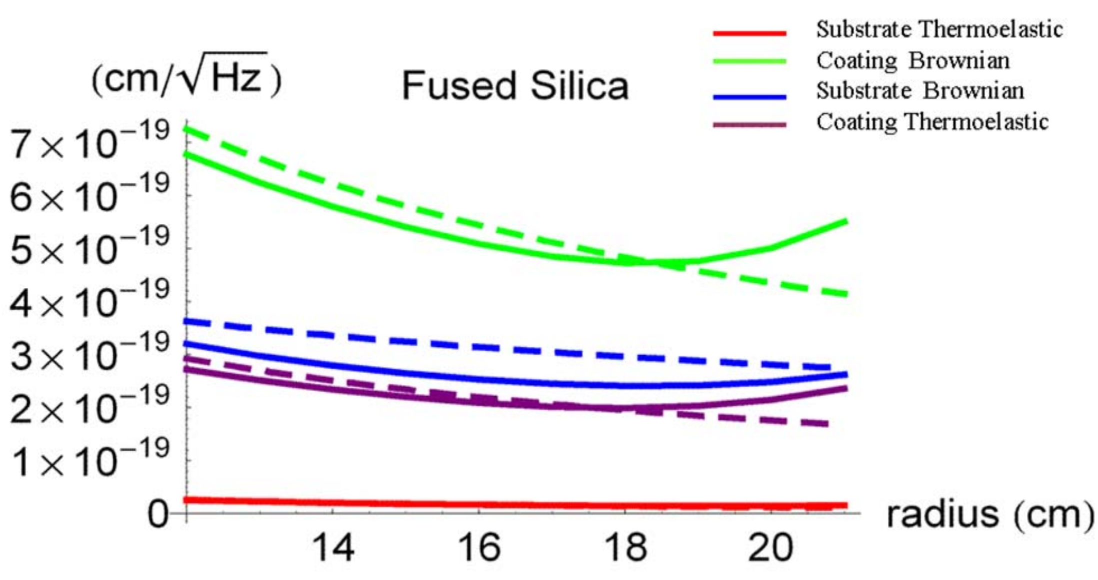

Fig. 2 Displacement noise using Gaussian beam, for different thermal noise contributions as functions of the mirror radius. Solid lines correspond to cylindrical test mass whereas dashed lines correspond to semi-

infinite mirror. The points in the dashed lines are calculated using the same beam radii as the finite size calculations.

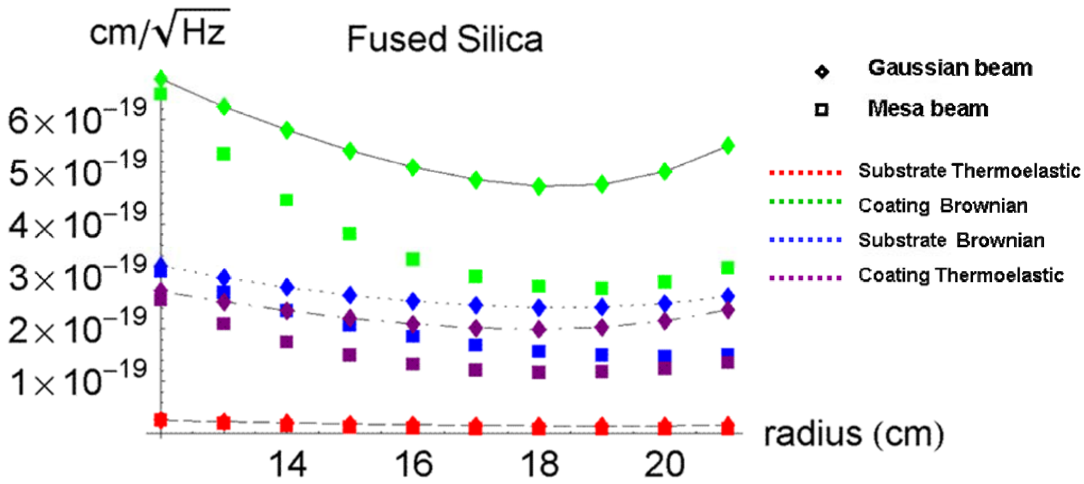

Fig. 3 Comparison between Gaussian (squares) and Mesa beam (rhombi) noise performance, separating the four different thermal noise contributions.

Fig. (3) shows the displacement noise of all the analyzed thermal noise contributions, for Gaussian and Mesa beam in the case of Fused Silica substrate. The dominant noise, the coating Brownian thermal noise, undergoes a reduction of a factor ≈ 1.7 for a mirror radius of 18 cm. The substrate thermal noise is reduced by a factor ≈ 1.55, whereas the coating thermo-elastic and the substrate thermo-elastic are reduced by factor ≈ 1.71 and ≈ 1.92 respectively.
Fig. (4) shows the total mirror thermal noise for Gaussian and Mesa beam. It is evident that the minimum thermal noise occurs for a mirror radius of about 18 cm for Gaussian beam and for about 19 cm for Mesa beam. The corresponding mirror aspect ratios ($2a/H$) are 2 for Gaussian beam and 2.4 for Mesa beam. The gain in sensitivity is a about a factor 1.7 switching from Gaussian to Mesa beam at the minima of thermal noise.

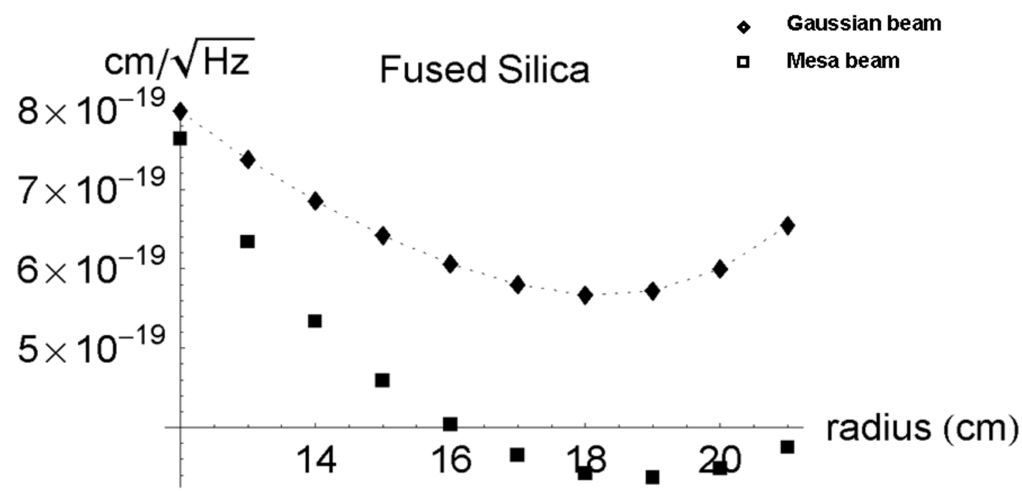

Fig. 4 Total thermal noise $\left(\sqrt{S_X}\right)$ for Gaussian and Mesa beam.

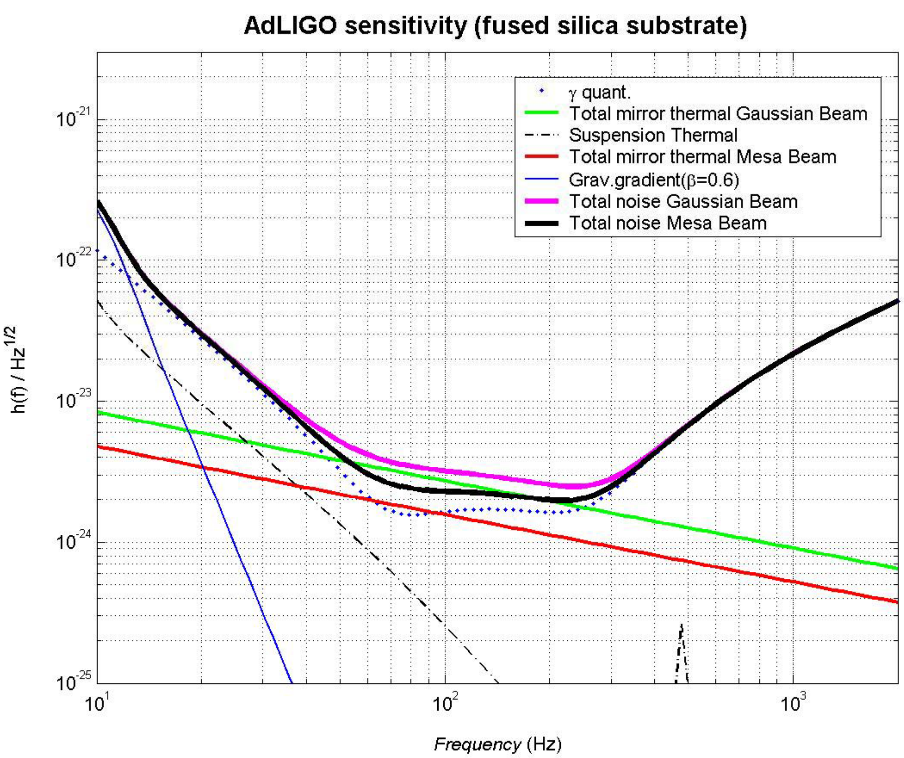

Fig. 5 Estimation of the sensitivity improvement, obtained by simply replacing the baseline Gaussian beam profile with a Mesa beam in Advanced LIGO.

Fig.(5) shows the expected sensitivity of Advanced LIGO interferometer in the Gaussian or Mesa beam configuration. The estimated range for NS-NS binary systems increases from 177 Mpc for Gaussian beam to 228 Mpc for Mesa beam. This is a

remarkable factor if we consider that we didn't optimize the other interferometer's parameters to take full advantage of the reduced mirror thermal noise floor.
In this evaluation the Thermo-refractive noise of the coating was ignored. Its effect will be dealt in the last section.

Sapphire substrate:

We have conducted the same kind of analysis for mirrors with Sapphire substrate.
In this case the dominant noise is the substrate thermo-elastic contribution and the advantages of using Mesa bean have been already analyzed in [3] for this particular noise source. Here we compute the various thermal noise contribution for finite size test mass and show the relative gain for each thermal noise employing a Mesa beam instead of a standard Gaussian.

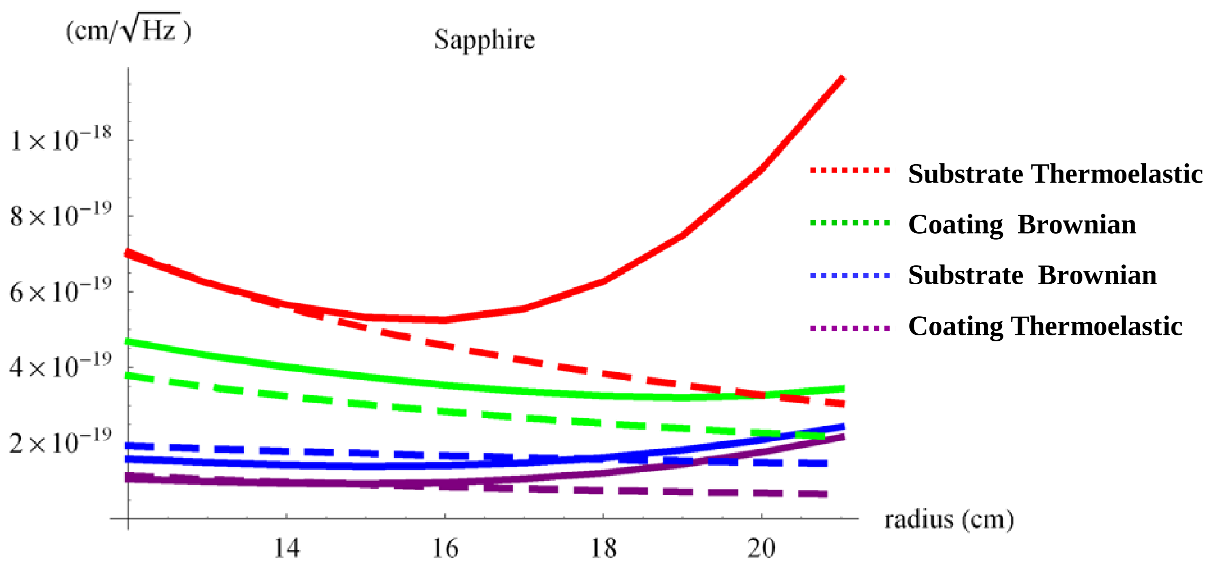

Fig. 6 Displacement thermal noises for semi-infinite (dashed lines) and finite cylindrical (solid lines) test mass.

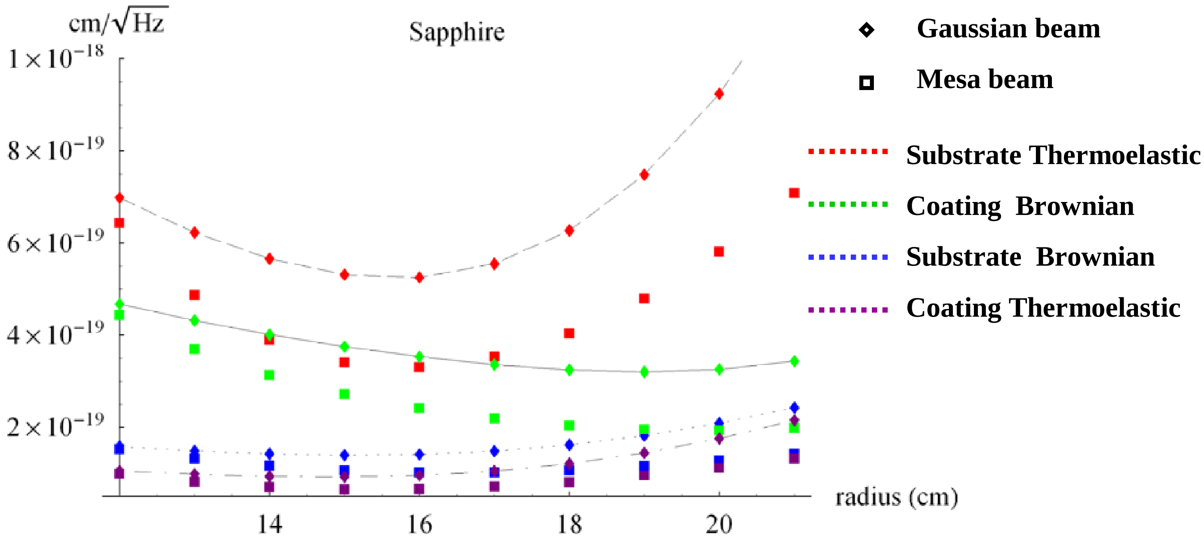

Fig. 7 Comparison between Gaussian and Mesa beam for different thermal noise contributions.

For Sapphire substrate the minimum of thermal noise occurs for a mirror radius of about 16 cm, pretty close to the Advanced LIGO baseline design. The corresponding mirror aspect ratio ($2a/H$) is about 2.6. The total thermal noise reduction for Mesa beam is about a factor 1.6 around the minimum (see Fig. (8)) .

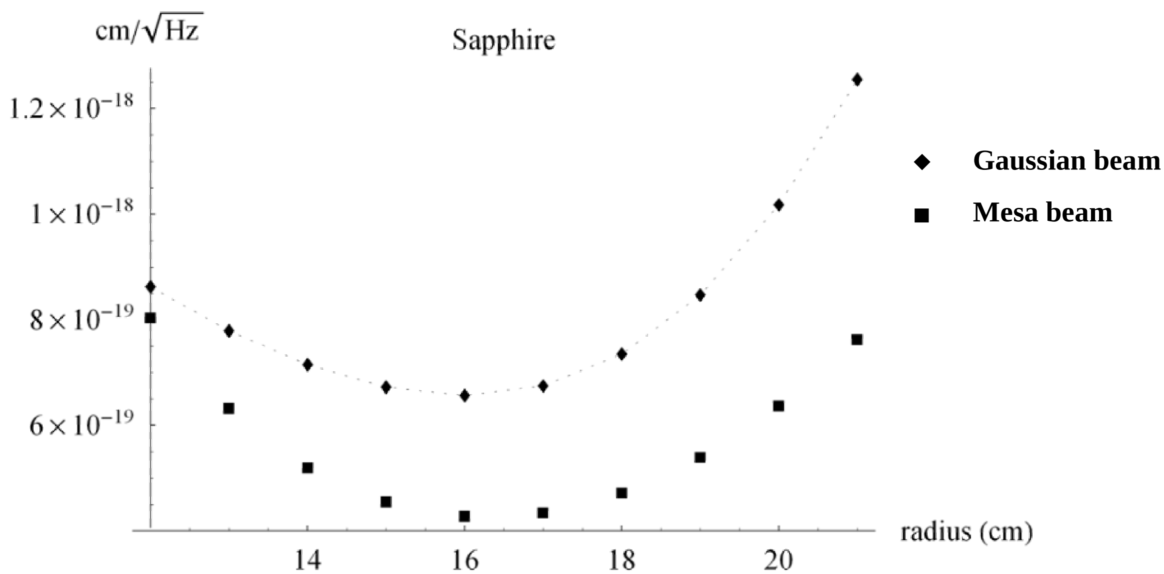

Fig. 8 Total thermal noise $\left(\sqrt{S_X}\right)$ for Gaussian and Mesa beam.

Coating Thermo-refractive noise

The thermo-refractive noise is generated by temperature fluctuations that couple with phase fluctuations of the laser (and therefore with measured displacement) thanks to the non-null coefficient $\beta = \frac{dn}{dT}$ where n is the refractive index.

Following [16],[13], the spectral density of the equivalent displacement noise can be written as

$$S_X(\omega) = \lambda^2 \beta_{eff}^2 S_T(\omega) \qquad (13)$$

Where S_T is the spectral density of temperature fluctuation in the coating and $\beta_{eff} = \frac{n_1 n_2 (\beta_1 + \beta_2)}{4(n_1^2 - n_2^2)}$ for a multilayer quarter-wavelength optical coating.

The spectral density of the temperature fluctuation can be calculated using the Langevin approach for semi infinite mirror for arbitrary beam power profile and inserted in (13)

$$S_X(\omega) = \lambda^2 \beta_{eff}^2 \frac{4k_b T^2 K}{\rho C} \int_{-\infty}^{\infty} dq_z \int_0^{\infty} \frac{q_\perp dq_\perp}{(2\pi)^2} \frac{2q^2}{K^2 q^4 + \omega^2} \frac{1}{1 + q_\perp^2 d^2} \left|\tilde{g}(q_\perp)\right|^2 \qquad (14)$$

$$q^2 = q_\perp^2 + q_z^2 = q_x^2 + q_y^2 + q_z^2$$

$\tilde{g}(q_\perp) = 2\pi \int_0^{\infty} r dr\, f(r) J_0(q_\perp r)$ is the Hankel transform of the normalized power distribution over the mirror surface.

In this section we want to give just an estimation of the coating thermo-refractive noise reduction using a Flat Top beam instead of a Gaussian beam and for this purpose we will approximate the real mesa beam as a perfect cylindrical beam

$$f_{FT}(r) = \frac{1}{\pi b^2} \quad for \quad r \le b, \quad 0 \quad for \quad r > b$$

$$f_G(r) = \frac{2 e^{-2r^2/w^2}}{\pi w^2}$$

In both cases the Hankel transform can be analytically performed but the integral in (14) must be numerically evaluated (unless other approximations are accepted, as done in [16]).

For this comparison we chose a value of $b = 4w_0$ ($w_0 = 2.6$ cm) which correspond to the "standard" radius of the integration disc for Mesa Beam and compare this ideally flat beam with the Advanced LIGO Gaussian baseline design $w = 6$ cm. The displacement noise is reduced by a factor 1.7 in the case of a Flat Top beam.

$$\sqrt{\frac{S_X^{GB}}{S_X^{FT}}(f = 100Hz)} \approx \sqrt{3}$$

Conclusion

We presented a full thermal noise budget calculation for a GWID test mass using Gaussian and Mesa beam.
We have shown, using a model of test mass with finite dimensions, that all the contribution to displacement noise due to thermal fluctuation are reduced considerably by using Mesa beam instead of the standard Gaussian one. Coating Brownian and thermo-elastic and substrate Brownian and thermo-elastic are reduced by different factors depending on the mass aspect ratio and beam radius. Moreover we have shown that the minima of total thermal noise occur for different mirror aspect ratio, depending on the substrate material and beam geometry. At the minima of the noise curves we have a gain in sensitivity of a factor 1.7 for Fused Silica substrate and 1.6 for Sapphire substrate. We also addressed the problem of coating thermo-refractive noise and we have shown that the use of an ideally flat beam profile reduces the displacement noise of a factor 1.7.

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