LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
-LIGO-
CALIFORNIA INSTITUTE OF TECHNOLOGY
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Technical Note **LIGO-** **T040203-00-R** **11/Oct./04**

Study of mirror thermal noise as a function of mirror aspect ratio and size

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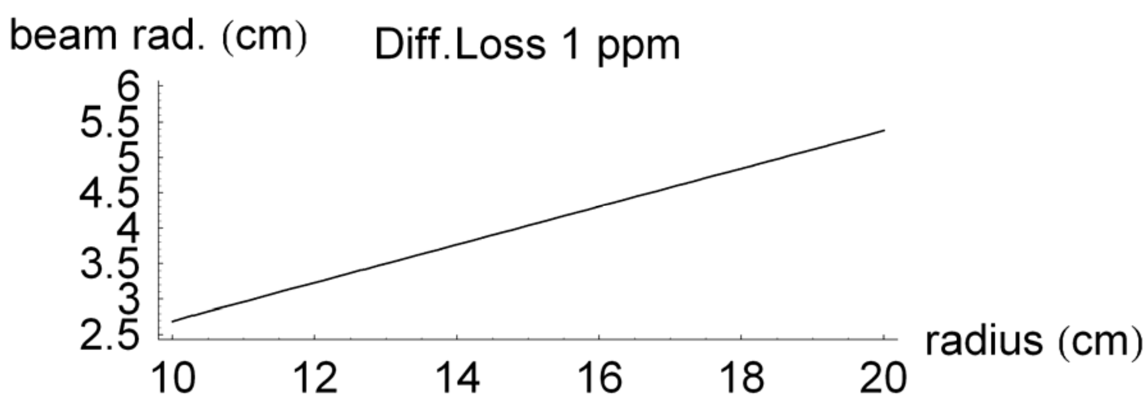
INTRODUCTION
We have conducted a comparative study of the thermal noise in different mirror configurations, both for Fused Silica and for Sapphire.
The study separates the evaluation of the bulk, thermoelastic, and coating brownian thermal noise.
The paramaters used in this evaluation are the following:

Parameters : (c.g.s. units)	**Fused Silica:**	**Sapphire:**	**Coating (Ta_2O_5 + SiO_2):**
Density	2.2	4	
Young modulus	$7.2\ 10^{11}$	$4\ 10^{12}$	$1.1\ 10^{12}$
Poisson ratio	0.17	0.29	0.2
Loss angle	$3\ 10^{-9}$	$5\ 10^{-9}$	10^{-4}
Linear thermal expansion coeff.	$5.5\ 10^{-7}$	$5\ 10^{-6}$	
Specific heat per unit mass (V const.)	$6.7\ 10^{6}$	$7.9\ 10^{6}$	
Thermal conductivity	$1.4\ 10^{5}$	$4\ 10^{6}$	
Total thickness	variable	variable	$6\ 10^{-4}$

We also fixed the Mirror mass to 40 Kg
We dynamically adjusted the Gaussian beam radius (radius at which the power drops down by a factor 1/e respect to the maximum) to maintain a fixed diffraction loss = 10^{-6} (clipping approximation).
The optimization (1 ppm diffraction loss) Gaussian beam spot radius versus mirror radius is illustrated in the following figure.



The mirror thickness is also dynamically adjusted as a function of the mirror radius in order to maintain the total 40 Kg mass fixed.
We made the calculation at the frequency 100 Hz and, assuming the quasi-static approximation and material properties independent from the frequency,
the strain sensitivity for the substrate thermoelastic noise will scale as

$\text{Noise}_{\text{thermoelastic}} \propto 1/f$

while the strain sensitivity for substrate and coating thermal noise will scale as

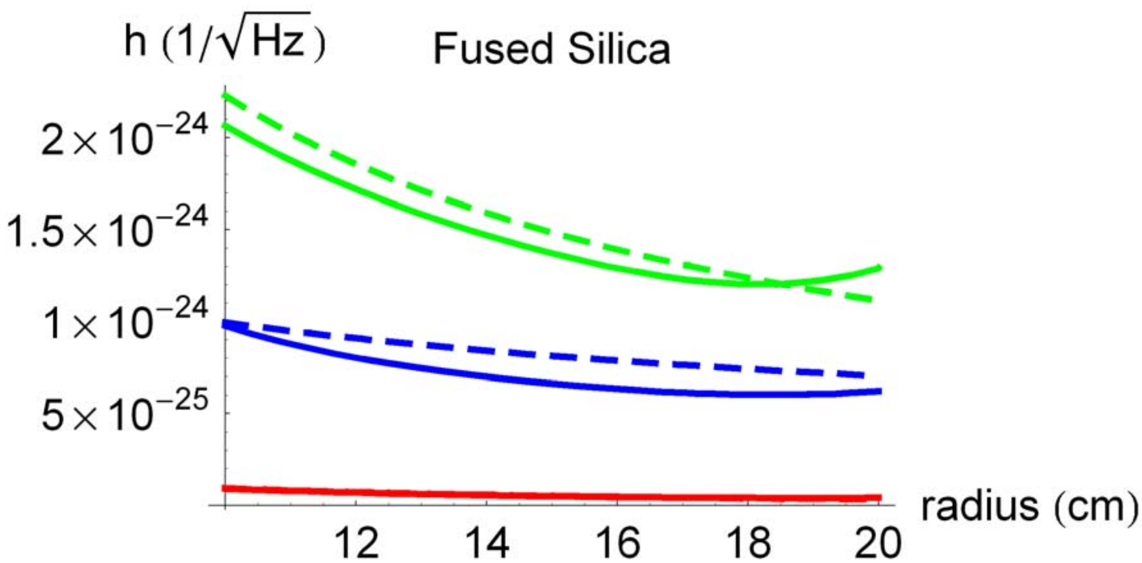
$\text{Noise}_{\text{brownian}} \propto 1/\sqrt{f}$

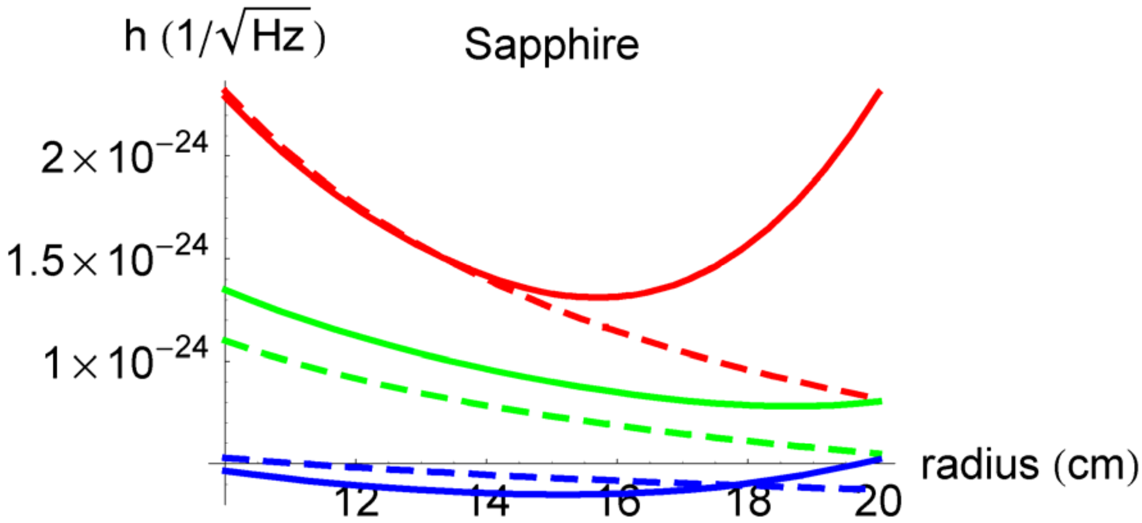
As a comparison we calculated the thermal noise in the infinite mirror cases (dashed lines) using the beam radius chosen for the finite mirror case to satisfy the 1 ppm diffracton losses constraint.

In the first two graphs we kept the different contributions separated and used the following conventions and scales:

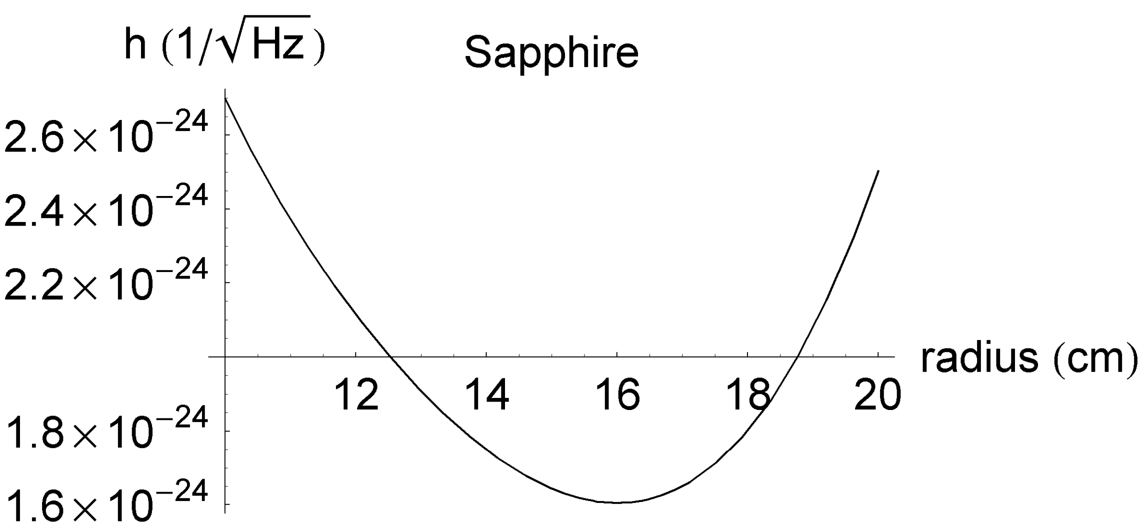
thermoelastic noise	RED
coating thermal noise	GREEN
substrate thermal noise	BLUE

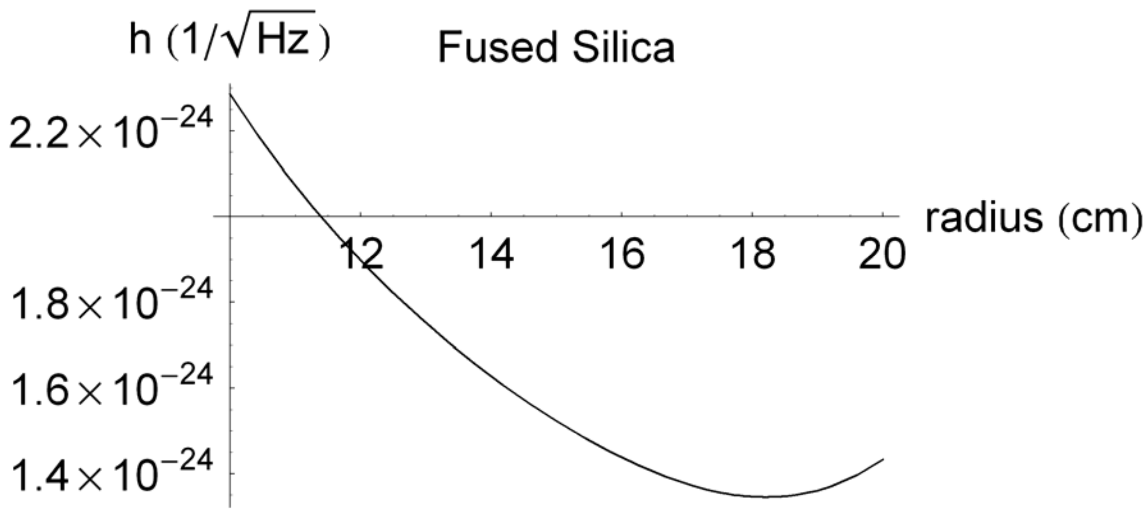
In the graphs h is the strain sensitivity (equivalent noise) for one mirror (ETM) assuming a 4 Km LIGO cavity.





To calculate the optimizations we added quadratically the three noise sources, the result is shown in the following graphs.





The calculations confirm the mirror aspect ratio choice for sapphire.
It also shows that, although the present choice of mirror diameter applied to a fused silica mass is not optimal, the difference between the present choice and an optimal choice of mirror and beam spot diameters is not dramatic.
The sensitivity difference between the present choice of mirror radius (15.7 cm) and the optimized geometry (18.3 cm with 40 Kg mass), with optimized beam spot and mirror diameter, is the following.
A fused silica mirror with a radius of 18.3 cm (optimized) yields a lower thermal noise with respect to a 15.7 cm mirror radius (present choice). This corresponds to a ~ 8% decrease in h and ~24% gain in detection volume (rate).

For FS the minimum of TN corresponds to a thickness of 17.3 cm and spot size of 4.92 cm, while with mirror radius of 15.7 cm we get a thickness of 23.5 cm and a spot size of 4.22 cm.

Note that the difference in spot radius from the adv.LIGO nominal spot size is in part due to a different way of defining the spot, not only to the spot size optimization. Also the assumed Q-factor parameters can be changed. But this is not expected to change the essence of the comparison between mirror aspect ratios for both materials.

Future developments include the addition of the thermoelastic coating noise, although the choice of parameter is problematic (especially for the Ta2O5 glass parameters which are not well known, and then repeat the analysis for the Flat-Top Beam Profile.
It is also interesting to change the diffraction loss requirements, but this will require a more general optimization of the entire interferometer parameters.