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**Comparison of Thermal Distortions in Sapphire and Fused Silica**

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## Introduction

Thermal distortions impair the interferometer performance in several ways. Depending on the location and source of the distortion, the performance limit and the thermal compensation required to achieve it will vary.

### Arm cavity distortions:

The HR surfaces of the test masses will deform under thermal loading to a degree that will substantially change the arm cavity mode size. This distortion cannot be compensated except by applying heat directly to the test mass itself, and most efficiently to the HR surface. The following table shows what the spot sizes at the test masses will be for an interferometer designed to have the correct test mass radii of curvature at full power, for cold operation, and for two types of compensation during cold operation: both test masses compensated, and only the ITM compensated. These numbers assume 40ppm/cm bulk sapphire absorption, less than 2ppm/cm absorption in fused silica, and 0.5ppm absorption in the coating. In sapphire the coating absorption dominates the effect.

		ITM spot size	ETM spot size
Silica	Hot, or both compensated	6.0cm	6.0cm
	Cold	8.5cm	8.5cm
	Cold, ITM compensated	6.8cm	6.6cm
Sapphire	Hot, or both compensated	6.0cm	6.0cm
	Cold	7.0cm	7.1cm
	Cold, ITM compensated	6.3cm	6.2cm

The baseline plan is to compensate the HR surfaces of the TMs in much the same way that the ITMs are now compensated in initial LIGO. This will require of order 1W compensation power. In order that this not introduce radiation pressure noise above 1/10th the displacement noise requirement (here assumed  $10\text{-}20\text{m}/\sqrt{\text{Hz}}$ ) the relative intensity noise must be less than  $4.8 \times 10^{-7}/\sqrt{\text{Hz}}$ . In order that compensation not introduce thermoelastic noise above this requirement the RIN must be  $5.5 \times 10^{-9}/\sqrt{\text{Hz}}$  for sapphire and  $2.4 \times 10^{-8}/\sqrt{\text{Hz}}$  for silica<sup>1</sup>. These are well above the shot noise limit for either an incandescent or CO2 laser source, but we have no way to measure stability at this level due to limited dynamic range in the available photodetectors. There is therefore some risk to this design, and this risk is greater for sapphire.

### Recycling cavity distortions

There are two goals of thermal compensation in the recycling cavity. One is to reduce the aberrations sufficiently that the RF sideband power buildup does not saturate as a result of thermal distortion losses, regardless of the laser power input to the interferometer. The other is to allow efficient

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<sup>1</sup> It may be a surprise that the DC effect is larger for silica while the sensitivity to actuator noise is larger for sapphire. This occurs because the thermal conductivity of the material contributes to the DC effect but not to the noise coupling at audio frequencies. See Appendix B of LIGO-T030062-03-D.

extraction of the gravitational wave sidebands to the output port where they will be homodyned with the carrier.

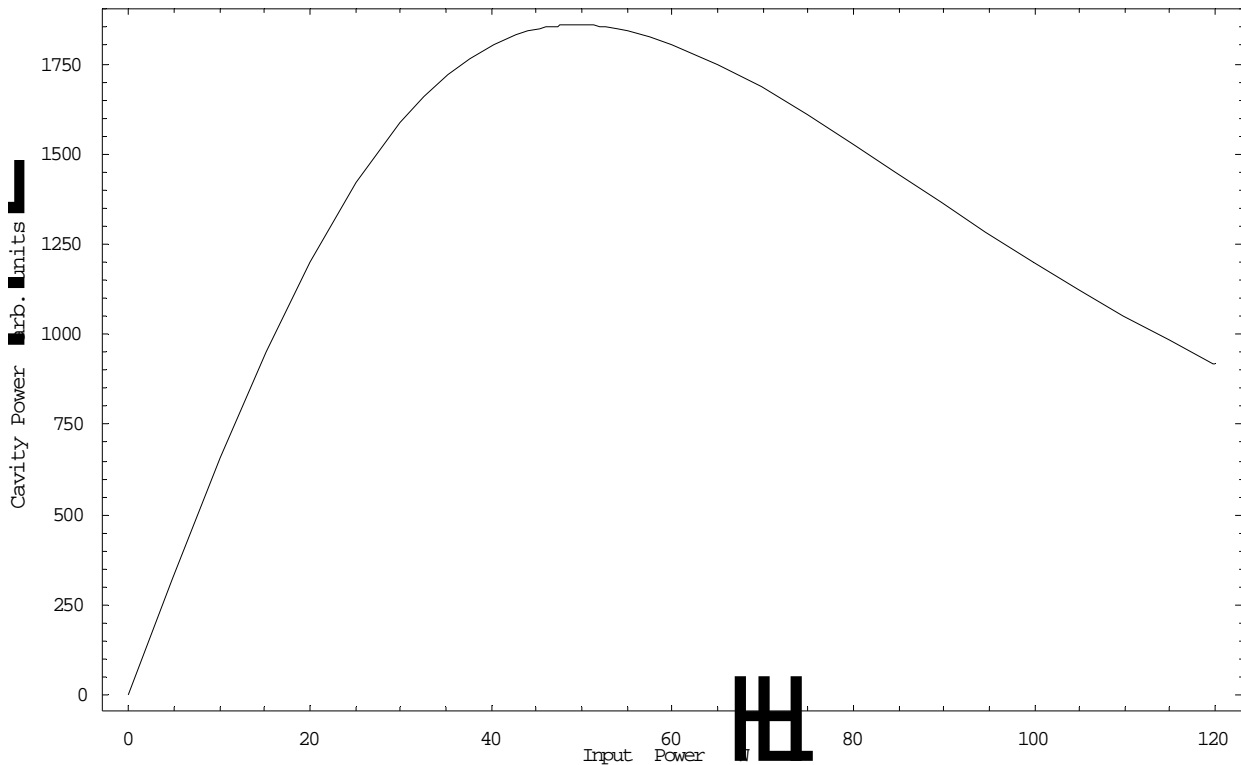
The round-trip RF sideband optical loss from the TEM00 mode expected from a sapphire ITM at full interferometer power is about 6%, assuming bulk absorption of 30ppm/cm, which is typical of the large samples to date. This also assumes that the HR surface is not compensated at high power and has the correct ROC at low power. If instead the design has the ROC correct at high power operation, the optical loss no longer includes the thermal expansion of the HR surface and therefore drops to 1.8%.

While this is large, it can be adequately reduced with a separate suspended thermal compensator plate. While a full model of the RF sidebands in the full interferometer can and should be done, a simple model of a Fabry-Perot with the recycling mirror as input mirror and perfectly reflecting back mirror should be representative. The power buildup in the cavity is then

$$P_{RF} = \frac{P_{in} R_{RM}}{\left(1 - \sqrt{T_{RM}} (1 - L)\right)^2}$$

where the loss is a quadratic function of the input laser power in the range of input powers we are considering,  $L = (\alpha P_{in})^2$ ; in this case  $\alpha = 2.0 \times 10^{-3} / W$ . Figure 4 shows how the RF sideband power will saturate at about 50W input power in the absence of thermal compensation. We may easily insert thermal compensation into this model by multiplying  $L$  by a factor between 0 and 1, and we can scale the absorption of sapphire from its nominal 30ppm/cm value by scaling  $\alpha$  by the same factor. Doing this, we find that the RF sideband power buildup just saturates at 120W input if the thermal compensation reduces the loss by a factor of 5.5. Ryan Lawrence was able to demonstrate a factor of 68 reduction in laboratory tests, so 30ppm/cm absorption can be compensated. This factor of 68 reduction becomes just adequate by this standard when the sapphire absorption is increased to 104ppm/cm.

Any potential interaction between simultaneous HR surface compensation and plate compensation has not yet been modeled. This analysis should also be repeated more thoroughly using FFT or Melody.



**Figure 1: RF cavity power vs. input power in presence of thermal lensing**

In the case of a fused silica ITM, at high power operation the RF sideband optical loss at high power is 93%, and clearly the cavity will require a large amount of thermal compensation to build up significant RF sideband power. This degree of compensation should just barely be attainable with a shielded ring heater acting on a compensator plate given the demonstrated performance mentioned above. Theoretical performance of a ring heater can be a factor of 100 better, so further experiments may improve things.

The efficiency of extracting the GW sidebands from the arm cavity with the signal recycling cavity will depend on these distortion losses being reduced to a level well below the transmissivity of the signal recycling mirror, or 5%. This should be possible with either fused silica or sapphire, though it will be much easier with sapphire.

### Bulk inhomogeneities

The absorption of fused silica seems to be very uniform, and since it contributes so little to the total absorbed power compared to the coating absorption we ignore it here. Note that later we will show the inhomogeneity of coating absorption to be a serious matter.

For sapphire, the bulk inhomogeneities have more influence, since the bulk absorption is the larger part of the total absorption. When the uniform 30ppm/cm absorption assumed above for sapphire is replaced by the absorption profile measured for a Pathfinder sapphire, the loss increases from 6% to 11%. Inserting this loss into the formula above (by changing  $\alpha$ ) shows that a factor of 11 loss suppression is necessary to meet the RF sideband power buildup requirement. Ryan Lawrence's proof-of-principle experiment demonstrating inhomogeneous compensation achieved a factor of 8.5 suppression, so the Pathfinder sapphire would not satisfy requirements with an inhomogeneous

compensator. It may be possible to improve the performance with further study of inhomogeneous compensation techniques.

### Coating absorption inhomogeneities

Spots of high absorption in the coating of the test masses can also cause aberrations that impair interferometer performance, in both silica and sapphire. The following table shows the results of a calculation that studies the effect of a gaussian spot of 4mm waist that absorbs in excess of the coating average, on silica, at full interferometer power. The resulting bump on the surface of the test mass will scatter power from the arm cavity mode. The effect is largest if the spot is near the center of the interferometer beam.

Spot location	Absorption causing 1ppm loss
Centered spot	.25ppm
off 2cm in x	.37ppm
off 4cm in x	1.0ppm
off 6cm in x	4.9ppm
off -2cm in x	.35ppm
off -4cm in x	1.0ppm
off -6cm in x	4.9ppm
off 2cm in y	.35ppm

The next table repeats the analysis for sapphire.

Spot location	Absorption causing 1ppm loss
Centered spot	.57ppm
off 2cm in x	.81ppm
off 4cm in x	2.3ppm
off 6cm in x	11ppm

These numbers are indicative only; they are probably accurate to about 30%, but they do not consider changes to the arm cavity mode resulting from the change in ROC of the optic's surface, nor are the simultaneous effects of multiple spots considered. Modeling shows that the amount of loss scales as the net power absorbed for these small spots (i.e., a spot half the size with twice the absorption will absorb the same net power from the interferometer beam and scatter the same net power from the arm cavity mode). Note that twice the absorbed power will yield twice the thermal deformation and four times the scatter from the arm cavity mode.

The effect of coating absorption inhomogeneity in the arm cavity will be very difficult to compensate in Advanced LIGO. Both input and end test masses are equally vulnerable, and all might need to be

compensated (it may be possible to compensate a bump on one test mass by actuating on the other test mass, but this seems unlikely). This compensation would require a CO<sub>2</sub> laser with an intensity profile tailored to each test mass's inhomogeneity, and a sensor capable of measuring the HR surface deformation of each optic. Given that this is not a tested approach, the following analysis should set restrictions on the allowable coating absorption inhomogeneity and not be used to design a compensator. Given the microroughness scatter loss requirement of <20ppm per HR surface in the arm cavity, we can express coating absorption inhomogeneity requirements, e.g., as 'if coating absorption inhomogeneity is dominated by a single central spot, that spot shall not absorb more than 30mW.' Note that sapphire is only slightly better by this measure.

Coating absorption inhomogeneities will also have an effect inside the recycling cavity. If a 4mm spot at the center of the coating absorbs at 1.2ppm rather than .5ppm, the excess power will be about 2.4mW. Modeling of the thermorefractive aberration in fused silica due to this heating yields an overlap integral of 99.88%, or 0.12% loss in the recycling cavity, assuming all homogeneous thermal lensing has been ideally corrected. CO<sub>2</sub> laser compensation of inhomogeneity causing this level of loss was modeled and demonstrated experimentally by Ryan Lawrence, where he was able to reduce this loss by a factor of ten. By analogy to the surface distortion results, off-axis spots should be less critical by similar factors.

The final requirement on inhomogeneous thermal aberration is not yet well defined. One reasonable approach is to require that inhomogeneous thermal aberrations not exceed the residual static refractive index inhomogeneity after the compensating polish. This is specified in the COC DRD as less than 10nm rms for adequate coupling of carrier light into the arm cavity, along with the requirement that the RF sideband power buildup not be significantly reduced. Precisely what 'significantly' means in this context is not yet defined. Nevertheless, the overall amplitude of thermal aberration in the above case is peaked at about 10nm in a small region around the center of the optic, so the rms is presumably far less. It would appear that, so long as point absorbers in the ITM do not dissipate more than about a few mW and are rare, CO<sub>2</sub> laser compensation of coating inhomogeneity in fused silica seems a viable option.